

**University of Toronto**  
**2001-2002 Physics Olympiad Preparation Program**

**Problem Set 4: Optics and Waves**

*Due Monday 11 February 2002*

**1) Yes, Officer!**

A car driven by a physicist is stopped by a police officer for running a red light. The physicist explains that the light appeared yellow because of the Doppler shift. (The wavelengths of red and yellow light are about 690 nm and 600 nm respectively.) Does the physicist deserve the ticket?

*[David]*

**2) You push, I'll pull.**

Radiation pressure is a very important factor for interstellar dust clouds<sup>1</sup>, comet tails<sup>2</sup>, and stellar and planetary system formation<sup>3</sup>. Once a cloud of gas and dust collapses and a star ignites, its radiation blows away smaller particles.

How big must a spherical dust particle be to be currently attracted by the sun instead of being repelled. *i.e.* What is its minimum radius,  $R$ ?

*Hints:* Assume the dust has a typical mineral density of about density  $3 \text{ g/cm}^3$  and that the dust totally absorbs the sunlight.

*[David]*

**3) 500 channels - but nothing to watch!**

For years I have been waiting for my favourite TV show, Fraggles Rock<sup>4</sup>, to be available again in Toronto. (That part isn't made up, but from here on I may wander just a teeny bit from reality.) I recently learned that the show is playing on a small station broadcasting on Channel 2 (57 Mhz), which is not on Toronto cable. The station is 50 km away, but fortunately my house is on a hill and with a good antenna I can pick up the station by direct line of sight.

One day in the middle of the show I notice that the signal is fading in and out, going from maximum to minimum and back again 8 times per minute. I then learn from the radio that one of my atmospheric physics colleagues has lost control<sup>5</sup> of a humongous balloon<sup>6</sup> and it is currently just halfway between me and the TV station. The balloon is 20 km up and slowly rising at a

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<sup>1</sup> [http://www.seds.org/messier/more/m045\\_i349.html](http://www.seds.org/messier/more/m045_i349.html)

<sup>2</sup> <http://antwrp.gsfc.nasa.gov/apod/ap001227.html>

<sup>3</sup> [http://www.seds.org/messier/more/m042\\_hst2.html](http://www.seds.org/messier/more/m042_hst2.html)

<sup>4</sup> [http://www.henson.com/television/series/fraggle\\_home.html](http://www.henson.com/television/series/fraggle_home.html), <http://home.no.net/fraggel/fragglorock.htm>

<sup>5</sup> <http://www.atmosp.physics.utoronto.ca/MANTRA/press.html>

<sup>6</sup> [http://www.msc-smc.ec.gc.ca/events/balloon/launch/launch17\\_e.cfm](http://www.msc-smc.ec.gc.ca/events/balloon/launch/launch17_e.cfm)

constant speed, and the signal I am receiving is the sum of the direct line of sight signal plus the reflected signal from the balloon. How fast is the balloon rising?

[David]

#### 4) An intense experience!

Nonlinear optical materials have an index of refraction that is dependent on the intensity of the incident beam. For a linear non-linear optical material,  $n=n_1+n_2I$ , where  $I$  is the intensity of the beam,  $n$  is the index of refraction, and  $n_1$  &  $n_2$  are positive, real constants.

Most laser beams have a Gaussian distribution of intensity in the  $xy$  plane normal to the direction of the beam,  $z$ . If for simplicity we only consider one axis we have

$$I(x) = I_0 e^{-\frac{x^2}{w^2}}$$

- Plot the intensity versus  $x$ . What parameter determines the size of the beam in this formula?
- If a Gaussian beam hits a nonlinear material, how does it travel inside it? Show qualitatively that the beam has to focus down. How does the beam continue after it focuses?
- Estimate the distance from the surface where the beam focuses. Use the fact that different rays which meet each other at the focal point have to travel the same optical distance (Optical distance=Index of refraction \* distance). If it helps simplify your answer, you may assume  $n_1 \gg n_2 I_0$ .
- How do your answers change if  $n_2$  is negative?

[Yaser]

#### 5) All the world in a drop of water

A microscope can be made using a drop of water as a lens.<sup>1</sup>

- What is the focal length of roughly 5mm diameter drop of water supported by a horizontal piece of transparent plastic?
- If you put your eye very close to the water drop and look through it you can see quite impressive magnification of objects. (I thought the ridges on my fingers were fascinating, but I am known to be easily entertained. By “very close” I mean your eyelashes may be touching the plastic.) Estimate the magnification of your water drop microscope.

