2000-2001 Physics Olympiad Preparation Program

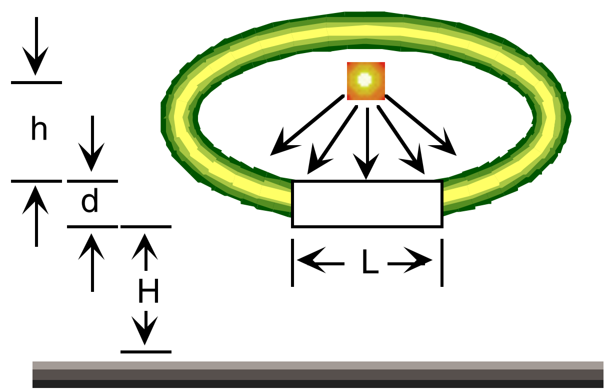
— University of Toronto —

Problem Set 4: Optics and Waves

Due February 16, 2001 (revised date)

1) Sub hubbub

In our deep-sea research submarine *POPSub* there is a frosted (not clear) light bulb of radius R. With you in it, the sub is at a height H from the bottom of the sea, examining life on the ocean floor. The light bulb is at a height h from the square ($L \times L$) window which is d thick and made of a glass with index of refraction n.



a) Calculate the area at the bottom of the sea which is illuminated.

b) As you look out, how big does the spot *look*? *[Yaser]*

2) "Out, out damned spot!"

Each time there's a solar eclipse, a certain number of people suffer eye damage, or even are blinded, because they look directly at the sun. It isn't even necessarily safe at maximum, when the sun is blocked. To be safe, one needs to use the sort of goggles that prevent the same kind of eye-damage in welders, whose torches can be very very bright; there are other special filters and glasses, but some you might think are good aren't actually enough, since, *e.g.*, invisible infrared or ultraviolet may pass through and cause damage .

When you think about using a magnifying glass, in the sun, to burn wood or set fire to paper, it really isn't very surprising your eyes are in danger. Direct sunlight is about 1 kW m^{-2}, at the earth — how about calculating the intensity of light *focussed* from the sun using a lens?

Your eyeball is roughly the size of the circle you make with thumb and forefinger placed tip-to-tip, as when you make an 'OK' sign. Make a guess, from that, of roughly what the focal length is of the lens-system of your eye, and let's figure the numbers:

a) Estimate the size of your eye's *pupil* when it's at its smallest, and figure out the power in watts that would shine through if you looked straight at the sun (don't do this by experimenting!). Now, using the focal length of the lens of your eye, determine the size of the image of the sun formed at the back of your eye. Then find the intensity of light that this gives.

It could be worse: if the sun gave the same intensity of light but was much smaller or much farther away, the focussed image would be much smaller and the intensity much higher. That's because all the rays of sunlight would be practically parallel, and focus nearly in the same place. But lasers give nearly parallel rays, so they *will* focus nearly to a spot…

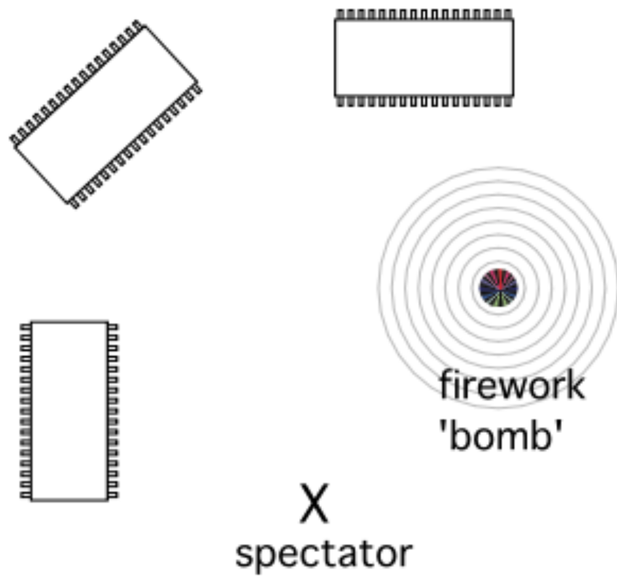
b) Consider a laser-pointer with 5 mW total output and a beam about 1 mm across. It doesn't focus to a point, quite, because of *diffraction*. So, if all the light goes into your eye, what is the focussed intensity at the back of your eye?

This value is about the maximum intensity for a Class IIIa laser. This can cause temporary loss of vision, like a camera-flash going off in your face; beyond this power eye-damage can result. *[Robin]*

Laser Safety: http://www.adm.uwaterloo.ca/infohs/lasermanual/documents/tblcont.html
Classifications: http://www.adm.uwaterloo.ca/infohs/lasermanual/documents/section8.html

3) Musical buildings

I used to enjoy watching fireworks on the 4th of July in Rochester, NY, where I went to graduate school. I noticed something quite interesting, though: when the loud 'bomb' fireworks went off, making a noise like a cannon, the buildings would play music. Reflected from several different buildings, the noise sounded something like the grating on a bridge when your car drives over — a brief rough note of music.



This was because several buildings in Rochester had long vertical ribs in their architecture, running the height of the buildings, separating sets of windows. The 'bang' sound would reflect off each rib, and the reflections coming to me would make a note like running a pencil along a comb.

a) What frequency should you hear reflected? Consider a building with rib-spacing D at a distance L_1 from the 'bomb' and L_2 from the spectator. You may need to know also the angles θ_1 and θ_2 from the building of these two paths. You may need to make L_1 and L_2 large, to simplify.

b) There's a different 'picture' of how this works: that the single bang of a firework 'bomb' is a sound with many many different frequencies in it, rather than a single note — something like white light, as opposed to coloured light. What frequency would you expect to hear, standing in the same place as (a), if all the different wavelengths of sound simply diffracted from the 'grating' made of building-ribs? *[Robin]*

4) A high-tech diet, high in optical fiber...

