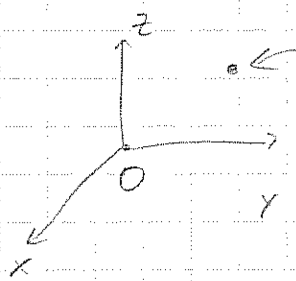
PHY 450 S "Relativistic electrodynamics"

www.physics.utoronto.ca/~poppitz/e-poppitz/PHY450.html

Principles of relativity & special relativity

In classical & quantum (non-gravitational) physics, we postulate the existence of space & time - the arena where all physical phenomena take place.

Space: 3-dimensional Euclidean space $\mathbb{R}^3$



a point P has cartesian coordinates (x, y, z)

$x \in (-\infty, \infty)$
$y \in (-\infty, \infty)$
$z \in (-\infty, \infty)$

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the distance between two points $P(x, y, z)$ & $P'(x', y', z')$ is the Euclidean distance: $\| PP' \| = \left[(x-x')^2 + (y-y')^2 + (z-z')^2 \right]^{1/2}$

O w/ x, y, z axes above is called a coordinate system; O can be chosen to be any point in space; distances do not depend on the choice of coordinate system

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time is a parameter with respect to which the positions of free particles change at a constant rate; mathematically, we describe the motion of free particles by means of describing $(x(t), y(t), z(t))$: its coordinates as a function of time, such that its acceleration vanishes:

$$\frac{d^2 x(t)}{dt^2} = 0$$

$$\frac{d^2 y(t)}{dt^2} = 0$$

$$\frac{d^2 z(t)}{dt^2} = 0$$

Here x, y, z are the coordinates of the free particle in an "inertial frame" (a frame where $\frac{d^2 \vec{r}}{dt^2} = 0$ holds for free particles).

$\left[\vec{r} = (x, y, z) = \text{"3-vector"} \right]$

Since for a free particle, w/ say $x = v_0 t$, $y = 0$, $z = 0$, x-coordinate changes at a fixed rate (v_0), one can use it (its position) as a "clock" to measure time.

Note: our def. of inertial frame as one where $\ddot{\vec{r}} = 0$ for free particle is $\equiv$ to the usual one that "in an inertial frame, a moving body not acted upon by external forces proceeds with constant velocity" (... since $\ddot{\vec{r}} = 0$ means $\frac{d}{dt}\dot{\vec{r}} = \frac{d}{dt}\vec{v} = 0$, i.e. $\vec{v} = \text{const}$)

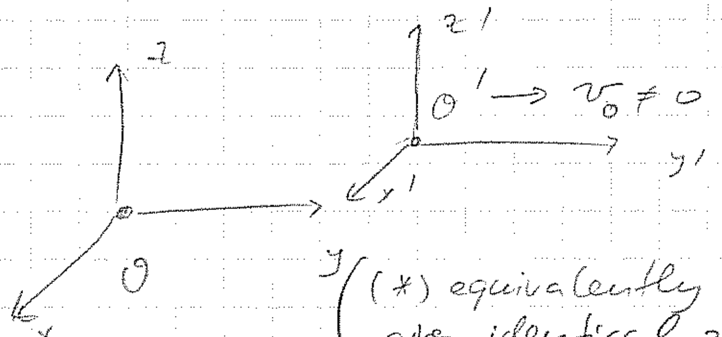
The principle of relativity says that the laws of nature are identical in all inertial frames of reference.(*)

for us, "laws of nature" means equations that describe the dynamics,

(e.g. Newton's law or Schrödinger equation)

so relativity means that the equations of motion have identical form in all inertial frames, when expressed in terms of the coordinates of the given inertial frame ... (i.e. "invariant")

example:



O' moves w.r.t. O w/ $\vec{v}_0$ along y axis

((*) equivalently: identical experiments in two frames give identical results.)

