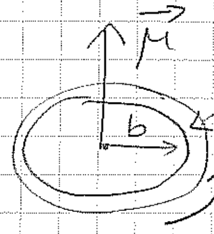
In tutorial, you'll study another kind of radiation —



conductor, wire of radius b

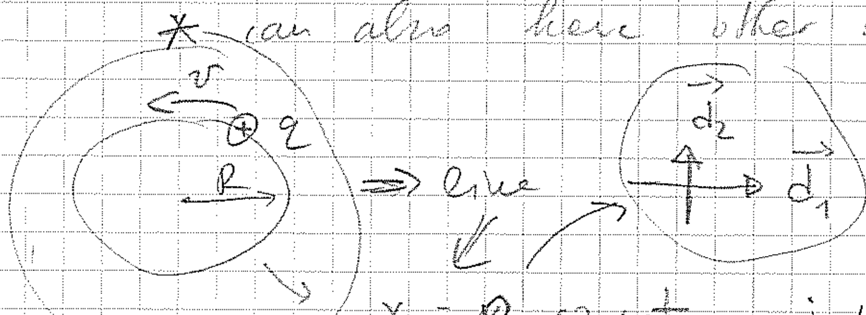
I - electrical current

$$|\vec{\mu}| \equiv I \pi b^2 = \text{magnetic dipole moment}$$

- if current is constant $\equiv$ magnetostatics, $\vec{B}$, no $\vec{E}$, (constant in time)
- if I changes, e.g. $I = I_0 \sin \omega t$, then there's radiation
- charge density is zero (neutral conductor), so $\vec{A}$ is created

In h.w. * will study radiation from "half-wave" antenna —

* can also have other situations:



$x = R \sin \omega t$; $\omega = \frac{v}{r}$;

$y = R \cos \omega t$;

As the accelerated charge moves, it loses energy (as radiated power comes from its kinetic energy) -- -- "radiation friction" — will come to study this.

But 1st let's go back and study in more depth the energy and momentum of the EM field --

Question: what 4-index object do these quantities fit into?

$$\left\{ \begin{array}{l} \mathcal{E} = \text{energy density} = \dfrac{\vec{E}^2 + \vec{B}^2}{8\pi} \\ \vec{P} = \text{momentum density} = \dfrac{\vec{S}}{c^2} \end{array} \right.$$

, as we heuristically argued before.

↳ to find this out, recall that in classical particle mechanics energy (E) and momentum ($\vec{P}$) conservation were quantities conserved due to space-time symmetries - translations in time and space. For, e.g. t-translation invariance, we had $L = L(q, \dot{q}, \cancel{t})$ no explicit t-dependence and we used

$$\frac{d}{dt}L = \frac{\partial L}{\partial q}\dot{q} + \frac{\partial L}{\partial \dot{q}}\ddot{q} = \swarrow \text{use Euler-Lagrange}$$

$$= \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)\dot{q} + \frac{\partial L}{\partial \dot{q}}\frac{d}{dt}\left(\dot{q}\right) =$$

$$= \frac{d}{dt}\left(\dot{q}\,\frac{\partial L}{\partial \dot{q}}\right)$$

hence, for $q, \dot{q}$ obeying EOM, we concluded

$$\frac{d}{dt}\left(\dot{q}\,\frac{\partial L}{\partial \dot{q}} - L\right) = 0$$

hence $E = \dot{q}\,\frac{\partial L}{\partial \dot{q}} - L$ = const $\Longleftrightarrow$ conserved

For the EM field, we have a very similar story $\Longrightarrow$ now dynamical variables are not $q(t)$ (& its time derivatives) but $A^i(x)$.

$$\left(\underset{\left(ct,\,\vec{x}\right) = x^j}{\overset{x}{\parallel\!\!|}}\right)$$

the action of the EM field is

$$S[A] = \int d^4x \left(-\frac{1}{16\pi c} F_{ij}F^{ij}\right)$$

$$= \int d^4x \left(-\frac{1}{16\pi\epsilon} (\partial_i A_j - \partial_j A_i)(\partial^i A^j - \partial^j A^i) \right)$$

<u>Note</u> $\Rightarrow$ there's no explicit $(\vec{x}, ct)$ dependence — — x^i dependence occurs only through the dependence of A_i on x.

