2000-2001 Physics Olympiad Preparation Program

— University of Toronto —

Problem Set 5: Electricity and Magnetism

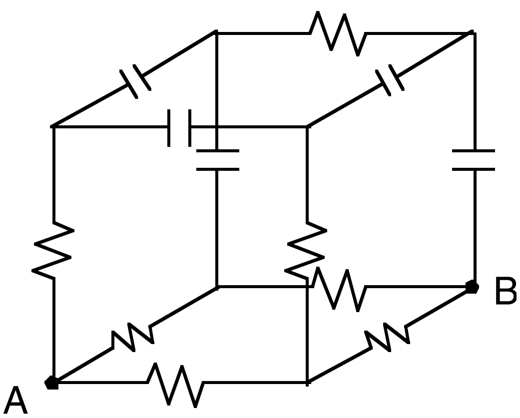
Due March 9, 2001 (revised date)

1) Bridging the gaps

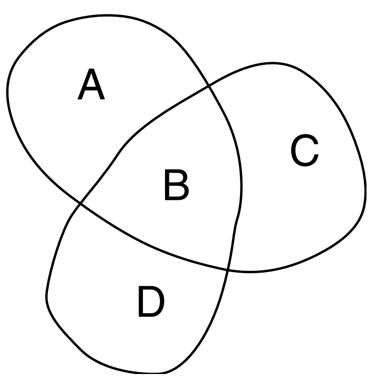
Consider an insulated wire of length L wound on a metal cylinder. The insulation breaks at some hidden place, and the wire inside touches the cylinder. Using a battery, an ammeter and two variable resistors, figure out where along the wire is the break. *[Peter]*

2) Roadkill this question

In the circuit drawn at right, all circuit elements lie on the edges of a cube. All resistors have identical resistance R, all capacitors have identical capacitance C. The voltage between points A and B is V. Figure out what charge is produced on the capacitor next to B. *[Yaser]*



3) Loopy circuits

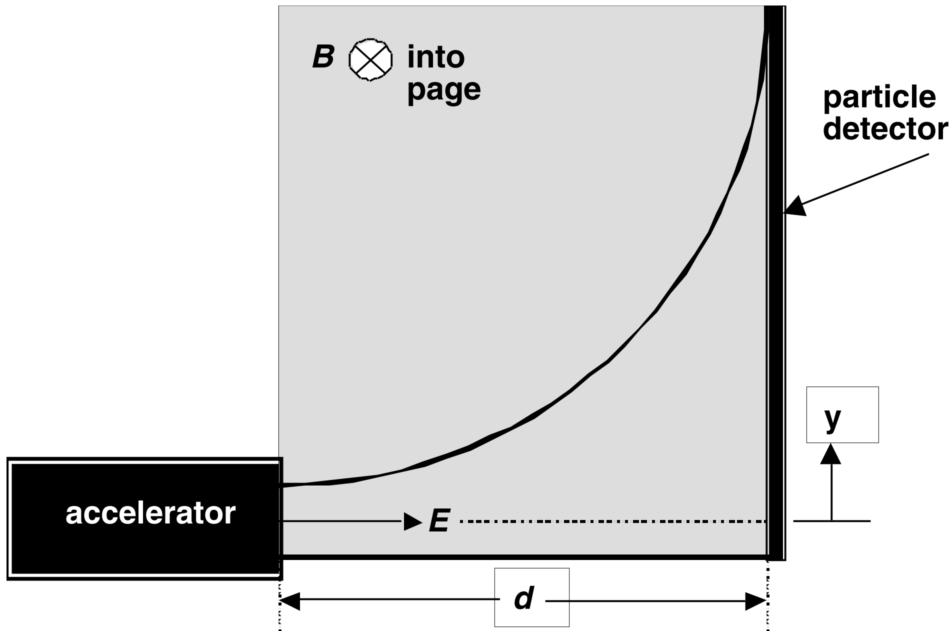


A wire loop is laid out on a plane to make the closed circuit shown at right. A uniform magnetic field B is oriented perpendicular to this plane — sticking directly out of the page. This magnetic field changes at the rate $\mathrm{d}B/\mathrm{d}t$.

What is the induced electromotive force in the wire? Find the answer in terms of the area of each section, *A*, *B*, *C*, and *D*. *[Yaser]*

4) Use the Force, Luke!