the <u>Gallilean relativity principle</u>

says that equations of motion are invariant under <u>"Gallilean transformations"</u> (as described above)

$$x' = x$$
$$y' = y - v_0 t$$
$$z' = z$$

$\equiv$ Gallilean transformations of coordinates

($y' = y - v_0 t$ assumes at $t=0$ $O' \equiv O$)

Because "t" is the same "absolute" time in all inertial frames, velocities of particles in these two frames (O & O') are related by

$$v_x' = v_x$$
$$v_{y'} = v_y - v_0$$
$$v_z' = v_z$$

or in vector notation

$$\vec{v}\,' = \vec{v} - \vec{v}_0 .$$

velocity in O' — velocity in O — velocity of O' wrt O

→ i.e. laws of physics are same in any two inertial frames –
– & more specifically, the frames are related by

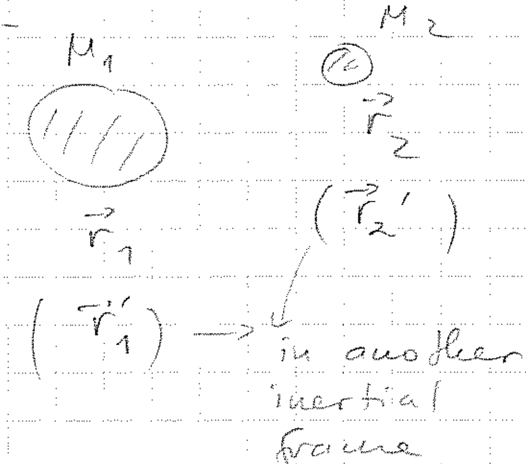
$$\left\{ \begin{array}{l} \vec{r}\,' = \vec{r} - \vec{v}_0 t \\ \vec{v}\,' = \vec{v} - \vec{v}_0 \end{array} \right. \quad \left(\text{2nd equation} \equiv \frac{d}{dt} \text{ of 1st equation} \right)$$

→ Gallilean transforms of coordinates & velocity.

<u>Example:</u> consider Newton's law ...

$$M_1 \ddot{\vec{r}}_1 = - \frac{\partial}{\partial \vec{r}_1} V(\vec{r}_1 - \vec{r}_2)$$

$$M_2 \ddot{\vec{r}}_2 = - \frac{\partial}{\partial \vec{r}_2} V(\vec{r}_1 - \vec{r}_2)$$



$$\boxed{\text{e.g. } V(\vec{r}_1 - \vec{r}_2) = - G_N \frac{M_1 M_2}{|\vec{r}_1 - \vec{r}_2|}}$$

attractive Newton's law

or

$$- \frac{\text{const.}}{|\vec{r}_1 - \vec{r}_2|^6}$$

attractive van der Waals ...

Now, in another inertial frame, we have

$$\vec{r}_i = \vec{r}_i\,' + \vec{v}_0 t \quad \left(\begin{array}{c} \text{Gallilean} \\ \text{inverse} \\ \text{transforms} \end{array} \right)$$

$i = 1$ or 2

$$\ddot{\vec{r}}_1 = \ddot{\vec{r}}_1\,', \text{ since } (\vec{v}_0 t)^{\cdot\cdot} = 0$$

$$\vec{r}_1 - \vec{r}_2 = \vec{r}_1\,' + v_0 t - (\vec{r}_2\,' + v_0 t) = \vec{r}_1\,' - \vec{r}_2\,'$$

$$\frac{\partial}{\partial \vec{r}_1} = \frac{\partial}{\partial (\vec{r}_1\,' + \vec{v}_0 t)} = \frac{\partial}{\partial \vec{r}_1\,'} \quad \left[\begin{array}{l} \text{because of linearity, think} \\ \text{about this one!} \end{array} \right]$$

So we can transform

$$M_1 \ddot{\vec{r}}_1 = - \frac{\partial}{\partial \vec{r}_1} V(\vec{r}_1 - \vec{r}_2)$$

$$\downarrow$$

$$M \ddot{\vec{r}}_1{}' = - \frac{\partial}{\partial \vec{r}_1{}'} V(\vec{r}_1{}' - \vec{r}_2{}')$$