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analogy:

$$\int dt \left(\frac{m\dot{q}^2}{2} - V(q) \right)$$

only depends on t thru t-dependence of q; we explored this (≠ EOM) to find conserved quantity: $\frac{d}{dt}E = 0$

$\hookrightarrow$ difference here is that for systems w/ a continuum distribution of d.o.f., conserv. laws take the form

$$(*) \quad \frac{\partial}{\partial x^i} C^{i(A)} = 0$$

labels conserved quantity

for if $*$ holds, we have

$$\frac{\partial}{c\,\partial t} C^{0(A)} + \vec{\nabla} \cdot \vec{C}^{(A)} = 0$$

conservation law for fields (fluids, EM)

$$\Downarrow$$

$$\frac{\partial}{\partial t}\left(\frac{1}{c} \int_V C^{0(A)} d^3x \right) = - \oint_{\partial V = S} d^2\vec{\sigma} \cdot \vec{C}^{(A)}$$

i.e. amount of $C^{0(A)}$ in V can change only because this quantity escapes V

So, the fact that the EM field action has no explicit dependence on x^j ($ct, \vec{x}$) means that we're to expect that there are 4 conserved quantities (energy & momentum densities) and their associated fluxes (energy flux & momentum flux)

Let's see how this works — really similar to particle case, except for remark on bottom of p 169 —

$$\frac{d}{dx^l}\left(-\frac{1}{16\pi c} F_{ij} F^{ij}\right) =$$

two terms are identical

$$= -\frac{1}{16\pi c}\left[(\partial_l F_{ij}) F^{ij} + F_{ij}(\partial_l F^{ij})\right] = -\frac{1}{8\pi c}(\partial_l F_{ij}) F^{ij}$$

$$= -\frac{1}{8\pi c}(\partial_l \partial_i A_j - \partial_l \partial_j A_i) F^{ij} =$$

$$= -\frac{1}{8\pi c}\left[\partial_l \partial_i A_j \cdot F^{ij} - \partial_l \partial_j A_i \cdot F^{ij}\right]$$

$F^{ij} = -F^{ji}$

$$= -\frac{1}{8\pi c}\left[\partial_l \partial_i A_j \cdot F^{ij} + \partial_l \partial_j A_i \cdot F^{ji}\right]$$

→ relabel dummy indices ($j \to i$, $i \to j$)

$$= -\frac{1}{8\pi c}\left[\partial_l \partial_i A_j \cdot F^{ij} + \partial_l \partial_i A_j \cdot F^{ij}\right]$$

EOM $\partial_i F^{ij} = 0$

$$= -\frac{1}{4\pi c}(\partial_l \partial_i A_j) F^{ij} = -\frac{1}{4\pi c}\partial_i\left[\partial_l A_j F^{ij}\right] + \underbrace{\frac{1}{4\pi c}\partial_l A_j \partial_i F^{ij}}_{=0}$$

So, we now used the free-field EOM

$$\partial_i F^{ij} = 0 \qquad \left(= \frac{4\pi}{c} j^j \text{, if there are currents; we'll generalize later} \right.$$

and found

$$\partial_\ell \left(-\frac{1}{16\pi c} F_{ij} F^{ij} \right) = \partial_i \left[\partial_\ell A_j \, F^{ij} \left(-\frac{1}{4\pi c} \right) \right]$$

$$\parallel$$

$$\partial_k \left(\delta^k{}_\ell \left(-\frac{1}{16\pi c} F_{ij} F^{ij} \right) \right) = \partial_k \left[F^{kj} \, \partial_\ell A_j \left(-\frac{1}{4\pi c} \right) \right]$$

hence

$$\partial_k \underbrace{\left(-\frac{1}{4\pi c} F^{kj} \partial_\ell A_j + \delta^k{}_\ell \frac{1}{16\pi c} F^{ij} F_{ij} \right)} = 0 \qquad (171.1)$$

this looks like a set of 4 conserved quantities

$$\partial_k T^k{}_{\underline{\ell}} = 0$$

$$\hookrightarrow \underbrace{\ell = 0, 1, 2, 3}$$

$T^0{}_\ell$ = densities

$T^\alpha{}_\ell$ = fluxes... according to our discussion on p. 169, bottom.

but this needs a bit more work ...

