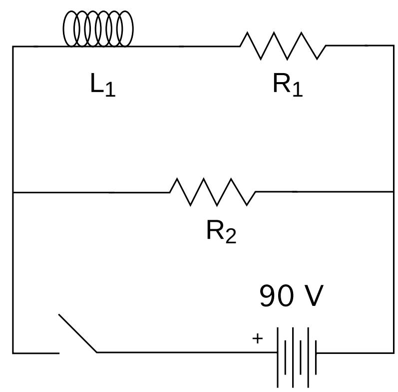
2000-2001 Physics Olympiad Preparation Program

— University of Toronto —

Problem Set 6: AC Circuits and Electronics

Due *April 6, 2001 (revised date)*

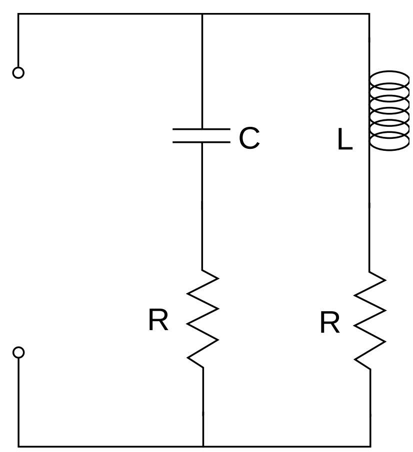
1) Inducting Henry



A circuit contains an inductor L_1 = 70 mH in series with a resistor R_1 = 50 Ω, both of which are in parallel with a R_2 = 350 Ω resistor and a 90 V battery. The switch is closed for 110 μs and then opened.

What is the value of the current in both resistors at the moment the switch is opened at 110 μs? *[Sal]*

2) Inducing Resistance



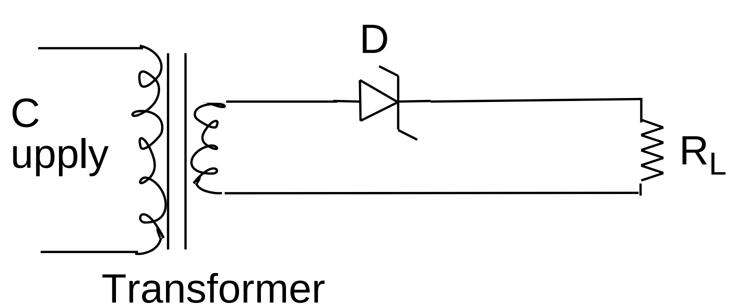
a) Graph the current that would flow from a power supply in the circuit at right (across terminals marked with circles) if the following voltage were applied:

$$V(t) = \begin{cases} 0 & t < 0 \\ V_o & t \geq 0 \end{cases}$$

b) What condition on L, R, and C will make the impedance seen by the power supply one which is purely *resistive*? (You may want to see the POPTOR Primer™ at the back of this set). *[Robin & Cambridge Tripos]*

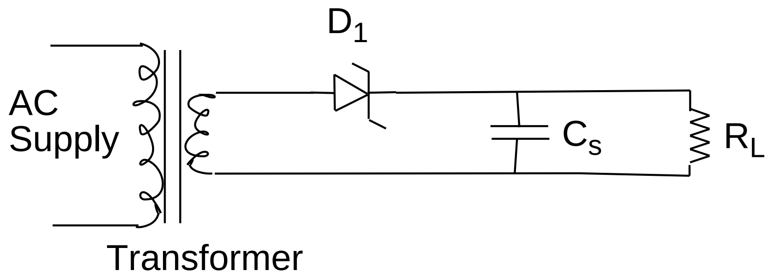
3) Rectifying a situation

In its simplest description, a *diode* is an electrical component that permits current to flow in only one direction. It can be used as a *rectifier* to change alternating current (AC) to direct current (DC).



a) In the circuit shown at right, a step-down transformer converts 110V AC to 6.3V AC. *D* is a diode and the load resistor R_L is 100 Ω. If the frequency of the AC power supply is 60 Hz, plot the voltage across R_L versus time.

b) In order to get a smoother output voltage, capacitors can be used as shown in the circuit at right. If C_s is 330 μF, plot the voltage across R_L for three values of R_L, so that



