Confining Strings On $R^3 \times S^1$

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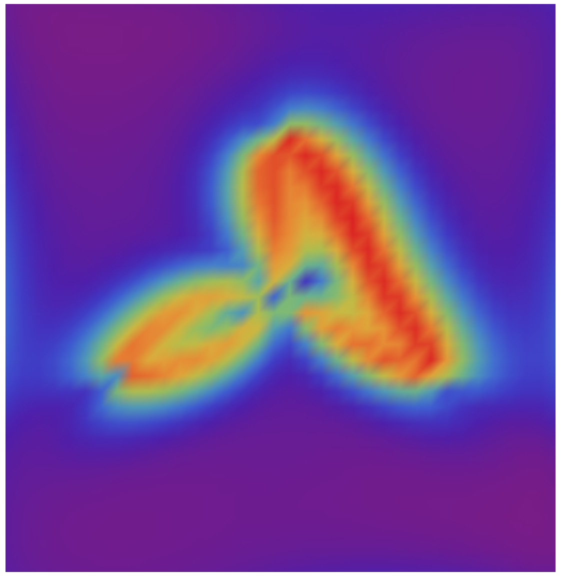
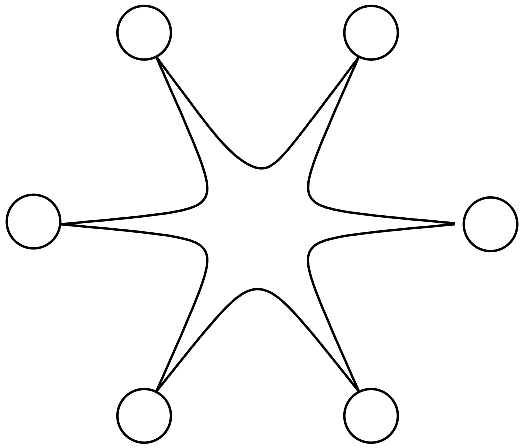
1501.06773, with Mohamed Anber EPFL (Lausanne)
Tin Sulejmanpasic NCSU

also 1508.00910
and in (slow) progress



3-quark SU(3) “baryon” in QCD(adj)



6-monopole SU(6) “dual baryon”, from Shifman-Yung 0703267
(ref. also motivated by other analogies to their work...)

Motivation/Summary/Outline I:

Confining strings may seem ubiquitous and 'old'... but are analytically understood - within continuum QFT, starting from the microscopic QFT degrees of freedom, and in a controlled manner - only in a few cases.

- Seiberg-Witten theory: N=2 super YM with N=1 soft mass, abelian confinement Douglas Shenker; Hanany Strassler Zaffaroni mid/late 1990s

- monopole confinement in abelian Higgs model and in related (dual) models with nonabelian strings Gorsky, Shifman, Yung 2004-2014-

→ (here) confinement on $R^3 \times S^1$, abelian Unsal, Shifman, Yaffe,... 2007-

Lattice - numerical experiment - confining flux tubes exist, for sure, spectrum etc.

String theory - strings are there in dual theory, to begin with
one only has to work to make them give linear potential (so they don't fall to horizon)
- under control in regimes quite far from asymptotically-free QFT

It is interesting to study the few understood QFT cases, their relations to each other, to string, and to lattice...

Motivation/Summary/Outline II:

In this talk, I will study the last case above:

- confinement on $R^3 x S^1$, abelian Unsal, Shifman, Yaffe,...

Many properties of theories with semiclassical confinement in this setup have been understood

SYM: Seiberg, Witten/Aharony, Intriligator, Hanany, Seiberg, Strassler late 1990s

SYM, with new insight, & non-SYM: Unsal w/ Yaffe, Shifman... since 2007

but confining strings have not been studied in any detail.

We shall see that confining strings in these theories have properties distinct from other theories with abelian confinement (e.g. SW) and show surprising similarities to various dual theories with (non-) abelian confinement of monopoles discussed previously.

Motivation/Summary/Outline III:

1. a lightning review of confinement on R^3x S^1:
   deformed Yang-Mills theory and QCD(adjoint)/SYM
   Unsal w/ Yaffe, Shifman...

   experts: hopefully not too bored

   non-experts: can't explain all, will assert a few facts

   - but if these are accepted, study of strings will be clear

2. confining strings in deformed YM and QCD(adj):
   domain walls, mesons, and baryons

3. comparison to other understood cases and the transition to the nonabelian regime

4. for the future:
   lattice & transition to nonabelian confinement?

1. confinement on R^3 x S^1, size of circle- L:

We study SU(N) in the regime $NL\Lambda \ll 1$

QCD(adj): YM with n_f adjoint Weyl fermions; $n_f = 1$ is SYM

dYM: pure YM with particular double-trace "deformation"

Before describing dynamics, some remarks on "philosophy":

"This is not the real world, even without quarks:
partially compactified theory, abelianized dynamics…
all different from physical theories on R^4:

why bother?"

"…why bother?"

answers from:

Government of Canada Gouvernement du Canada
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Application - Subatomic Physics Envelope - Individual

Description of Proposed Research Poppitz

i. The picture of the confining vacuum and of the thermal deconfinement transition emerging from these calculable examples is beautiful and elegant. This fact alone is very satisfying to a theorist.

ii. One might hope that upon studying a solvable example, new unexpected and interesting features of more general utility will be encountered.

iii. Once an analytical approach is understood within its region of validity, it is tempting to push it to, and even beyond, its limits—i.e. the approach might contain qualitative lessons for phenomenological models of the real strongly coupled system.

"…why bother?" (overview of recent activity)

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"topological molecules" show importance and peculiarity of beyond-the-leading order semiclassics

formal issues: resurgence: P - NP ambiguity cancellation and transseries in QM, QFT(hints) ?

role of complexified path integrals - for now understand ex's in QM; appear more general?

more physical issues: stabilization of center symmetry and deconfinement by "neutral bions"

theta-dependence of deconfinement transition properties first observed here, lattice confirmed

this talk: towards understanding abelian to non-abelian confinement transition?

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"MILM" (monopole-instanton-liquid model) of deconfinement

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formal issues: resurgence: P - NP ambiguity cancellation and transseries in QM, QFT(hints) ?
Argyres, Dunne, Cherman, Unsal, …
role of complexified path integrals - for now understand ex's in QM; appear more general?
Behtash, Dunne, EP, Schaefer, Sulejmanpasic, Unsal…

more physical issues: stabilization of center symmetry and deconfinement by "neutral bions"
EP, Schaefer, Unsal…
theta-dependence of deconfinement transition properties first observed here, lattice confirmed
Unsal, Anber, EP… D'Elia, Negro…

this talk: towards understanding abelian to non-abelian confinement transition?
Anber, EP, Sulejmanpasic

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"MILM" (monopole-instanton-liquid model) of deconfinement Shuryak, Sulejmanpasic…

1. confinement on R^3 x S^1, size of circle- L:

We study SU(N) in the regime $NL\Lambda \ll 1$

QCD(adj): YM with n_f adjoint Weyl fermions; $n_f = 1$ is SYM

dYM: pure YM with particular double-trace "deformation"

Assertions...

i.) in each case, dynamical* abelianization at 1/(NL)

$SU(N) \to U(1)^{N-1}$ W-bosons' mass $\frac{1}{NL} \gg \Lambda$

no light states charged under the N-1 massless "photons"

since only adjoint fields, massless states after breaking neutral under Cartan

in the regime we study, perturbative IR dynamics boring:
free U(1)s + light neutral Cartan subalgebra "gauginos" in QCD(adj)

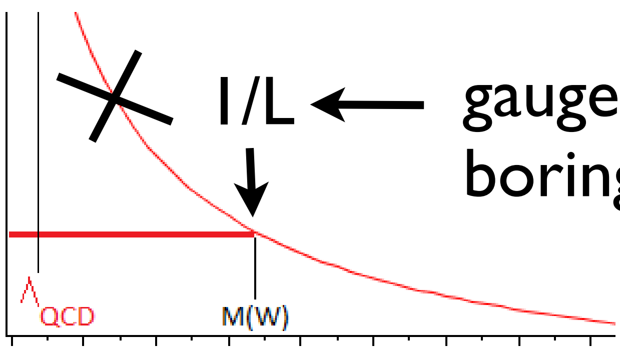
* not by "maximal abelian gauge" !