A *mass spectrometer* is a device that measures the mass of charged particles (called ions) by steering particles with the same mass to the same point in space. Mass spectrometers are analogous to prisms, for light: a prism spreads a beam of white light into its component colors (or wavelengths); in a mass spectrometer a beam of ions with different masses is spread out according to mass. There are many uses for mass spectrometers such as in radiocarbon dating, where mass spectrometers measure the ratio of numbers of carbon isotopes ^{13}C and ^{14}C. The figure at right shows the geometry of a simple mass spectrometer: A given ion is accelerated by an electric field to a specific energy — and hence velocity. As the ion leaves the accelerator, it enters a region containing a uniform magnetic field that is perpendicular to its direction of motion. As the ion moves through the magnetic field, it experiences a centripetal force that causes its path to curve along a circle. A detector set along the y-axis, in this figure, can determine the intercept of the ion-path and that axis.



a) Derive an expression relating the energy, charge and mass of the ion to its location (along the y-axis) on the detector. Assume non-relativistic energies.

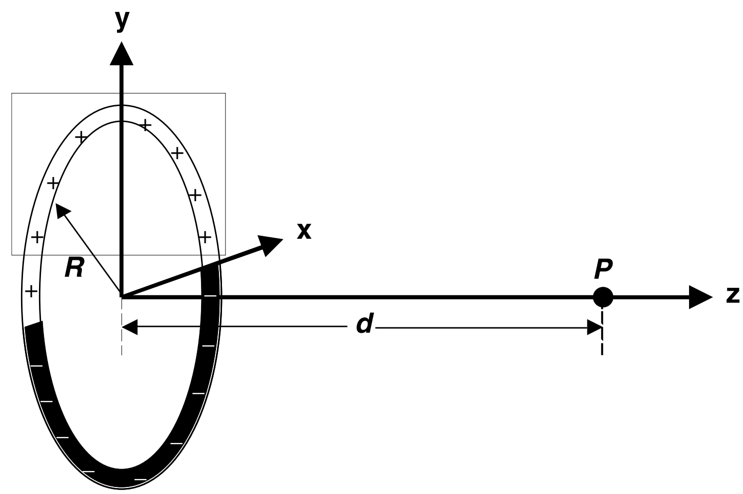
b) What minimum path-radius can this apparatus accommodate.

c) If $B = 2$ Tesla, what is the minimum measurable mass for a ion with charge q = +1 e (where e is the magnitude of charge on an electron), accelerated to an energy 25 eV? What is the maximum mass? Do you think there may be any difficulty in trying to detect extremely heavy ions? If so, why? Note: an *electron volt* (eV) is another unit for energy. *[Brian]*

HINTS: $\bar{F} = q\bar{v} \times \bar{B}$ is the force experienced by a charged particle moving through a magnetic field. In this equation, q is the charge of the particle, v is the velocity of the particle and B is the magnetic field. The right-hand rule for cross products may be useful in determining which *way* the particle curves.

5) A ringing charge

A thin ring of radius R positioned in the x-y plane has a positive *linear charge density* $+\rho$ [C/m] on the top half and an equal-magnitude negative linear charge density $-\rho$ [C/m] on the bottom half. Derive an expression for the *magnitude* and *direction* of the electric field at point P, a distance d away from the center of the ring.



HINT: what is contribution to the electric field at point P from any small segment of the ring? *[Brian]*

6) Scoping out electrical charge

Two identical spheres of mass m are identically charged, then each suspended from the same point by two non-conducting threads. Take each thread to have zero mass and fixed length L. The electrostatic forces acting on each object will act to push them apart.

THE THEORY

a) Derive an expression for the charge q on the spheres, found from their mass, the length of strings, and the angle θ between the threads.

THE EXPERIMENT

You will need the following:

1) 2 pieces of light string or sewing thread, and sewing needle
2) Styrofoam™ (or similar) packing material — packing ‘peanuts’ or blocks
3) utility knife or paring knife or razor blade
4) A protractor or ruler
5) A bit of tape, or push-pin

Procedure:

From the Styrofoam block cut out two cubes and try to cut the corners off to make something nearly spherical. Try fairly small cubes — about a centimeter or less. Using the sewing needle and thread, thread equal lengths of thread through the centers of the spheres. Using the tape or push-pin, attach the free ends of the thread together and suspend the whole thing freely — I tied the two ends together and then taped the end to the edge of my bookshelf.

Charge the spheres up — I rubbed mine against my hair, but you could rub a balloon against a sweater and then touch it for a few seconds to the styrofoam. The suspended charged balls will now hang with some space between them.

b) Now use the results of your measurements, and part (a) above, to determine the charge on the Styrofoam spheres. [*Brian*]

INFOBITS™ — Useful Bits of POPTOR Information

Remember to check the POPTOR web-page for hints and any necessary corrections! www.physics.utoronto.ca/~poptor