$R_L C_s$ infinitely large

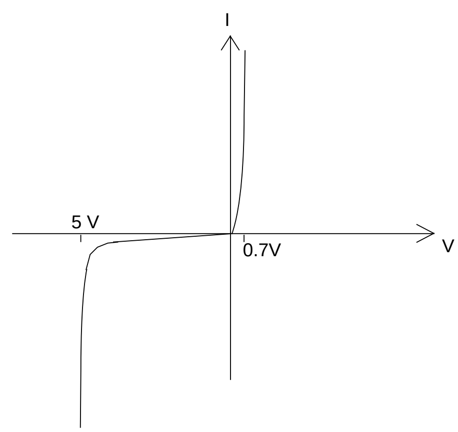
$R_L C_s = 1/6$ sec

$R_L C_s = 1/60$ s

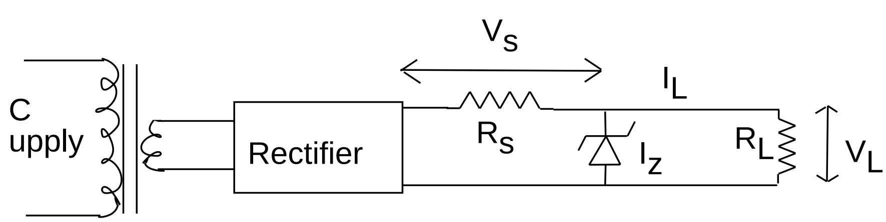
[Yaser]

4) Zeners and the art of electronics maintenance

A *Zener diode* maintains a more or less constant voltage across it in the reverse direction independent of current passing through it for a wide range of currents. This property may be used to stabilize a power supply against variations in input supply voltage and in load current drawn.



Assume that there is a Zener diode, which the I-V curve is as below. This diode is placed in the circuit, which is shown. Transformer gives an output voltage reduced from the AC supply by a factor of 11. Rectifier is used to change the AC voltage to DC.



a) If R_s is 100 Ω, for R_L = 100 Ω, what is the voltage across it if the voltage of AC supply is varied from 85 volts to 130 volts?

b) Assume that the voltage of AC supply is 110 volts. What is the voltage across R_L if we vary its resistance from 50 Ω to 150 Ω? *[Yaser]*

5) Active courage

a) By replacing an op-amp with its ideal model (see the POPTOR Primer™ below), derive an expression for V_{out} as a function of V_{in} and use this expression to calculate the ratio $\frac{V_{out}}{V_{in}}$.

b) What is the limit of the equation $\frac{V_{out}}{V_{in}}$ you derived in part (a) as the gain A approaches infinity? What effect does the op-amp have on the output signal with respect to the input signal?

c) Is the equation in part (b) a good approximation to the equation in part (a) if the gain A is equal to 10,000? (less than 1% is good)

d) Why doesn't the op-amp amplify the input voltage by the op-amp gain A?

e) BONUS (Worth an *insane* number of marks): When the op-amp gain A is very large, what does the voltage on the negative terminal V_1 become? What happens to the voltage V_1 when V_2 is non-zero (not grounded)?

[The result of this bonus question illustrates the real usefulness of op-amps: op-amps in negative feedback mode allow one to arbitrarily set the voltage of any point in a circuit *without sinking current (i.e., with infinite effective resistance).* This is the basis for circuits such as the Miller Integrator.] *[Brian]*

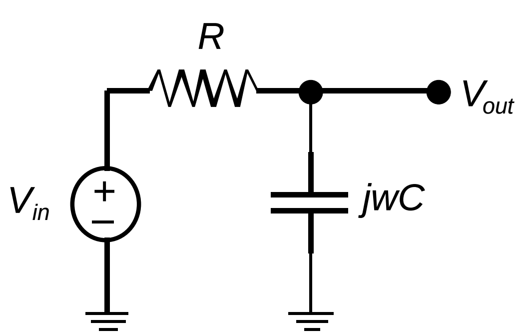
6) Freaky Filters for "Phreaky" Physicists

Various types of frequency filter appear in many devices. Here are some examples:

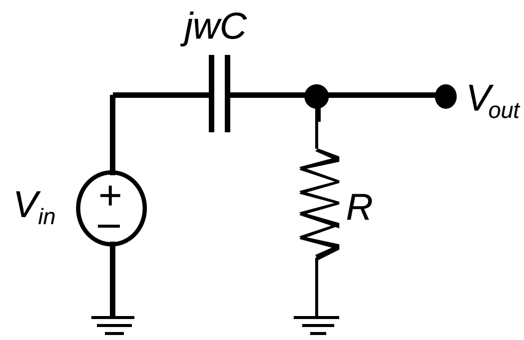
Electrical – radio antenna
Mechanical – automotive shock absorbers
Optical – sunglasses
Acoustical – earplugs
Data – Net-Nanny

This problem will concentrate on understanding how simple first-order electrical filters work. But before getting into solving the actual problem, you'll need to understand the useful concept of *impedance*: See the POPTOR Primer™ below, for a briefing, if you don't know about impedances very well.

For the two filters on the next page, answer the following questions.



Filter A



Filter B

a) Derive expressions for $\frac{V_{out}}{V_{in}}$ (these should complex functions of ω).

b) From part (a) produce expressions for the magnitude and phase of $\frac{V_{out}}{V_{in}}$ as a function of frequency.

c) What is the magnitude and phase of $\frac{V_{out}}{V_{in}}$ when $\omega = 0$ and as ω approaches infinity (HINT: you may want to use l'Hôpital's Rule)?

d) Two-way speakers usually have a *tweeter* for high-frequency sounds and a *woofer* for low to mid-frequency sounds. The reason why two-way speakers are used is because a single speaker cannot efficiently produce both high and low frequency sounds (FYI: speakers act like filters too!). The tweeter and woofer each have a separate filter that filters the incoming signal from the amplifier. If you were building your own two-way speaker, which filter (A or B) would you use on the tweeter and which filter would you use on the woofer? Since the magnitude of the filter responses do not cut off sharply, the tweeter and woofer will have some overlap in the sound frequencies they produce. Use the equations for magnitude developed in part (b) to calculate the *crossover frequency* for the two filters if $R = 10{,}000\ \Omega$ and $C = 0.02\ \mu F$.

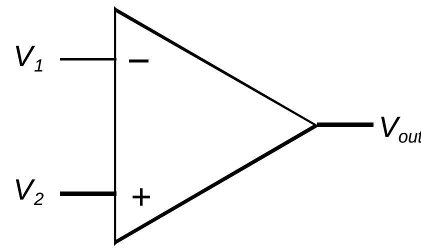
BONUS (Worth a reasonable number of marks): What effect will the phase have on the music played through our two-way speaker? *[Brian]*

Remember to check the POPTOR web-page for hints and any necessary corrections!

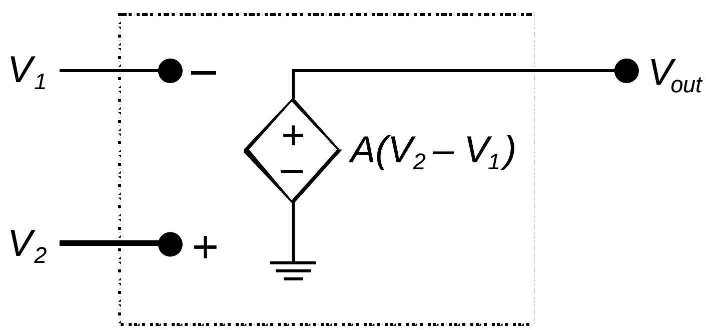
www.physics.utoronto.ca/~poptor

Active Circuits

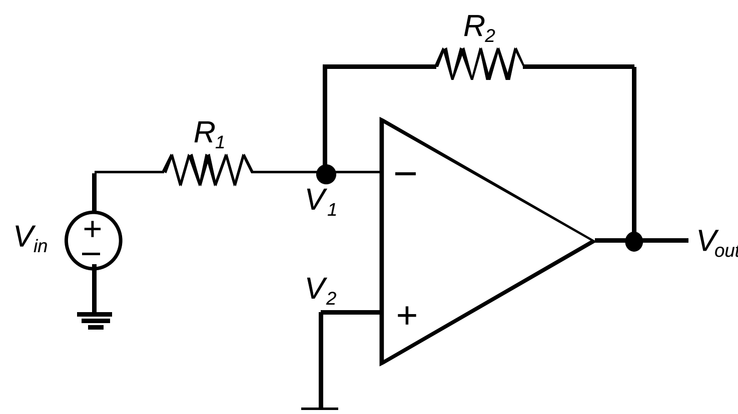
For many of you this may be your first encounter with an active circuit element. Unlike resistors, capacitors, inductors and diodes which are passive circuit elements, active components like op-amps and transistors have the ability to put energy into a circuit in a controlled manner based on electrical input from the circuit itself!