I. confinement on R^3 x S^1, size of circle- L:

Assertions, contd.:

i.) in each case, the theory abelianizes at a scale 1/(NL)
in the regime we study, perturbative IR dynamics boring:
free U(1)s + Cartan components of gauginos in QCD(adj) $\frac{1}{NL} \gg \Lambda$

$$W = P e^{i \oint_{S_1} A_4 \, dx^4} \qquad \text{SU(2): } \langle A_4^{\text{Cartan}} \rangle \sim \frac{\pi}{L}$$



gauge coupling "freezes" (Higgsed) to a small value a scale 1/L
boring perturbative IR dynamics

energy scale

SYM: Seiberg, Witten/Aharony, Intriligator, Hanany, Seiberg, Strassler
SYM & non-SYM: Unsal w/ Yaffe, Shifman...

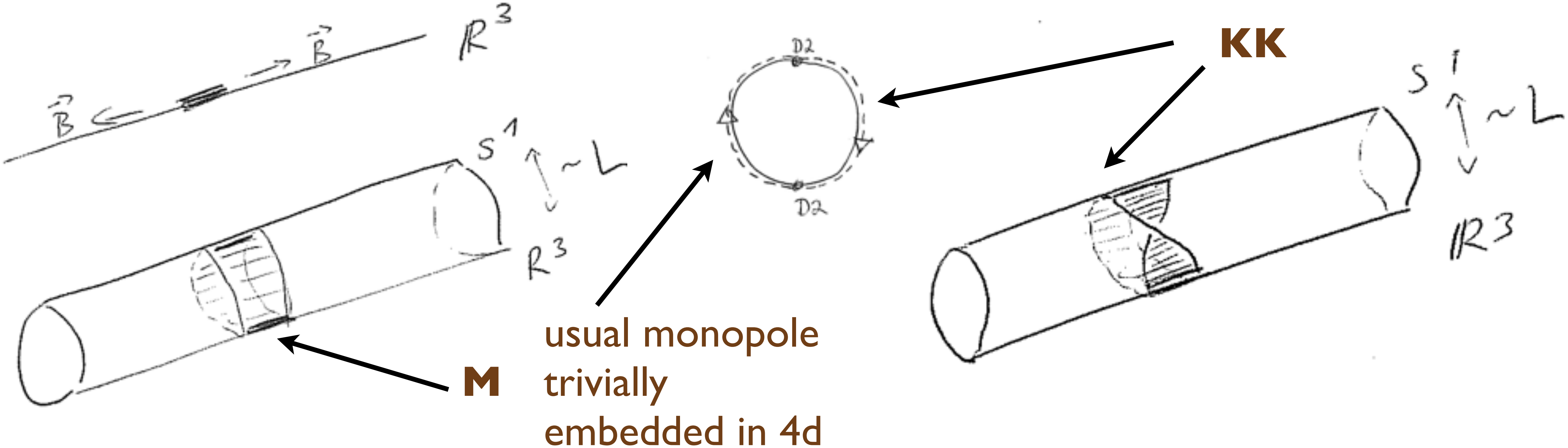
ii.) nonperturbatively, however, the dynamics is quite rich

the $SU(N) \to U(1)^{N-1}$ theory has instanton solutions

these change the IR behavior of the theory and
generate a mass gap (Polyakov mechanism in a locally 4d setting)

1. confinement on R^3 x S^1, size of circle- L:

Wilson line breaks SU(2) to U(1) so there are monopole-instantons



For SU(N), 4d BPST instanton dissociates into N constituents:

$$SU(N):\ e^{-S_0} = e^{-\frac{8\pi^2}{g_4^2(L)\,N}}$$

(large-N survive!)

As opposed to 4d BPST instantons, have long-range “magnetic field”.
Dilute monopole-instanton gas - as in SM to obtain ‘t Hooft vertex (qqql)^3 = 3d dilute - but Coulomb! - gas

[this is all non-experts need to accept/believe/ to understand study of strings]

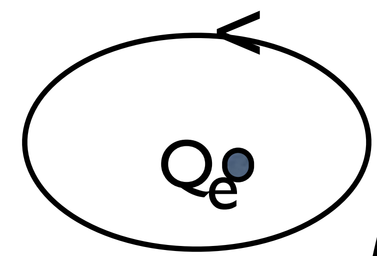
1. confinement on R^3 x S^1, size of circle- L:

to write Z - the partition function of dYM/QCD(adj), need

sigma = dual photon field $e^2 d\sigma = *F$

$$e^2 = \frac{g^2}{L}, \quad g^2 \sim g_4^2(1/L)$$

electric coupling ~ 4d coupling at 1/L



$$\oint d\sigma = Q_e$$

$$\partial_0 \sigma \sim \frac{L}{g^2} F_{12}$$

time derivative = 3d magnetic field

$$\partial_i \sigma \sim \frac{L}{g^2} \epsilon_{ij} E_j, \quad j = 1, 2$$

spatial gradient = 3d electric field

monodromy of sigma around a spatial loop = electric charge inside

Main result

[Polyakov, 1970's]:

$$Z \sim \int \mathcal{D}\sigma \; e^{-\int dx L_{eff}(x)}$$

$$Z[j] = \langle e^{i \int dx j(x) \rho_m(x)} \rangle, \quad L_{eff}(x) = e^2 (\partial_i \sigma)^2 - \zeta \cos(\sigma(x) + j(x))$$

for SU(2), only one dual photon (Cartan)

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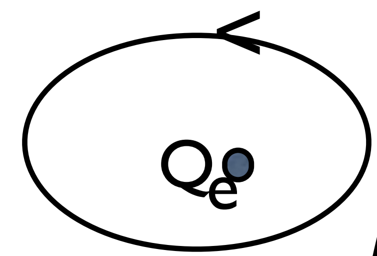

$$\oint d\sigma = Q_e$$

$$\partial_0 \sigma \sim \frac{L}{g^2} F_{12} \xrightarrow{\text{“magnetic charge operator”}} Q_M = \int d^2x \, \partial_0 \sigma$$

time derivative = 3d magnetic field

$$e^{\pm i\sigma(y)}$$

at fixed time,

$$\left[Q_M, e^{\pm i\sigma(y)}\right] = \pm\, e^{\pm i\sigma(y)}$$

creates unit magnetic vortex

“’t Hooft vertex” = monopole operator

Main result
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1. confinement on R^3 x S^1, size of circle- L:

$$L_{eff}^{dYM} = \frac{g^2}{L}(\partial_i \vec{\sigma})^2 - \sum_i^N \zeta \cos \vec{\alpha}_i \cdot \vec{\sigma} \qquad \zeta \sim L^{-3} e^{-\frac{4\pi^2}{g^2}\frac{2}{N}}$$

monopole-instanton fugacity

two important scales!

$$M \sim \frac{1}{L}$$ W-boson mass

$$m \sim M e^{-\frac{\mathcal{O}(1)\pi^2}{g^2}}$$ dual photon mass

$$L_{eff}^{dYM} = M \left[(\partial_i \vec{\sigma})^2 - \sum_i^N m^2 \cos \vec{\alpha}_i \cdot \vec{\sigma} \right]$$

same as before, except N-1 dual photons and N monopole-instantons

$\vec{\alpha}_1 = (1,-1,0,0,...0)$ $\vec{\alpha}_2 = (0,1,-1,0,...0)$ ... $\vec{\alpha}_{N-1} = (0,0,0,...0,1,-1)$ $\vec{\alpha}_N = (-1,0,0,...0,1)$

monopole-instanton charges (under U(1)^N, convenient basis)=all simple+lowest root

$$L_{eff}^{QCD(adj)} = M \left[(\partial_i \vec{\sigma})^2 - \sum_i^N m^2 \cos(\vec{\alpha}_i - \vec{\alpha}_{i-1(\mathrm{mod}N)}) \cdot \vec{\sigma} \right]$$

I. confinement on R^3 x S^1, size of circle- L:

formulae reveal different confinement mechanisms in dYM and QCD(adj):

monopole instantons vs "magnetic bions"

monopole instantons

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monopole-instanton charges (under $U(1)^N$, convenient basis)=all simple+lowest root

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magnetic bions

"topological molecules", or "bound states" of monopole instantons and anti-monopole instantons

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$$L_{eff}^{dYM} = M\left[(\partial_i\vec{\sigma})^2 - \sum_i^N m^2 \cos\vec{\alpha}_i\cdot\vec{\sigma}\right]$$

a crucial - for strings - property, most easily seen QCD(adj) L_eff:

$\vec{\alpha}_j\cdot\vec{\sigma} \to \vec{\alpha}_{j+1(\mathrm{mod}N)}\cdot\vec{\sigma}$ Z_N Weyl symmetry (due to center stability)

later also denote by $\vec{\sigma} \to P\vec{\sigma}$

$$L_{eff}^{QCD(adj)} = M\left[(\partial_i\vec{\sigma})^2 - \sum_i^N m^2 \cos(\vec{\alpha}_i - \vec{\alpha}_{i-1(\mathrm{mod}N)})\cdot\vec{\sigma}\right]$$

magnetic bions

"bound states" of monopole instantons and anti-monopole instantons

II. confining strings in QCD(adj) and dYM:

$$L_{eff}^{dYM} = M \left[(\partial_i \vec{\sigma})^2 - \sum_i^N m^2 \cos \vec{\alpha}_i \cdot \vec{\sigma} \right]$$

$$L_{eff}^{QCD(adj)} = M \left[(\partial_i \vec{\sigma})^2 - \sum_i^N m^2 \cos(\vec{\alpha}_i - \vec{\alpha}_{i-1(\mathrm{mod}N)}) \cdot \vec{\sigma} \right]$$

probe for confinement - area law for quarks in representation $\mathcal{R}$

$$W_{\mathcal{R}}(C) = \mathrm{tr}_{\mathcal{R}}\, P e^{i \oint_C A_k dx^k} \sim e^{-\mathrm{Area(C)}\Sigma_{str.}}$$

in the abelian regime of small L, simplify:

$$W_{\mathcal{R}}(C) = \sum_{H \in \mathcal{R}} \mathrm{tr}_{\mathcal{R}}\, e^{i\vec{H} \cdot \oint_C \vec{A}_k dx^k} = \sum_{\vec{\nu} \in \mathcal{R}} e^{i\vec{\nu} \cdot \oint_C \vec{A}_k dx^k} =$$

$$= \sum_{\vec{\nu} \in \mathcal{R}} e^{i\vec{\nu} \cdot \int_{S:\partial S = C} \vec{B}_{\mathrm{normal}} d^2x} = \sum_{\vec{\nu} \in \mathcal{R}} e^{i\vec{\nu} \cdot \vec{\Phi}(S(C))}$$

: all we need is magnetic flux through C

II. confining strings in QCD(adj) and dYM:

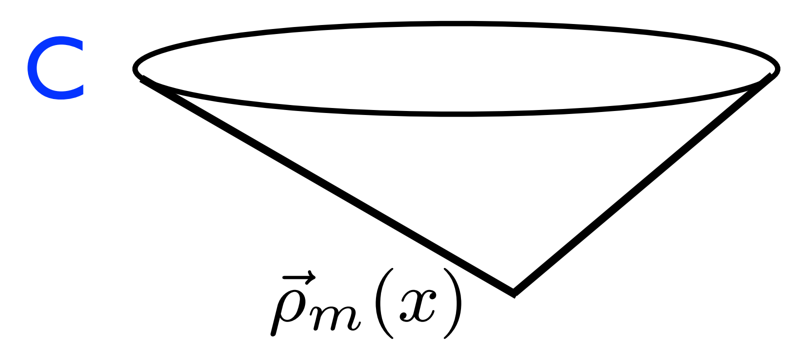
$$\langle W_{\mathcal{R}}(C)\rangle = \sum_{\vec\nu\in\mathcal{R}} \langle e^{i\vec\nu\cdot\vec\Phi(S(C))}\rangle$$: all we need is magnetic flux through C

but in the monopole gas, magnetic flux is due to monopole-instantons

picture: euclidean gas of monopoles with density $\vec\rho_m(x)$

monopoles at all x contribute to the flux thru C, must add them all

$$\vec\Phi(C,x) = \vec\rho_m(x)\eta_C(x)$$



4pi jumps don’t matter Dirac/GNO:

$$\vec\rho\cdot\vec\nu \in \frac{1}{2}Z$$

solid angle that C spans from x

flux thru C due to monopoles at x

$$\langle e^{i\vec\nu\cdot\vec\Phi(S(C))}\rangle = \langle e^{i\vec\nu\cdot\int d^3x\,\vec\rho_m(x)\eta_C(x)}\rangle$$

II. confining strings in QCD(adj) and dYM:

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$$\langle e^{i\vec{\nu}\cdot\vec{\Phi}(S(C))}\rangle = \langle e^{i\vec{\nu}\cdot\int d^3x\vec{\rho}_m(x)\eta_C(x)}\rangle$$

now recall that correlation functions of the density are generated by

$$Z[j] = \langle e^{i\int dx j(x)\rho_m(x)}\rangle, \quad L_{eff}(x) = e^2(\partial_i\sigma)^2 - \zeta\cos(\sigma(x)+j(x))$$

$$Z \sim \int \mathcal{D}\sigma\, e^{-\int dx L_{eff}(x)}$$

$\vec{\sigma}(x) \to \vec{\sigma}(x) + \vec{\nu}\eta_C(x)$ in potential term

all goes through for a multimonopole gas:

II. confining strings in QCD(adj) and dYM:

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dYM

$$\langle W(\vec{\nu})\rangle = \int \mathcal{D}\sigma \ \exp\left[-M\int_{R^3}(\partial\vec{\sigma})^2 - Mm^2\int_{R^3}\sum_{i=1}^{N}\left\{\begin{array}{c}\cos\vec{\alpha}_i\cdot(\vec{\sigma}+\vec{\nu}\eta_C)\\ \cos(\vec{\alpha}_i-\vec{\alpha}_{i-1(\mathrm{mod}N)})\cdot(\vec{\sigma}+\vec{\nu}\eta_C)\end{array}\right.\right]$$

QCD(adj)

Wilson loop-quarks with charges $\vec{\nu}$

Semiclassically, $\langle W(\vec{\nu})\rangle \sim e^{-S[\vec{\sigma}_{class.}]}$, where $\vec{\sigma}_{class.}$ solves:

dYM $\quad \nabla^2\vec{\sigma} - m^2\sum_{i=1}^{N}\vec{\alpha}_i\sin\vec{\alpha}_i\cdot(\vec{\sigma}+\vec{\nu}\eta_C) = 0$

QCD(adj) $\quad \nabla^2\vec{\sigma} - m^2\sum_{i=1}^{N}(\vec{\alpha}_i-\vec{\alpha}_{i-1(\mathrm{mod}N)})\sin(\vec{\alpha}_i-\vec{\alpha}_{i-1(\mathrm{mod}N)})\cdot(\vec{\sigma}+\vec{\nu}\eta_C) = 0$

These equations are great for numerics, for any contour C, via Gauss-Seidel relaxation - diffusion process in (discrete, fictitious) “time” t relaxes to minimum of action $\frac{\partial\sigma}{\partial t} = -\frac{\delta S}{\delta\sigma} = \nabla^2\sigma - 2m^2\sin(\sigma+\frac{1}{2}\eta_C)$

II. confining strings in QCD(adj) and dYM:

$$\langle W_{\mathcal{R}}(C)\rangle = \sum_{\vec{\nu}\in\mathcal{R}} \langle e^{i\vec{\nu}\cdot\vec{\Phi}(S(C))}\rangle = \sum_{\vec{\nu}\in\mathcal{R}} \langle W(\vec{\nu})\rangle$$

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Simply put, we are looking for solutions of the equations of motion with dual photon monodromy $\vec{\nu}$ around C (recall monodromy=electric charge!)

- let's get some intuition from simple cases...

II. confining strings in QCD(adj) and dYM:

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some intuition from simple cases: SU(2) $\vec{\sigma}$ is one-dimensional vector

$\alpha_1 = -\alpha_2 = 1$ magnetic charge of monopoles (electric charge of W bosons)

$\nu_1 = -\nu_2 = \frac{1}{2}$ electric charge of fundamental quarks (consider + sources only)

dYM $$\nabla^2\sigma - 2m^2\sin(\sigma + \frac{1}{2}\eta_C) = 0$$

QCD(adj) $$\nabla^2\sigma - 4m^2\sin 2(\sigma + \frac{1}{2}\eta_C) = 0$$

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some intuition from simple cases: SU(2) $\vec{\sigma}$ is one-dimensional vector

dYM

$$\nabla^2\sigma - 2m^2 \sin(\sigma + \frac{1}{2}\eta_C) = 0$$

QCD(adj)