consider 1st term

$$(172.1) \equiv \partial_k \left(F^{kj} \partial_\ell A_j \right) = \Big[\text{use } \partial_\ell A_j = \tfrac{1}{2}(\partial_\ell A_j - \partial_j A_\ell) + \tfrac{1}{2}(\partial_\ell A_j + \partial_j A_\ell)$$

$$\equiv \tfrac{1}{2} F_{\ell j} + \tfrac{1}{2}(\partial_\ell A_j + \partial_j A_\ell)$$

$$= \frac{1}{2}\partial_k \left(F^{kj} F_{\ell j}\right) + \frac{1}{2}\partial_k\left(F^{kj}\partial_\ell A_j\right) + \frac{1}{2}\partial_k\left(F^{kj}\partial_j A_\ell\right)$$

$$= \frac{1}{2}\partial_k\left(F^{kj}F_{\ell j}\right) + \frac{1}{2}F^{kj}\partial_k\partial_\ell A_j + \underbrace{\frac{1}{2}F^{kj}\partial_k\partial_j A_\ell}$$

(used $\partial_k F^{kj} = 0$)

$$\underbrace{F^{kj}}_{\text{antisymmetric}}\ \underbrace{\partial_k \partial_j}_{\text{symmetric}} (\text{anything}) = 0$$

$$= \frac{1}{2}\partial_k\left(F^{kj}F_{\ell j}\right) + \underbrace{\frac{1}{2}\partial_k\left(F^{kj}\partial_\ell A_j\right)}_{\text{but this is } \equiv \text{ l.h.s of } (172.1)}$$

hence

$$\partial_k\left(F^{kj}\partial_\ell A_j\right) = \partial_k\left(F^{kj}F_{\ell j}\right)$$ — plug into (171.1) & drop $\frac{1}{c}$.

$$\partial_k \underbrace{\left(-\frac{1}{4\pi} F^{kj} F_{\ell j} + \delta^k{}_\ell \frac{1}{16\pi} F^{ij} F_{ij} \right)} = 0$$

$$T^k{}_\ell \equiv -\frac{1}{4\pi} F^{kj} F_{\ell j} + \delta^k{}_\ell \frac{1}{16\pi} F^{ij} F_{ij} \quad \Big| \times g^{\ell m} \ \& \ \sum_\ell$$

$$\left\{\begin{array}{l} T^{km} = -\frac{1}{4\pi} F^{kj} F^{m}{}_{j} + g^{km} \frac{1}{16\pi} F^{ij} F_{ij} \\ \partial_k T^{km} = 0 \end{array}\right\} \text{eq. } (172.1)$$

used: lack of explicit dependence on $(ct, \vec{x})$ of the EM field action $\equiv$ i.e. spacetime translation invariance!

("Noether's theorem for free EM field")

"Maxwell's stress-energy tensor"

- symmetric $T^{km} = T^{mk}$.
- conserved: $\partial_k T^{km} = 0$

$\downarrow$

a set of 4 conservation laws, one for each "m"

$m = 0$: energy conservation

$m = \alpha$: momentum conservation,

Let's write these laws in "normal" notation...

-- start w/ $m = 0$ $\quad g^{00} = +1$.

$$T^{00} = -\frac{1}{4\pi} F^{0\alpha} F^{0}{}_{\alpha} + \frac{1}{16\pi} F^{ij} F_{ij} =$$

$$= -\frac{1}{4\pi} F^{0\alpha} F^{0\alpha'} g_{\alpha\alpha'} + \frac{1}{16\pi} \cdot 2 \cdot F^{0\alpha} F^{0\beta} g_{00} g_{\alpha\beta} + \frac{1}{16\pi} F^{\alpha\beta} F^{\gamma\delta} g_{\alpha\gamma} g_{\beta\delta}$$

$$= + \frac{1}{4\pi} \sum_{\alpha} (F^{0\alpha})^2 - \frac{1}{8\pi} \sum_{\alpha} (F^{0\alpha})^2 + \frac{1}{16\pi} \sum_{\alpha\beta} (F^{\alpha\beta})^2 = \circledast$$

$$F^{0\alpha} = -(\vec{E})^{\alpha}$$

$$F^{\alpha\beta} = -\epsilon^{\alpha\beta\gamma} (\vec{B})^{\gamma}$$

$$\circledast = \frac{1}{8\pi} \vec{E}^2 + \frac{1}{16\pi} \underbrace{\sum_{\alpha\beta} \epsilon^{\alpha\beta\gamma} \epsilon^{\alpha\beta\delta}}_{2\delta^{\gamma\delta}} (\vec{B})^{\gamma} (\vec{B})^{\delta}$$

$$= T^{00} = \frac{1}{8\pi} (\vec{E}^2 + \vec{B}^2) \quad \Rightarrow \text{our familiar } \mathcal{E} = \text{energy density}$$