If you can calculate the answer to these questions I'll be very impressed, since the only way I know to get the answer is to do the experiment. (To calculate the answer you'd have to calculate the shape of the drop which depends on its mass, the acceleration due to gravity, the surface tension of water, the molecular forces at the water/plastic boundary, ...)

- Play around with different drops. Very roughly, discuss how the optical parameters (*e.g.* the focal length, magnification, and distortion) depend on the physical parameters (*e.g.* diameter) of your droplets?

Make sure you describe what you did to get your answers.

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<sup>1</sup> <http://www.microscopy-uk.org.uk/mag/art98/watermic.html>

*Hints:* The drop should be more or less circular as seen from the top. I have used shiny transparent tape, transparent food wrap, transparent CD or audio cassette cases.... (Plastic seems to work better than glass.) You want the drop to be as rounded as possible, not smeared out, e.g. like the drop on the left, not like the one on the right.



I use a drinking straw to gently deposit my drops on the plastic. Your droplet lens is fairly sensitive to vibration, so it is better to support your piece of plastic so you don't have to hold it and you can put things underneath your lens without touching it, but this isn't necessary. Make sure you have good lighting; I used either bright daylight or a flashlight. With a little practice, I could put samples (*e.g.* salt crystals) on the bottom of an audio cassette case and consistently put the right sized drop on the top which gave the right focal length.

The magnification is conventionally defined as the apparent size compared to the size seen at the naked eye at a distance of 25cm from the eye. I found pieces of linear graph paper very useful for studying magnification and distortion; I darkened some of the lines to make them more visible if looked at out of focus while looking through a droplet at another piece of graph paper in focus. Overhead lights provided a convenient light source for the focal length measurement.

*[David]*

## POPBits™ – Possibly useful bits of information

### Constants and units<sup>1,2</sup>

astronomical unit (mean earth-sun distance)	au	149 597 870 660±20 m
atomic mass unit: (mass <sup>12</sup> C atom)/12	<i>u</i>	(1.66053873±0.00000013)×10 <sup>-27</sup> kg
Boltzmann constant	<i>k</i>	(1.3806503±0.0000024)×10 <sup>-23</sup> J/K
elementary ( <i>i.e.</i> electron) charge	<i>e</i>	(1.602176462±0.000000063)×10 <sup>-19</sup> C
electron volt	<i>eV</i>	(1.602176462±0.000000063)×10 <sup>-19</sup> J
index of refraction of water	<i>n<sub>H2O</sub></i>	1.33
Newtonian gravitational constant	<i>G<sub>N</sub></i>	(6.673±0.010)×10 <sup>-11</sup> m <sup>3</sup> /kg/s <sup>2</sup>
solar luminosity ( <i>i.e.</i> light output)	<i>L<sub>☉</sub></i>	(3.846 ± 0.008)×10 <sup>26</sup> W
solar mass	<i>M<sub>☉</sub></i>	(1.98892 ± 0.00025)×10 <sup>30</sup> kg
speed of light in vacuum	<i>c</i>	299 792 458 m/s
standard acceleration of gravity at the earth's surface	<i>g</i>	9.80665 m/s <sup>2</sup>
Stephan-Boltzmann radiation constant	<i>σ</i>	(5.670400±0.000040)×10 <sup>-8</sup> W/m <sup>2</sup> /K <sup>4</sup>
tropical year (2001)	yr	31556925.2 s

### Great excuses for a party

High energy Nobel birthdays:

Hideki Yukawa <sup>3</sup> (1907)	January 23
Samuel Chao Chung Ting <sup>4</sup> (1931)	January 27
Abdus Salam <sup>5</sup> (1926)	January 29

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<sup>1</sup> <http://physics.nist.gov/cuu/Constants/index.html>

<sup>2</sup> [http://pdg.lbl.gov/2000/contents\\_sports.html](http://pdg.lbl.gov/2000/contents_sports.html)

<sup>3</sup> <http://www.nobel.se/physics/laureates/1949/press.html>

<sup>4</sup> <http://www.nobel.se/physics/laureates/1976/press.html>

<sup>5</sup> <http://www.nobel.se/physics/laureates/1979/salam-bio.html>