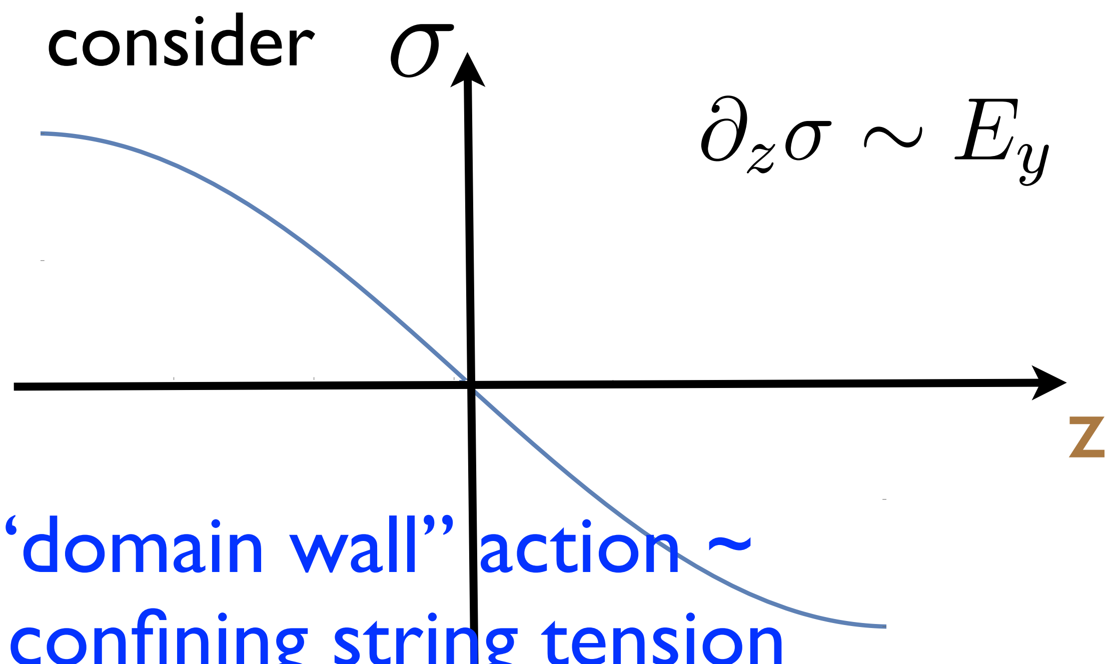
$$\nabla^2\sigma - 4m^2 \sin 2(\sigma + \frac{1}{2}\eta_C) = 0$$

dYM first:

let C be an infinitely large contour in the x-y plane and take the solid angle be $+2\pi$ above the plane and -2π below the plane

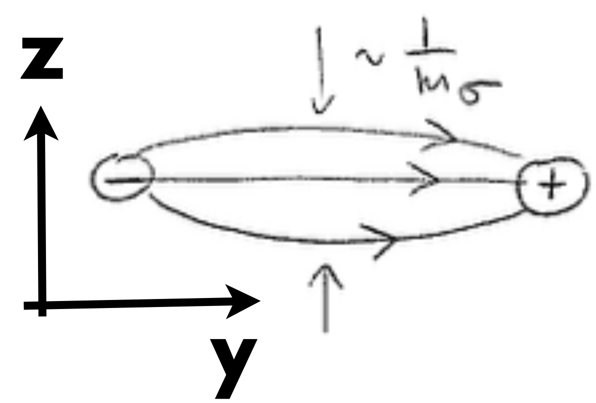
thus σ should have 2 π "monodromy" across z=0

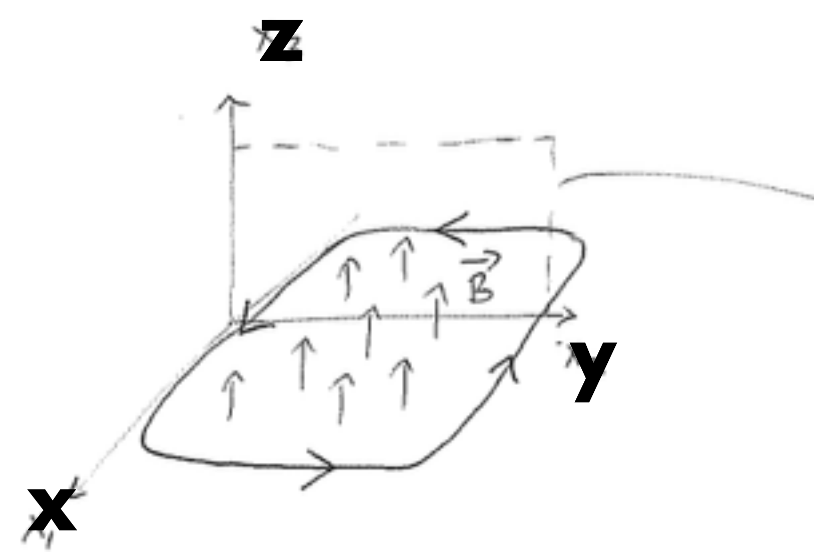
consider



$\partial_z\sigma \sim E_y$

"domain wall" action ~ confining string tension





$\partial_z\sigma \sim E_y$

σ "monodromy" around C=electric flux of confining string

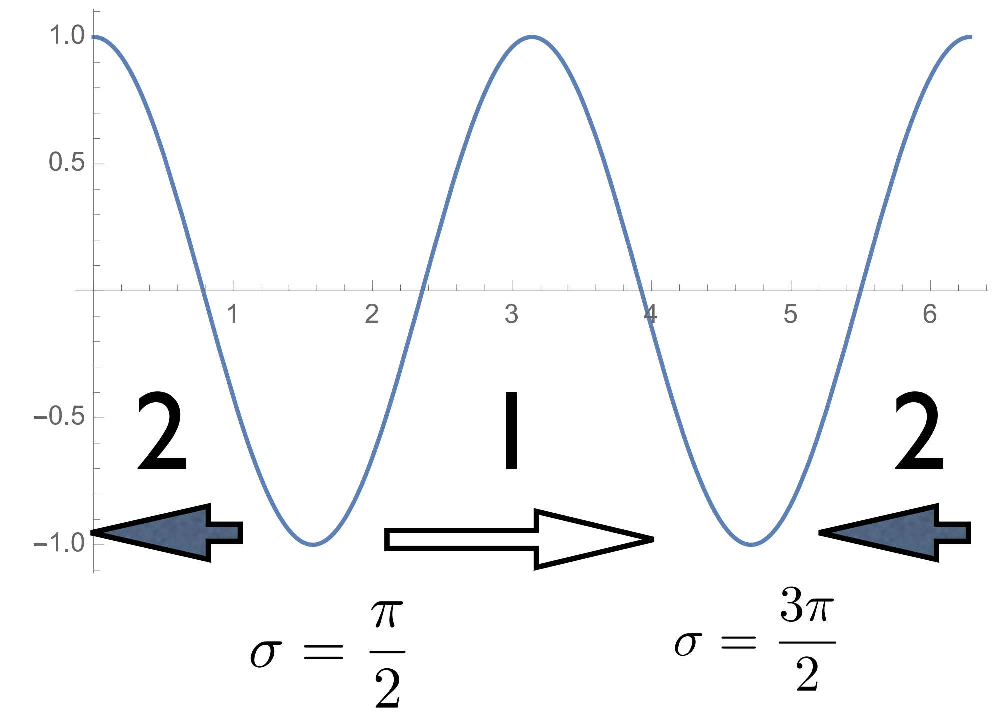
for quark, $+2\pi$

I I. confining strings in QCD(adj) and dYM:

QCD(adj)

$$\nabla^2\sigma - 4m^2 \sin 2(\sigma + \frac{1}{2}\eta_C) = 0$$

$$V(\sigma) \sim \cos 2\sigma$$



two vacua (broken chiral Z_2)
DW 1: el. flux π
DW 2: el. flux -π

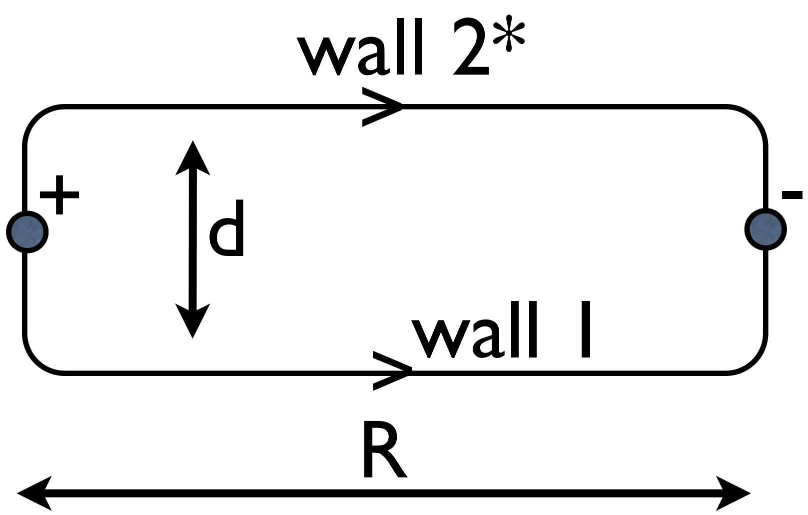
in SYM, both 1 and 2 DWs are BPS

e.g., both DWs have "1/2-quark" fluxes,
π not 2 π

no such charges allowed by Dirac;
(in fact these are genuine DWs separating Z_2 vacua)