$cT^{\alpha 0}$ should be called "energy flux",

since $\frac{\partial}{\partial (ct)} T^{00} + \frac{\partial}{\partial x^{\alpha}} T^{\alpha 0} = 0$

$$\Downarrow$$

$$\frac{\partial}{\partial t} \underbrace{\left(\int_V d^3x \, T^{00} \right)}_{\text{energy in } V} = - \underbrace{\left(\oint_{\partial V = S} d^2\sigma^{\alpha} \cdot (c T^{\alpha 0}) \right)}_{\text{energy flux thru } \partial V}$$

from (172.1):

$$T^{\alpha 0} = -\frac{1}{4\pi} F^{\alpha j} F^{0}{}_{j} \quad (\text{second term does } \underline{\text{not}} \text{ contribute as } g^{\alpha 0} = 0,$$

$$= -\frac{1}{4\pi} F^{\alpha\beta} F^{0\beta'} g_{\beta\beta'} = + \frac{1}{4\pi} F^{\alpha\beta} F^{0\beta} =$$

$$= -\frac{1}{4\pi} \epsilon^{\alpha\beta\gamma} (\vec{B})^{\gamma} (-(\vec{E})^{\beta}) = \frac{1}{4\pi} \epsilon^{\alpha\beta\gamma} (\vec{E})^{\beta} (\vec{B})^{\gamma}$$

hence $$c T^{\alpha 0} = \frac{c}{4\pi} (\vec{E} \times \vec{B})^{\alpha} = (\vec{S})^{\alpha} \quad (!)$$

Now, $\underline{cT^{\alpha 0} \text{ is the energy flux}} \equiv (\vec{S})^\alpha$

While, as we learned from EM wave (dim. analysis);

$$\frac{(\vec{S})^\alpha}{c^2} = \frac{1}{c} T^{0\alpha} \text{ is the } \underbrace{\text{momentum density}}$$

$\boxed{T^{0\alpha} \text{ is conserved due to space translation; coeff. } \tfrac{1}{c} \equiv \text{dim analysis}}$

since, $\partial_\kappa T^{\kappa\mu} = 0$, for $\mu = 0$ was $\partial_\kappa T^{\kappa 0} = 0$,

(already explained)

but for $\mu = \alpha$ it is

$$\partial_\kappa T^{\kappa\alpha} = 0 \Rightarrow$$

$$\Rightarrow \frac{\partial}{\partial(ct)}\left(T^{0\alpha}\right) + \frac{\partial}{\partial x^\beta} T^{\beta\alpha} = 0$$

hence

$$\frac{\partial}{\partial t}\left(\frac{1}{c}\int_V T^{0\alpha}\, d^3x\right) = -\int_V \frac{\partial}{\partial x^\beta}\left(T^{\beta\alpha}\right) d^3x$$

$$\shortparallel$$

$$= \underbrace{-\oint_{\partial V = S} d^2\sigma^\beta \cdot T^{\beta\alpha}}$$

(rate of change of α-th component of momentum of field in V)

(flux of α-th component of momentum thru $\partial V = S$)

$\underline{\text{momentum conservation for EM field}}$

hence $T^{\beta\alpha}$ = flux of α-th component of $\vec{p}$ thru unit area in β-direction

since $T^{\alpha\beta} = T^{\beta\alpha}$, flux of $(\vec{P})^\alpha$ thru $\hat{\beta}$
$= $ flux of $(\vec{P})^\beta$ thru $\hat{\alpha}$