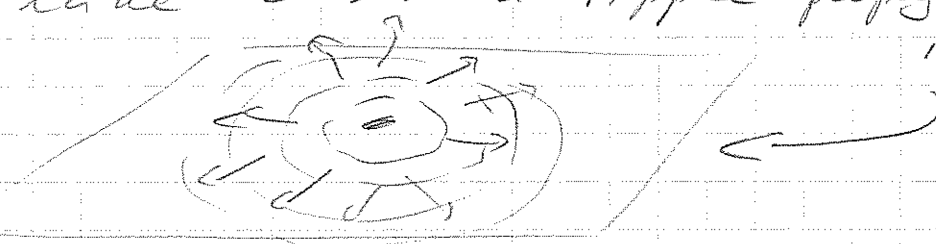
same as above, except $\vec{r}_{1,2} \to \vec{r}_{1,2}{}'$

The Gallilean principle of relativity was observed to hold for all interactions (described by gravitational attraction $V \sim \frac{1}{r}$; Coulomb's electrostatic law $V \sim \frac{1}{r}$ etc.);

UNTIL electromagnetism's unified nature and the prediction of the existence of E & M waves by Maxwell (& later verification).

As you may know — or will learn here, for sure — Maxwell's equations predict the existence of EM waves - a disturbance causing, say, $\vec{E}$ to change in time at some $\vec{x}$, causes a disturbance of both $\vec{E}$ & $\vec{B}$ which propagates outward

from $\vec{x}$ with speed "c" ($\simeq 3\times 10^8 \frac{m}{s}$ in vacuum)
– much like a stone thrown in a lake causes a ripple propagating outward



The speed "c" was expressed through ε_0 & μ_0 (in SI units) – and it wasn't clear what frame that speed is supposed to be in!!

[ε_0 & μ_0 are unphysical, really – an artefact of SI units; "c" is really the fundamental parameter in terms of which one should express Maxwell's equation ... (later...)]

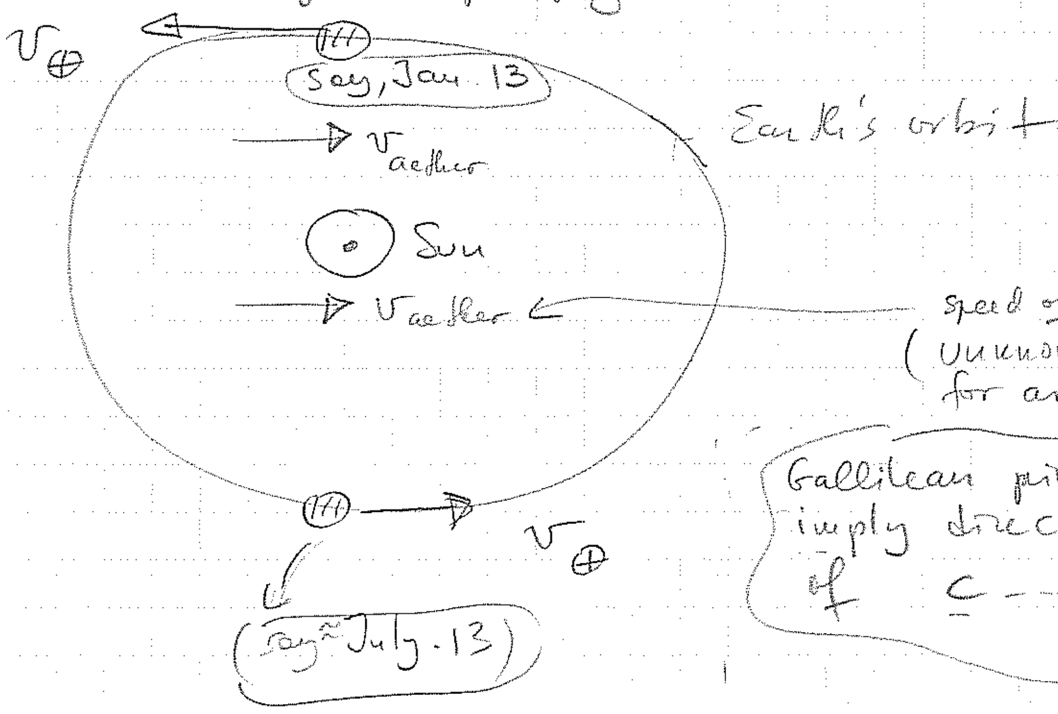
In particular, Gallilean relativity would imply that the speed of light would change depending on the frame:

$|\vec{c}\,'| = |\vec{c} - \vec{v}_0|$, definitely NOT $c' = c$ ...

<u>Note:</u> Poincare observed, along w/ Lorentz, that Maxwell's equations <u>were not invariant</u> under Gallilean transforms — unlike all previous equations of dynamics (e.g. Newton's).

((Furthermore, they constructed a set of coordinate transforms, called <u>"Lorentz transforms" under which Maxwell's equations were invariant</u> — as we'll see...))

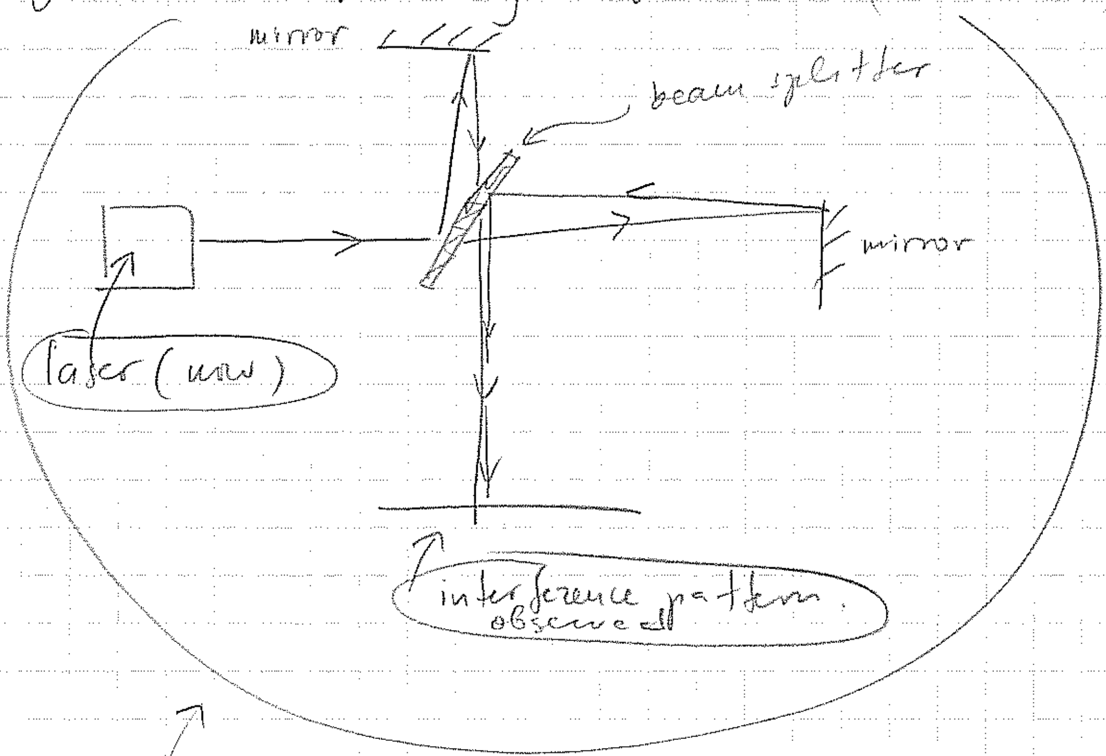
In the late XIX c it had been suggested that "c" is the speed of light wrt the rest frame of the "physical medium" through which light propagated — the "aether" ——



speed of aether w.r.t. Sun (unknown – but immaterial for argument)

Gallilean principle would imply directional dependence of c —!