So, whatever configuration has 2π monodromy - to confine quarks - must be composed of two walls... wall 1 followed by anti-wall 2* has correct flux

II. confining strings in QCD(adj) and dYM:



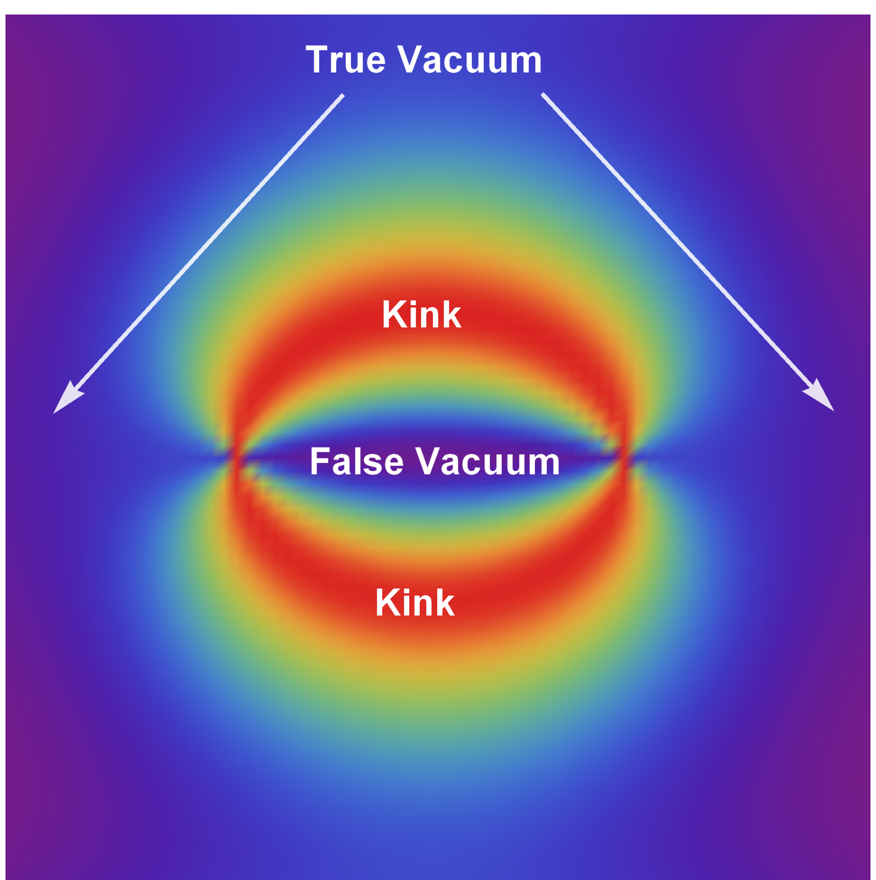
(T perpendicular to page)

wall action (model) 1-2* repulsion

$$S \sim MmT(R+d) + MmTRe^{-md}$$

$$md_* \sim \log mR$$ (semiclassically, w/out massless fermion exchange)

or via numerical minimization via Gauss-Seidel (logR growth of d holds)



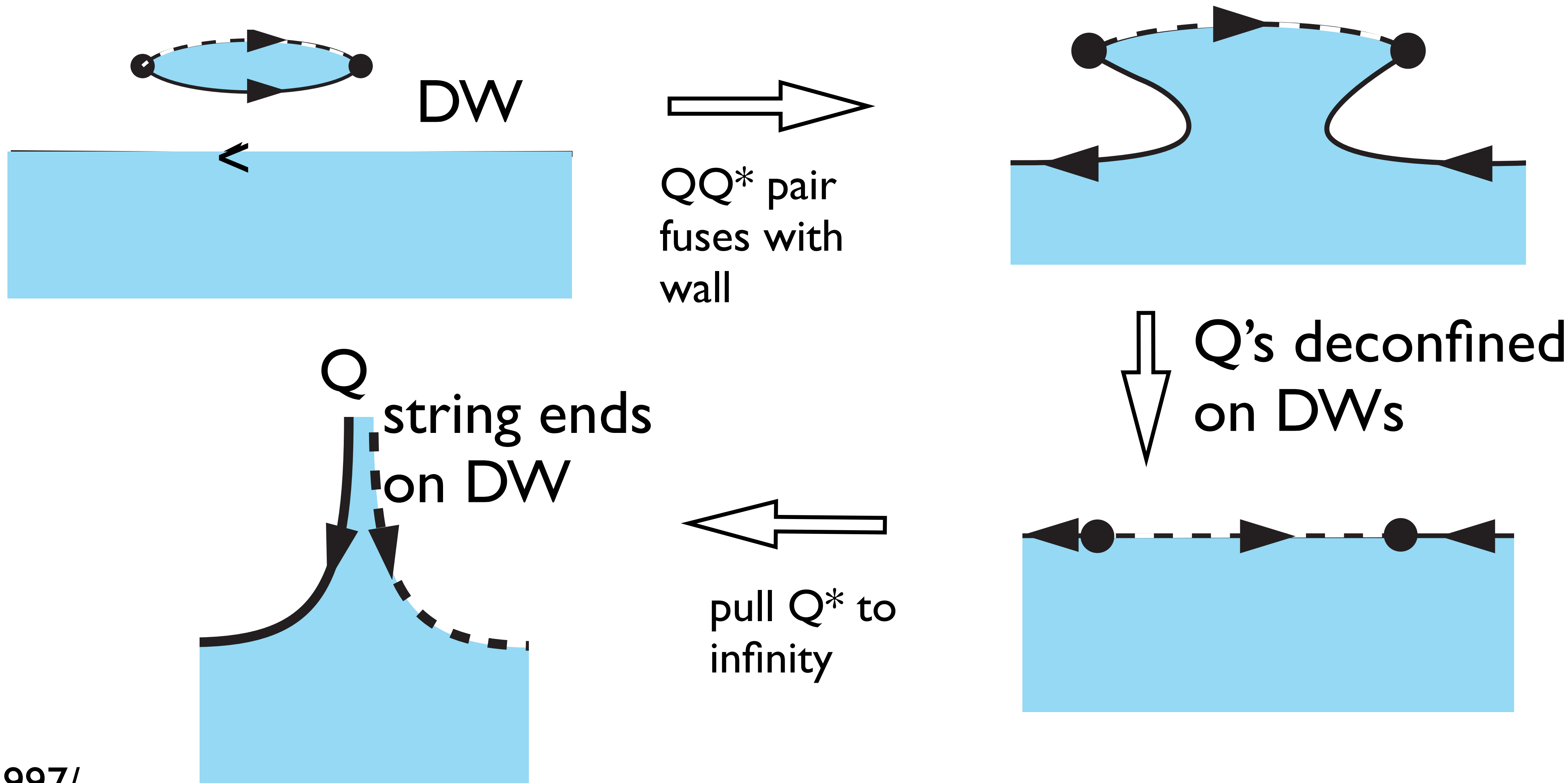
physically, the reason for the compositeness of the string is the composite nature of magnetic bions

(also, for all SU(N), as we'll see)

implications for DWs and strings... next:

II. confining strings in QCD(adj) and dYM:

the picture or strings “made out” of DWs also implies that confining strings can end on DWs



S.-J. Rey/Witten 1997/

1 MQCD: string (M2) ends on DW (some wrapped M5)

2 large-N SYM: BPS wall tension ~ N, not N^2, so “D-brane like” (think g_string ~1/N)

3 oblique confinement (heuristic!): wall supports free quarks so confining strings can end on it

here: pure QFT, no large-N, no SUSY/BPS (small-L instead), explicit, not heuristic, picture

an electric example of strings and branes “from flesh and blood” (Shifman-Yung all magnetic)

II. confining strings in QCD(adj) and dYM:

The story is even more fun in SU(N). Here, we don't know the solutions for single DWs (for SU(2), DWs 1 and 2 are explicitly known, SYM or QCD(adj)).