We now know

energy flux $= c\, T^{\alpha 0} = (\vec{S})^\alpha$

&

momentum density $= \frac{1}{c} T^{0\alpha} = \frac{(\vec{S})^\alpha}{c^2}$

What's $T^{\alpha\beta}$? from (172.1) we have

$$T^{\alpha\beta} = -\frac{1}{4\pi} F^{\alpha j} F^{\beta k} g_{jk} + g^{\alpha\beta} \frac{F^{ij} F_{ij}}{16\pi} =$$

$$= -\frac{1}{4\pi} F^{\alpha 0} F^{\beta 0} + \frac{1}{4\pi} \sum_\gamma F^{\alpha\gamma} F^{\beta\gamma} - \delta^{\alpha\beta} \frac{\left(-\sum_\gamma 2F^{0\gamma}F^{0\gamma} + \sum_{\gamma\delta} F^{\gamma\delta} F^{\gamma\delta}\right)}{16\pi}$$

$$= -\frac{1}{4\pi} E^\alpha E^\beta + \frac{1}{4\pi} \sum_{\gamma,\delta,\rho} \epsilon^{\alpha\gamma\delta} \epsilon^{\beta\gamma\rho} B^\delta B^\rho$$

$$- \frac{\delta^{\alpha\beta}}{16\pi} \left(-2\vec{E}\cdot\vec{E} + \sum_{\rho\mu} \sum_{\gamma\delta} \epsilon^{\gamma\delta\rho} \epsilon^{\gamma\delta\mu} B^\rho B^\mu \right) =$$

$$= -\frac{1}{4\pi} E^\alpha E^\beta + \frac{1}{4\pi} \sum_{\delta,\rho} \left(\delta^{\alpha\beta}\delta^{\delta\rho} - \delta^{\alpha\rho}\delta^{\beta\delta} \right) B^\delta B^\rho$$

$$- \frac{1}{16\pi} \delta^{\alpha\beta} \left(-2\vec{E}^2 + 2\sum_{\rho\mu} \delta^{\rho\mu} B^\rho B^\mu \right) =$$

$$= -\frac{1}{4\pi} E^\alpha E^\beta - \frac{1}{4\pi} B^\alpha B^\beta + \frac{1}{4\pi} \delta^{\alpha\beta} \vec{B}^2 + \frac{1}{8\pi} \delta^{\alpha\beta} \left(\vec{E}^2 - \vec{B}^2 \right) = \hookrightarrow$$

finally,

$$T^{\alpha\beta} = -\frac{1}{4\pi}\left(E^\alpha E^\beta + B^\alpha B^\beta - \frac{1}{2}\delta^{\alpha\beta}\left(\vec{E}^2 + \vec{B}^2\right)\right)$$

As we'll see, it is convenient to consider $-T^{\alpha\beta}$ $\Rightarrow$

$$\sigma^{\alpha\beta} \equiv -T^{\alpha\beta} = \frac{1}{4\pi}\left(E^\alpha E^\beta + B^\alpha B^\beta - \frac{1}{2}\delta^{\alpha\beta}\left(\vec{E}^2 + \vec{B}^2\right)\right)$$

often this one is called Maxwell's stress tensor

recall that $T^{\alpha\beta}$ $\Leftrightarrow$ flux of β th component of $\vec{P}$ thru an area $\perp \hat{\alpha}$

In full glory --

$\| T^{ij} \| =$

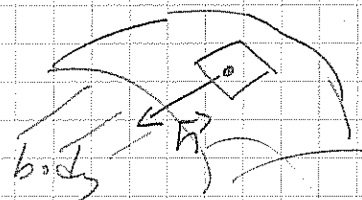
$\frac{\vec{E}^2 + \vec{B}^2}{8\pi}$	$\frac{(\vec{E}\times\vec{B})^1}{4\pi}$	$\frac{(\vec{E}\times\vec{B})^2}{4\pi}$	$\frac{(\vec{E}\times\vec{B})^3}{4\pi}$
these	$-\frac{1}{4\pi}\left((E^x)^2+(B^x)^2 - \frac{1}{2}\vec{E}^2 - \frac{1}{2}\vec{B}^2\right)$	$-\frac{1}{4\pi}\left(E^x E^y + B^x B^y\right)$	$-\frac{1}{4\pi}\left(E^x E^z + B^x B^z\right)$
are determined		$-\frac{1}{4\pi}\left((E^y)^2+(B^y)^2 - \frac{1}{2}\vec{E}^2 - \frac{1}{2}\vec{B}^2\right)$	$-\frac{1}{4\pi}\left(E^y E^z + B^y B^z\right)$
by symmetry of matrix			$-\frac{1}{4\pi}\left((E^z)^2+(B^z)^2 - \frac{1}{2}\vec{E}^2 - \frac{1}{2}\vec{B}^2\right)$