Op-amp stands for "operational amplifier" and is one of the simplest of all the active elements. The figure at right shows the circuit symbol for an op-amp. An op-amp is a three-terminal device that amplifies the difference in voltages between the "+" and "–" input terminals by a factor *A* and outputs the result ("*A*" is called the *gain* or *amplification factor*). The gain for a typical op-amp is usually between 10,000 and 1,000,000. Electrical characteristics of a "real-life" op-amp are very complex. However, for most applications it is more than reasonable to represent the op-amp by a simple model.



The circuit at left shows a model of an ideal op-amp – that is, if we could build a perfect op-amp, this model would completely describe its electrical response. Circuits containing op-amps can be analyzed using standard circuit techniques (such as Kirchoff's Voltage and Current Laws and Ohm's Law) by replacing all the op-amp symbols with its model.



The circuit on the right shows an op-amp in a *negative feedback* configuration. All negative feedback means is the output is fed back into the "–" input terminal; in this case, the negative feedback occurs through resistor R_2.

If you go on to take electronics courses at university or college, you will learn more about negative feedback. In the meantime, for a motivating explanation, negative feedback is used to eliminate non-linearity in circuit response and increase stability – but you do not need to know this in order to solve this question!

AC Impedance

Ohm's Law $V = IR$ relates the voltage V across a resistor R to the current I through the resistor. But circuits can include capacitors and coils in addition to resistors. How then can we analyze such circuits? Is there an Ohm's Law for capacitors & coils?

The answer is "yes" – it is the *impedance* version of Ohm's Law, $\boldsymbol{V} = \boldsymbol{IZ}$, where $\boldsymbol{Z}$ is called the impedance (boldface denotes a complex number, here). The impedance $\boldsymbol{Z}$ can be thought of as a complex-valued "resistance" having the form $a + jb$ where a is the "real" part and b is the "imaginary" part [1]. Don't get scared of complex-valued impedances. Complex numbers are only used to simplify the bookkeeping when determining things like magnitude and phase as you will soon find out [2]. The great thing about impedances is that the familiar techniques used to analyze DC circuits carry over!

When we want to extract a physically meaningful number from the impedance version of Ohm's Law, we just have to take the magnitude and phase of the complex expression. There are a few things about impedances you need to know about: The phase of a positive imaginary impedance is 90° and the phase of negative imaginary impedance is –90°. Similarly, the phase of a positive real impedance is 0° and the phase of a negative real impedance is 180°. The phase of an impedance with real *and* complex parts can be anywhere between 0° and 360°.

Experiments show that when a sinusoidal (sine-wave) voltage is applied across a capacitor, the current through the capacitor is 90° out of phase with respect to the applied voltage and *increases proportionally* with frequency. Similar experiments show that when a sinusoidal voltage is applied across an inductor (coil), the current through the coil is –90° out of phase with respect the applied voltage and *decreases inversely* with frequency. (N.B.: the frequency of the applied voltage and current are the same for both the capacitor and coil). Without proof (or you can take this to be an experimental result [3]), the impedance of a capacitor is $j\varpi C$ and the impedance of a coil is $\frac{-j}{\varpi L}$ where C is the capacitance, L is the inductance and ω is the frequency.

---

1. In circuit analysis the engineer's "*j*" is used to denote √(–1), instead of using the mathematician's "*i*" which can be confused with current.

2. In principle, we can solve these problems without complex numbers. However, doing so would induce a really bad case of trigonometric diarrhea.

3. The impedances of a capacitor and coil can be derived from Maxwell's equations. However, Maxwell's equations themselves cannot be derived, but are based upon empirical results.