Recall, the crucial - for strings - property

$\vec{\alpha}_j \cdot \vec{\sigma} \to \vec{\alpha}_{j+1(\mathrm{mod} N)} \cdot \vec{\sigma}$ Z_N Weyl symmetry (due to center stability)

More abstractly [in SU(3), this is a 120° rotation in weight space]

$$\vec{\sigma} \to P\vec{\sigma},$$

This implies that $\langle W(\vec{\nu})\rangle = \langle W(P\vec{\nu})\rangle$

i.e. confining string tensions for quarks with weights in the same Z_N Weyl orbit are the same, both for QCD(adj) and dYM.
Since P permutes the N weights of the fundamental, all strings confining fundamental quarks have the same tension.

II. confining strings in QCD(adj) and dYM:

Without details [can explain], in QCD(adj)/SYM, elementary DWs have monodromy $\frac{2\pi}{N_c}\vec{\rho}$ (the Weyl vector/N)

at the same time, the highest weight of the fundamental is

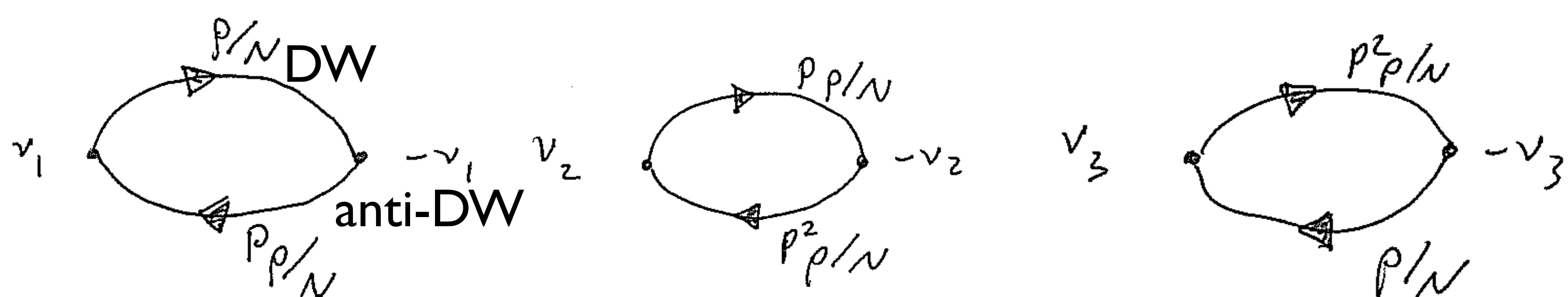
$$2\pi\vec{w}_1 = \frac{2\pi}{N_c}\vec{\rho} - \frac{2\pi}{N_c}P\vec{\rho}$$

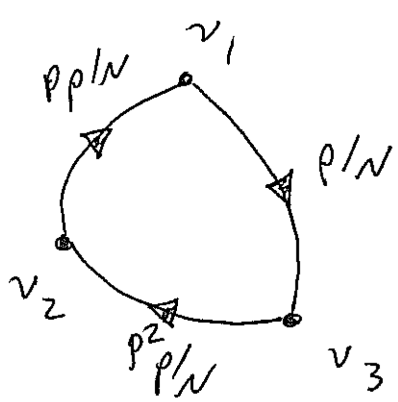
thus a string confining quarks (in the 0 vacuum) with charges $2\pi\vec{w}_1$ can be made of a wall and an P-antiwall

(this generalizes the SU(2) construction)

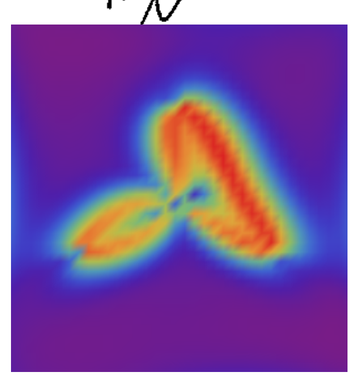
Strings confining the other two weights of the SU(3) fundamental are similarly constructed:

mesons -

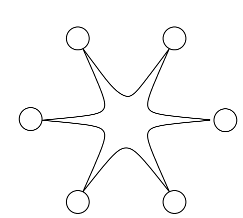




- baryon



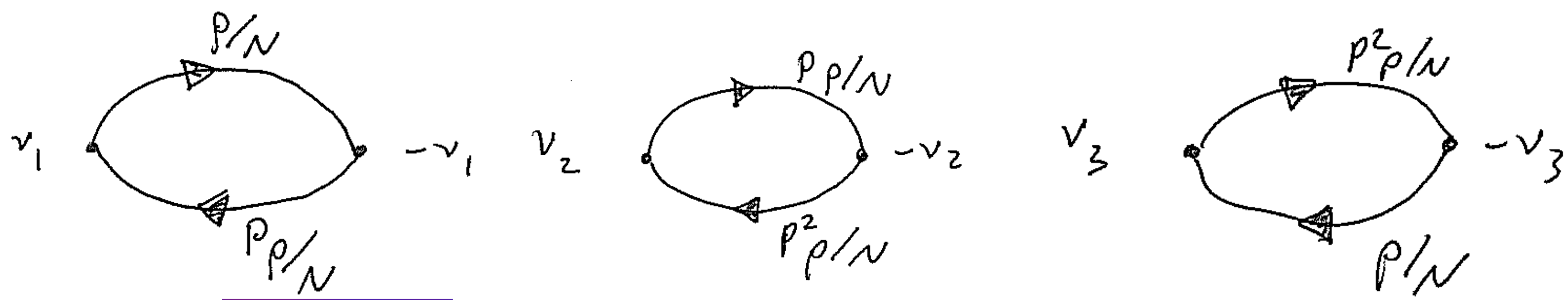
more like

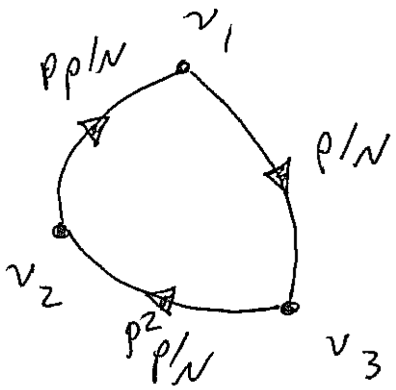


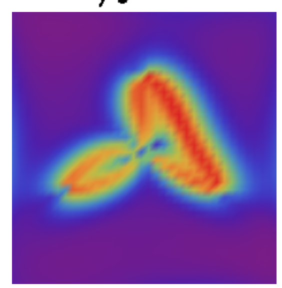
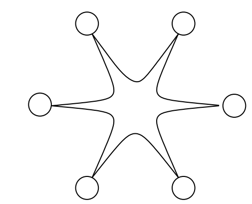
(N=3 here, but story really continues)

III. comparisons with other abelian and nonabelian confining strings...

mesons -

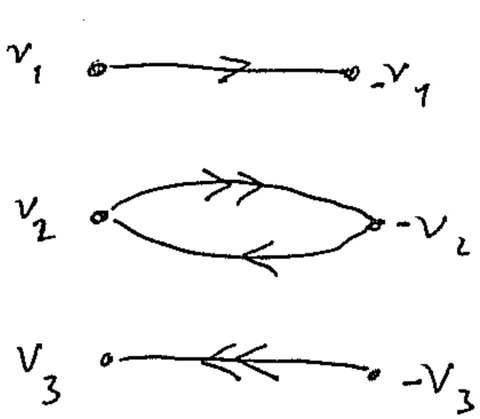




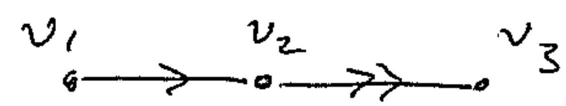
- baryon  cf the dual ones from 1st page 

(our one crude attempt at constructing baryons)

for fun, let's compare with Seiberg-Witten [for SU(3), SW has two dual ANO strings]



- nondegenerate mesons



only linear baryons

(more dramatic for N>3)

(k-th component of fundamental bound by k-string and an anti k-1-string)

qualitative difference because:

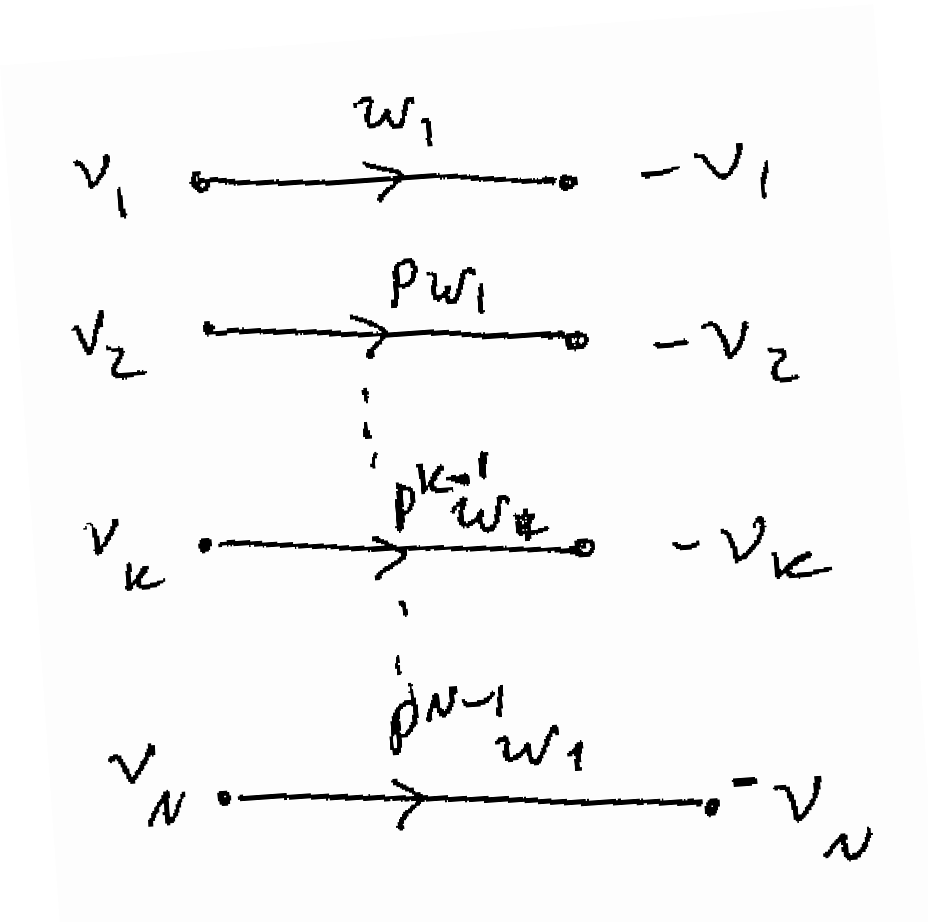
1. in SW there are N-1 condensing objects, in QCD(adj)/dYM there are N "condensing" monopole instantons
2. in SW Weyl group totally broken, in QCD(adj)/dYM a Z_N subgroup exact, due to center stability

III. comparisons with other abelian and nonabelian confining strings...

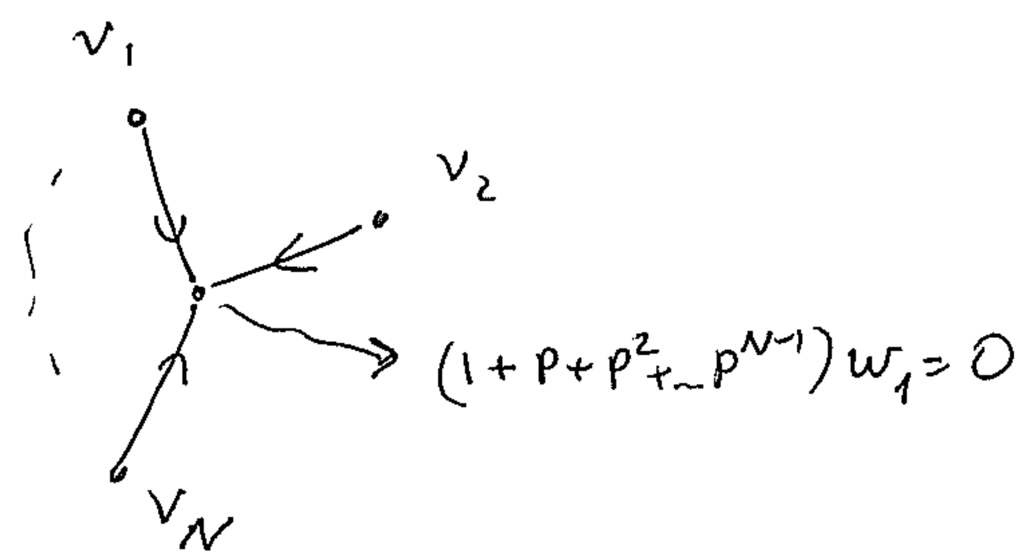
In dYM, we have “DWs” with flux w_1, w_2, ... w_{N-1} [the fundamental weights].

The vacuum is unique and these “DWs” are, in fact, confining strings.

For fundamental quarks, we also have Z_N degeneracy of strings:



also, “Y”-baryons exist, since
the sum of the N fluxes vanishes:



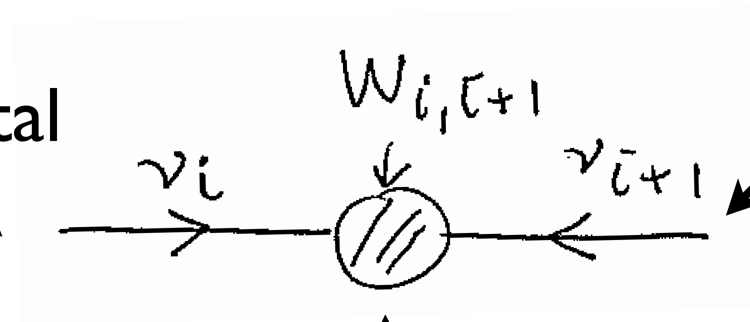
To be sure, just like in SW and QCD(adj), these are still abelian strings - distinct (if degenerate) meson Regge trajectories.

One can speculate about “integrating in” W-bosons, as entire heavy spectrum known- cf SW

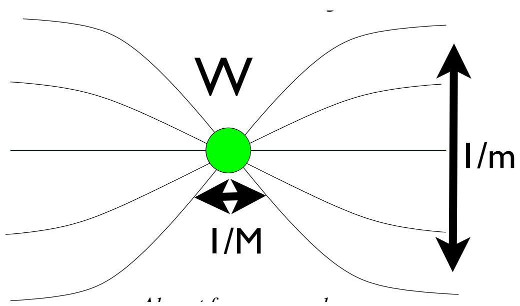
III. comparisons with other abelian and nonabelian confining strings...

One can speculate about "integrating in" W-bosons...[qualitatively similar in QCD(adj)/dYM]