interpretation

energy density	$\frac{1}{c}$(energy flux)$^{(1)}$	$\frac{1}{c}$(energy flux)$^{(2)}$	$\frac{1}{c}$(energy flux)$^{(3)}$
c (momentum)^1 density	(momentum)^1 flux thru$^{(1)}$	(momentum)^2 flux$^{(1)}$	(momentum)^3 flux$^{(1}$
c (momentum)^2 density	(momentum)^1 flux thru$^{(2)}$	(momentum)^2 flux thru$^{(2)}$	(momentum)^3 flux
c (momentum)^3 density	(momentum)^1 flux thru$^{(3)}$	(momentum)^2 flux thru$^{(3)}$	(momentum)^3 flux thru

If $d^2\vec{\sigma}$ is at the surface of a body, the rate of flow of $(\text{momentum})^\beta$ thru $d^2\vec{\sigma}$ is the $(\text{force})^\beta$ on the body:

$$df^\beta = \sum_\alpha d^2\sigma^\alpha \, T^{\alpha\beta}$$



inward normal as force on body = unit momentum of EM field transferred to body in unit time

$(\text{force})^\beta$ on $d^2\vec{\sigma}$ due to EM field

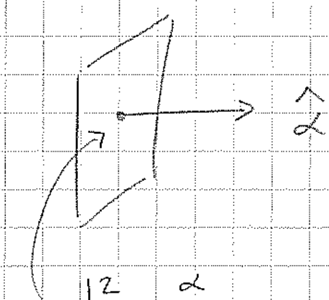
force on whole body due to EM field $F^\beta = \int_{\substack{\text{surface} \\ \text{of body}}} d^2\sigma \, n^\alpha \, T^{\alpha\beta}$

$T^{\alpha\beta}$ made out of $\vec{E}$ & $\vec{B}$ on the surface of the body

depends on whether there's reflection or not, of course ...
(since $\vec{E} = \vec{E}_{\text{incoming}} + \vec{E}_{\text{reflected}}$, etc...)

Let's see a few simple examples. Consider a plane wave travelling in $+\hat{x}$- direction.
<u>Suppose it is completely absorbed by a surface</u>

Now, if we have

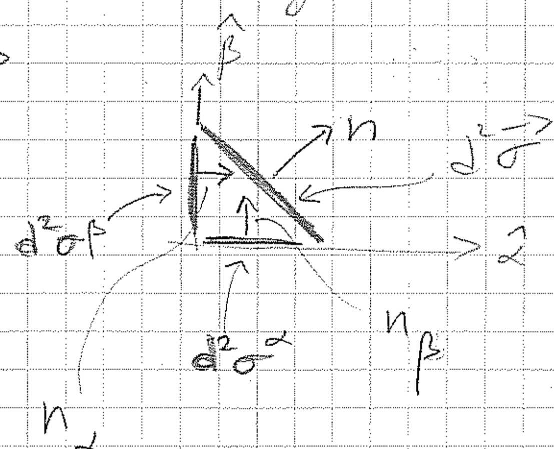


$d^2\sigma^\alpha$ = area $\perp$ $\hat{\alpha}$

We said that $T^{\alpha\beta} \equiv$ amount of $(\vec{P})^\beta$ that goes thru unit area $\underline{\perp}\ \hat{\alpha}$ in unit time

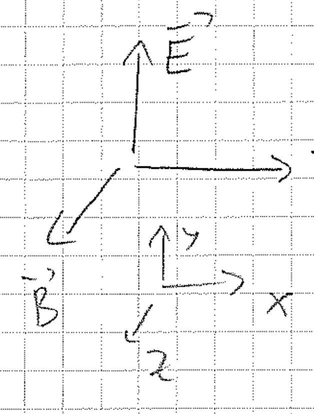
Hence $T^{\alpha\beta}\, d^2\sigma^\alpha$ = amount of (momentum)$^\beta$ that goes thru $d^2\sigma^\alpha$ in unit time (if no $\sum_\alpha$)

if we have a general surface element $d^2\vec{\sigma}$



$$d^2\vec{\sigma} = \sum_\alpha d^2\sigma\, n^\alpha$$
$$= \sum_\alpha d^2\sigma^\alpha$$

the β-component of momentum that goes thru $d^2\vec{\sigma}$ in unit time is $\sum_\alpha d^2\sigma^\alpha\, T^{\alpha\beta}$; if >0 momentum flows in direction of $\vec{n}$, if negative – in opposite direction.