The internet and most telephone communication depends most heavily not on copper wire, nowadays, but on *optical fiber*. Rather than electrical pulses in wire, for signals, optical fiber communications use laser pulses propagating in tiny flexible glass strands.

Think of the following model of a laser pulse propagating in an optical fiber. Take a glass rod with a circular cross-section of radius r. Say that light is launched into the fiber through the end, and that all rays lie within a cone of angle θ measured from the axis of the rod. The index of refraction of the fiber is n.

a) If the input pulse is very short-duration, what is the duration of the pulse of light coming out at the end of the fiber? Assume the length of the fiber to be L.

b) If the input light is a *train* (series) of pulses with duration τ, and time-separation Δt, what should be the *maximum* length of the fiber in order that the output pulses are still distinguishable. This is a real issue for communications, since we need to know the fastest digital signal that can be sent.

c) If the peak intensity of a single input pulse is I, what is its output peak intensity. *[Yaser]*

[check the POPTOR web-site for an example of sound in a tube doing the same thing as this...]

5) Wave goodbye

Hong and her brother Liang have a clothesline in their back-yard. They mentioned to me their experiments with their giant one-string guitar... If they hit the cable sharply with a stick, what they see is two waves that run away from the impact-place, one in either direction. They want to know why — can you explain for them?

What they see is called a *travelling wave*. The shape of the wave stays the same, as it simply moves along the wire as if it were sliding. A travelling wave has the form:

$$f(x,t) = f(x + ct)$$

or

$$f(x,t) = f(x - ct)$$

If you look closely, you'll see that the top one is a wave with shape *f(x)* travelling to the right (*increasing x* with time), and the lower one is the same shape, but travelling to the left (*decreasing x* with time). In fact, almost any shape will work.

There are some very interesting properties of such travelling waves:

1) if you have a shape that works as a wave, then the same shape would work as a wave if it were bigger or smaller by any factor *a.*

2) if you have two different shapes that work as travelling waves, then if you add the two together, the sum will also work as a wave.

These are properties of what we call a *linear* system. They're true, for instance, for sound waves in air (except for explosions and such). If they weren't true, we couldn't listen to music (think about that...).

a) Say that Hong and Liang suddenly put a 'dent' $f(x)$ in the shape of the clothesline by quickly striking with the stick. How do you figure out the two waves that run away in either direction? Look for a trick or idea that uses properties (1) and (2) above to help prove what happens; you don't have to use much math.

b) Say that the clothesline is tied firmly at both ends. Then the position of the cable at the end cannot move. What happens when a wave travels right to the end of the cable? Try the same kind of ideas that work for (a).

c) Say that at one end the clothesline is tied to a ring that slides up and down an upright pole; say that this slides perfectly without friction., and the ring has negligible mass. This is a different *boundary condition*. What happens when a wave travels right to the end of this cable? *[Sal/Robin]*

6) Ten yards for interference…

Many people have home theatres these days and although technology makes having a home theatre relatively simple, in order to get the maximum effect one still has to take into account the geometry of the setup. Consider a basic home theatre setup, with distance $2d$ between the speakers, and a distance L from the speakers to the opposite wall of the room. Assume the speakers are point sources, that each puts out exactly the same single-frequency pure sine wave (this must be a pretty boring movie, or a test of the Emergency Measures Systems…) and that the walls do not reflect any sound.

a) Find a general expression giving the position(s) P of maximum intensity along the opposite wall as a function of speaker spacing d and room length L. Give the position(s) P in terms of distance y from the centreline between the speakers. Do not assume $d \ll L$.

b) If $L = 5$ m, $d = 3$ m and both speakers are producing an in-phase 320 Hz sound wave, calculate the position in one direction of the first three points of maximum intensity along the opposite wall, as measured from the centreline between the speakers. The speed of sound in air is 331 m/s.

HINT: This question involves solutions to an equation that can't easily be written as a tidy equation $y = f(x)$, but more like $0 = f(y) + g(y)$. You're welcome to solve the equations graphically — plotting, and then looking for roots. You're also allowed to solve by hand iteratively, or get a computer to solve the equation numerically. *[Brian]*

INFOBITS™ — Useful Bits of POPTOR Information

You don't *need* this to solve question 5, but it's interesting.

Waves satisfy a differential equation (an equation of derivatives) called *the wave equation*:

$$\frac{\partial^2 f(x,t)}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 f(x,t)}{\partial t^2} = 0$$

It is actually a very simple equation — you just need to take the derivative twice. The unusual notation is because these are *partial derivatives*. The *partial* derviative with respect to x means that you take the derivative with respect to the x-variable (totally ignoring the t-variable). So, for instance,

$$\frac{\partial \sin(x+3t)}{\partial x} = \cos(x+3t)$$

but

$$\frac{\partial \sin(x+3t)}{\partial t} = 3\cos(x+3t) \qquad \text{(using the chain rule)}$$

You just pretend the other variable doesn't even exist.

A *travelling wave* can always be described by $f(x,t) = f(x \pm ct)$, if f is any function that can be differentiated properly. It gives the shape f(x) in space, travelling in the direction of *increasing* x for $f(x - ct)$ and *decreasing* x for $f(x + ct)$.

Remember to check the POPTOR web-page for hints and any possible corrections! www.physics.utoronto.ca/~poptor