a string confining i-th component of fundamental

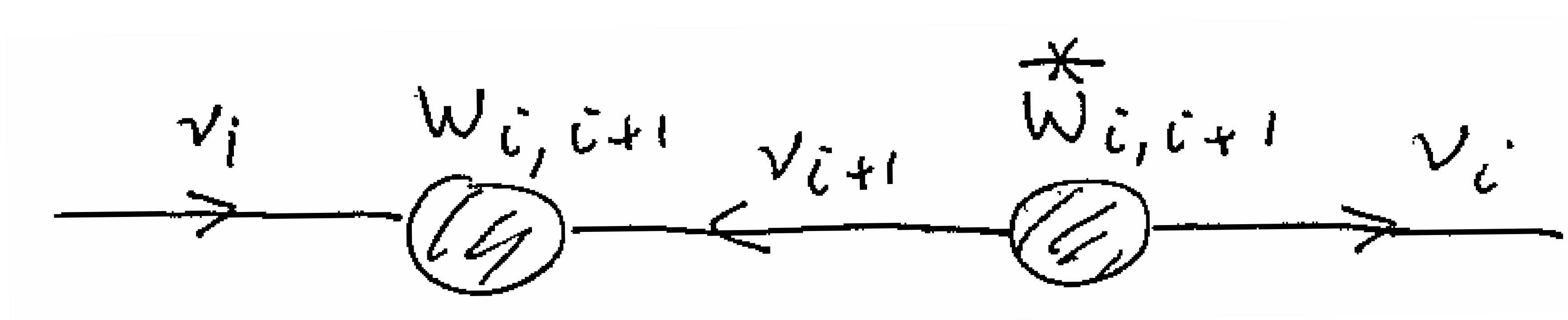


a degenerate anti-string confining i+1-th component of fundamental



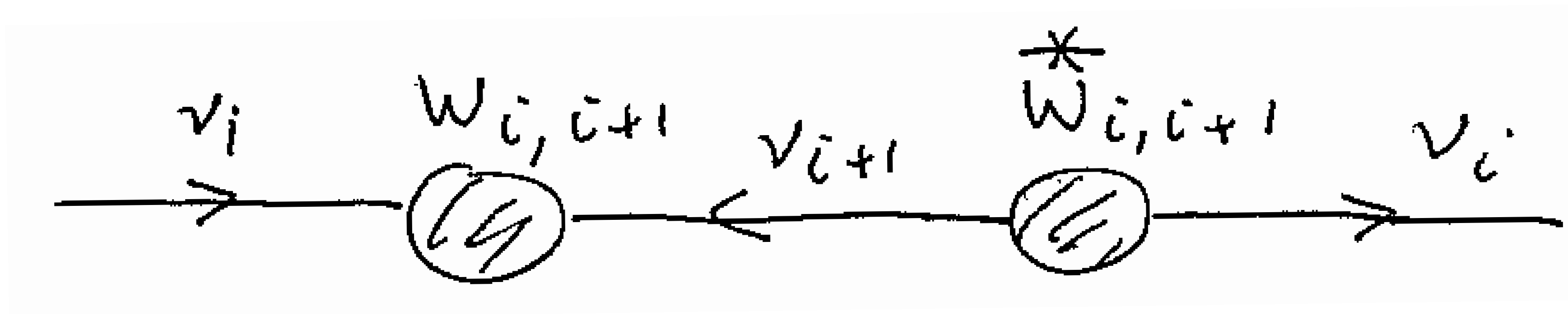
flux is exactly absorbed by W boson (no tension imbalance) - off-diagonal massive gauge boson - "nearest-neighbor" W's are the lightest, stable, and there are N degenerate species

Thus - like quarks on DWs in QCD(adj) - W-bosons in QCD(adj) and dYM are not confined on strings (at scales larger than the Debye screening length, 1/m):



III. comparisons with other abelian and nonabelian confining strings...

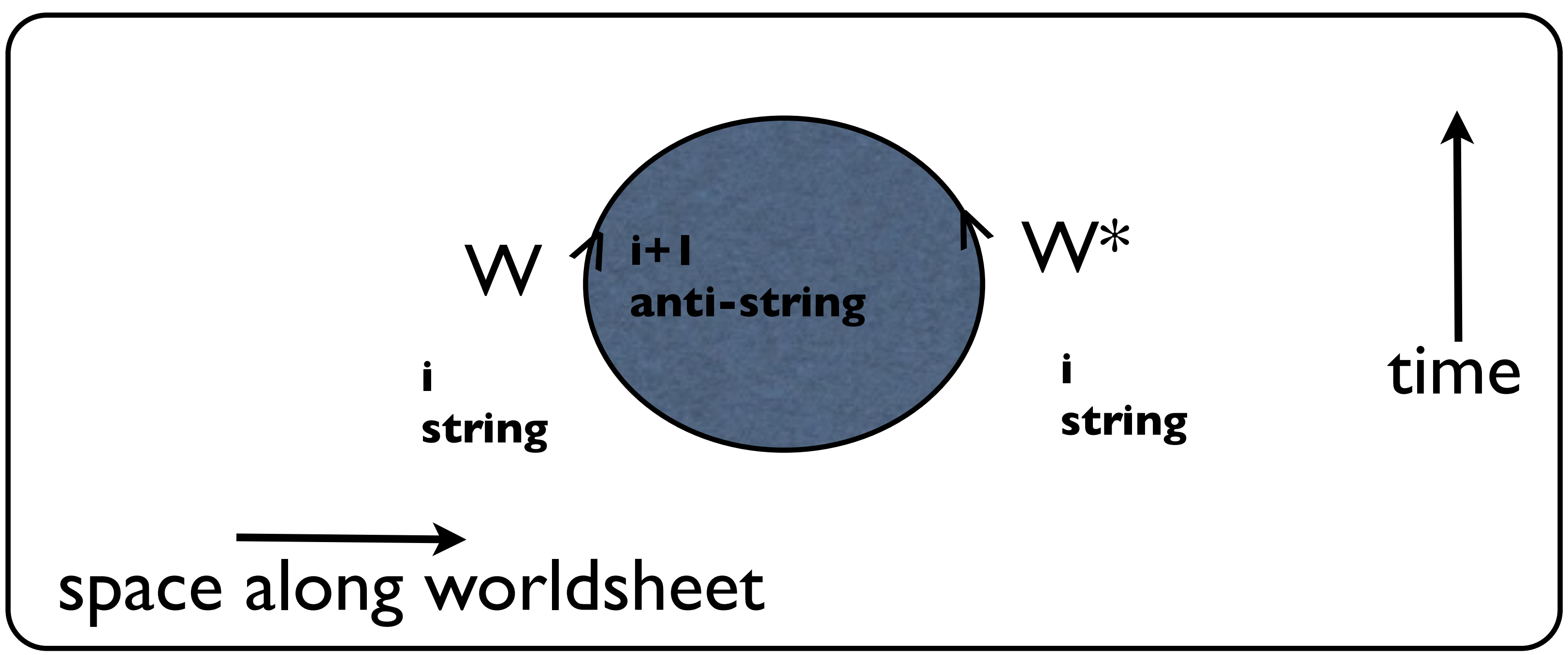
One can speculate about "integrating in" W-bosons...[qualitatively similar in QCD(adj)/dYM]



W-W* pairs on the string are massive (order M) excitations on the worldsheet

W is a "bead" on the string converting an i-string to an i+1 anti-string

On the Euclidean worldsheet, virtual W worldlines on the string look like boundaries (DWs?!) separating regions with an i-string flux to an i+1 anti-string flux



the abelian-regime picture is thus quite different from SW theory (also, here the effects of heavy Ws are calculable in principle…)

as S^1size increases, approach nonabelian regime. W-flux should "melt" on the worldsheet, restoring correct N-ality-only dependence of string tensions

nonabelian strings only characterized by Z_N center flux: how does this crossover proceed?

IV. future...

We've seen that even abelian confinement can be quite rich and diverse.

Interesting doable questions:

Taxonomy and properties of k-strings in this setup?

(in (slow) progress w/ students)

The picture of strings and DWs in dYM and QCD(adj) can be used to elucidate the recently discovered distinct global structure - discrete theta angles "p" Aharony Seiberg Tachikawa, Kapustin Seiberg 2013-2014 -of $[SU(N)/Z_k]_p$ theories in a physical manner.

already published in Anber, EP: 1508.00910… (another story)

IV. future...

We've seen that even abelian confinement can be quite rich and diverse.

Interesting hard questions:

Can the "double strings" in SYM be seen on the lattice?

perhaps less of a fantasy goal then massless QCD(adj) - e.g. Bergner,Piemonte 2014

How do the "double strings" in SYM morph into the ones in SW theory?

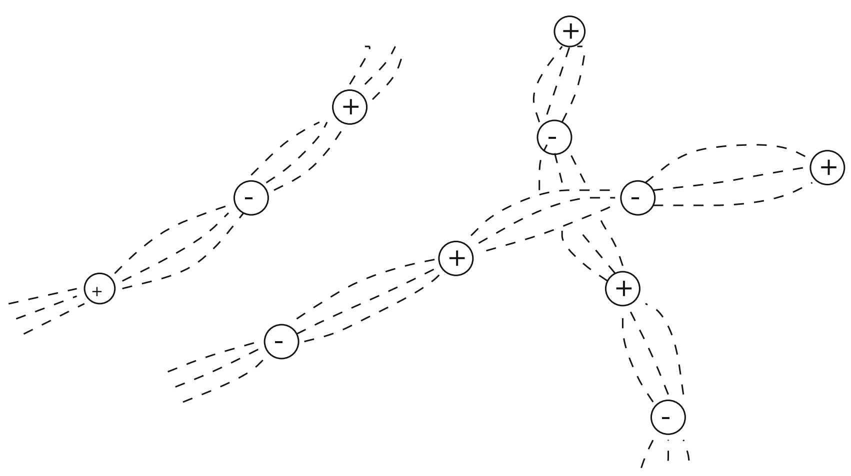
Is there a phase transition/crossover on the worldsheet upon transition from abelian to non-abelian regime? [only known study is of Hanany, Strassler, Zaffaroni within MQCD Seiberg-Witten: decays, otherwise same strings…] How would lattice look for one?

How is this abelian picture related to the center-vortex picture and how do the confining strings appear there? asked before in 3d Polyakov model by Greensite, Ambjorn…

(somewhat of a) fantasy: a time slice of SU(2) monopole gas with magnetic flux collimated into center vortices

- these disorder Wilson loop and give area law, N-ality good!

but, gauge fixing needed to see these in SU(2) phase!



it is not known (to me, at lest) how confining strings with Nambu-Goto like spectrum, as well seen in simulations, arise from center-vortex picture