from p. 123

$$\vec E = p\vec\beta \sin(cpt - px)$$

$$\vec B = \vec p \times \vec\beta \sin(cpt - px)$$

i.e.
$$E_y = p\beta \sin(cpt - px)$$
$$B_z = p\beta \sin(cpt - px)$$

$\to$ all other components of $\vec E, \vec B$ vanish

<u>hence</u>:

$$T^{xx} = -\frac{1}{4\pi}\left((\overset{0}{\overset{\shortparallel}{E^x}})^2 + (\overset{0}{\overset{\shortparallel}{B^x}})^2 - \frac{1}{2}\vec E^2 - \frac{1}{2}\vec B^2\right) =$$

$$T^{xx} = +\frac{1}{4\pi} p^2 \beta^2 \sin^2(\omega t - px) \quad , \quad \omega = cp$$

$\to$ this wave only transports $(\vec p)^x$ in $\hat x$

clearly these should be $= 0$

$$T^{yy} = -\frac{1}{4\pi}\left((E^y)^2 + (\underset{\overset{\shortparallel}{0}}{B^y})^2 - \frac{1}{2}\vec E^2 - \frac{1}{2}\vec B^2\right)$$

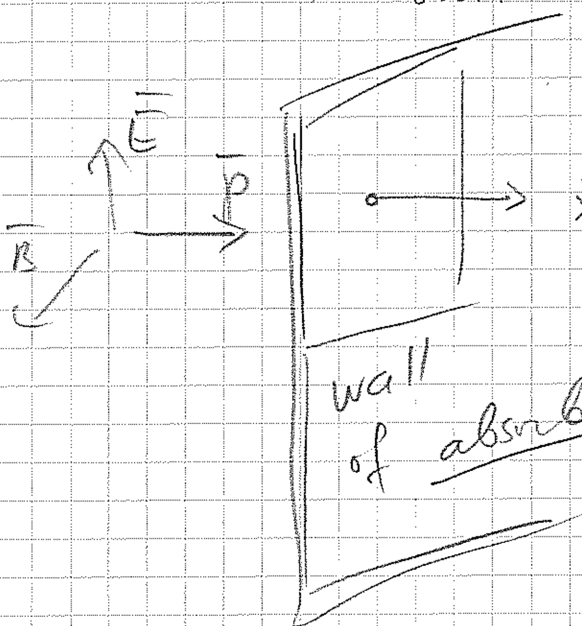
$$= -\frac{1}{4\pi}\left(\frac{1}{2}(E^y)^2 - \frac{1}{2}(B^y)^2\right) = 0 \quad (\text{no } y\text{-momentum} \perp \hat y$$

$T^{zz} = 0$ for same reason (no z-momentum $\perp \hat z$

clearly also $T^{xy} = T^{xz} = 0$ $\left(\sim E^x E^y \mp B^x B^y \right.$ or $E^x E^z + B^x B^z = 0$

as well as $T^{yz} = 0$.

cT^{xx} = amount of $(\vec{P})^x$ thru $+\hat{x}$ in unit time



(assumed perfect; otherwise need to also add reflected field in calculating $T^{\alpha\beta}$ on surface)

$$dF^x = T^{xx}\, d^2\sigma^x$$

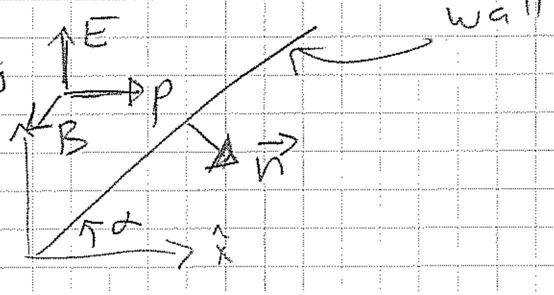
$$F^x = \int d^2\sigma^x\, T^{xx} =$$

$$= \int_{\substack{\text{area}\\ \text{of wall}}} dy\, dz\, \frac{p^2\beta^2}{4\pi} \sin^2(\omega t - px)$$

average over period

$$F^x = (\text{Area}) \frac{\beta^2 p^2}{4\pi} \times \frac{1}{2}$$

If the wall is tilted force has $\perp$ & $//$ components:



(tutorial & HW 6)