Michelson-Morley experiment (~1887)



rotate apparatus - with (or wrt) Earth & wrt aether -

if 'c' is different in different directions, interference pattern should shift

(looked for daily as well as seasonal variations
↓ roth of Earth ↓ roth around Sun)

⇒ (very ingenious, in fact, since c is large & $v_\oplus$ small, need great precision — $v_\oplus \approx \frac{150 \times 10^6 \text{ km}}{10^7 \text{ s}} \sim 15 \frac{\text{km}}{\text{s}}$ $v_\oplus/c \sim 10^{-4}$!)

* Played great role in accepting relativity, but apparently was not so crucial to Einstein's thinking (he didn't believe in aether...)

Einstein's resolution of this conundrum was radical and far-reaching. There are various equivalent ways to state it ---

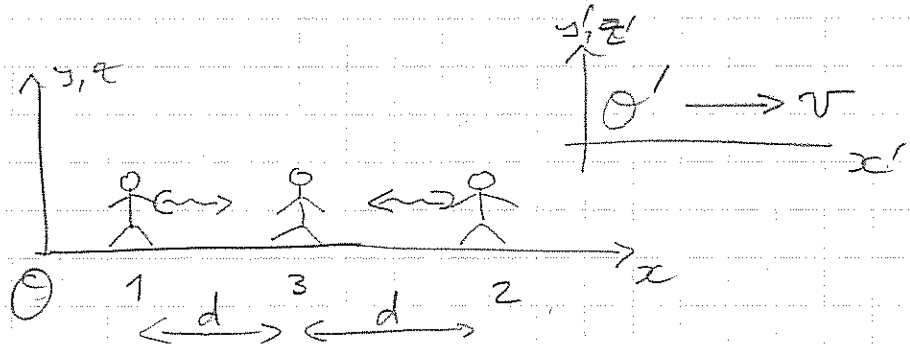
* replace Gallileo's transforms between coordinates in different reference frames by Lorentz transforms ($\equiv$ the symmetries of invariance under Maxwell's equations) & elevate these transforms to a fundamental property of nature
* postulate that speed of light is c in all inertial frames

(In this framework, Gallilean relativity emerges for $v \ll c$ — as a limiting case — so it is by no means "wrong" --)

Used in the latter form, Einstein's relativity implies that the notion of simultaneity is not absolute — i.e. viewed in different inertial frames events may be simultaneous in one and not in another frame of reference.

(thus time also transforms under "Lorentz transforms" unlike in "Gallilean transforms", as we'll see)

E.g.:



Experiment: 1 & 2 emit light received by 3 simultaneously; which was emitted first?

in frame O

(1, 2, 3 @ rest wrt O)

signals were emitted simultaneously

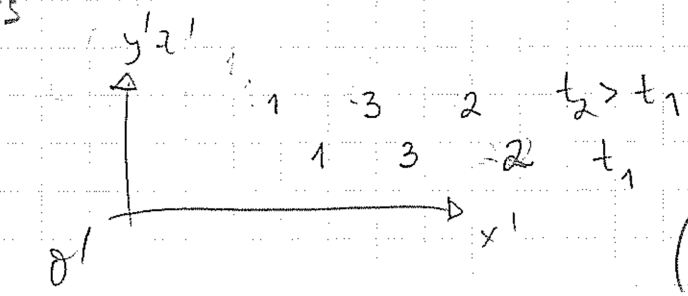
(since 1-3 & 2-3 distances are equal)

in frame O'

moving in +x with v, the floor where 1, 2, 3 are standing is moving to the left w/ v; thus 1 at earlier times was closer to the position of 3 at the time he receives signal – but since "c" is same, & 3 received signals at same time, this means 1 sent signal later than 2.

Thus, simultaneity is relative – & depends on the frame

→ in pic's



MORAL: we need to develop some math to describe this & any experiment

((– Not only position but time is relative –))