

University of Toronto

2001-2002 Physics Olympiad Preparation Program

Problem Set 5: Electricity and Magnetism

Due Monday 11 March 2002

1) Purple Haze!

My old friend Steve studies top quarks using the CDF¹ experiment which has a very large superconducting solenoidal magnet with a uniform horizontal field of 1.5 Tesla. In addition to physics, Steve also loves guitars.² Is it safe for Steve to play his guitar inside the CDF magnet? In particular, is he likely to shock himself or burn his fingers on the E₄ string if it is vibrating with a peak-to-peak amplitude of 1mm?

Hints: The E₄ string (330 Hz) on Steve's guitar is 66 cm long, 0.23mm in diameter, and has an electrical resistivity of $12 \mu\Omega$ cm, a thermal conductivity of 80 W/m/K, a linear mass density of 0.3g/m, and a specific heat of 500 J/kg/K. (You may not need all this data, but we didn't want to leave out anything potentially useful.) The string vibrates sinusoidally in its lowest mode, and both ends of the string are connected to pins at constant temperature (21°C). Assume that any heat produced in the string can only escape through the ends of the string or through Steve's fingers, *i.e.* thermal radiation and convection are negligible. The time scale for heat flow out of the wire is much longer than the oscillation period of the wire. Thermal conductivity and electrical conductivity are completely analogous and the equations have identical form, *i.e.* electrical conductivity relates charge flow and voltage differences, thermal conductivity relates heat flow and temperature differences.

[David]

2) Relatively electric!

Near the surface of the earth there is typically a vertical electric field of about -120 V/m. (The field can reach 20 kV/m underneath a thundercloud - this is why your hair may stand on end just before you get hit by lightning - but we'll ignore this interesting but irrelevant safety tip for the rest of the problem.)

- Why aren't people electrocuted by the typical 200 V potential difference between the air at their feet and the air at their head?
- What is the typical surface electric charge density on the earth's surface? Give your answer in units of electron charges per metre squared.
- What is the typical surface magnetic field (magnitude and direction) at the equator produced by this surface electric charge density, as measured by a magnetic field meter just above the surface of the earth

¹ <http://www-cdf.fnal.gov/cdf.html>

² <http://webbug.physics.uiuc.edu/courses/phys398/>

- i) if it is attached to the earth and rotates with it?
 - ii) just above the surface of the earth, letting the earth rotate under it?
- (d) Are either of your answers to the two parts of (c) consistent with the observed magnitude (typically $30\mu\text{T}$) or observed direction of the surface magnetic field at the earth's equator?

Hints: My coordinates are such that an electric field pointing up is positive, pointing down is negative. The surface of the earth, cows, and people, all have similar (non-infinite) resistivities. Don't worry about the fact that the meter sitting above the surface of the earth would be moving supersonically, *e.g.* assume it is in a jet flying in an eternal sunrise, or sunset, or noon. The north pole of a magnet is the one which is attracted to the north pole of the earth.

[David]

3) Spheres within spheres

An insulating sphere of radius R with uniform volume charge density of r has an electric field inside it equal to $\vec{E} = \frac{\rho}{3\epsilon_0} \vec{r}$ where r is the distance from the center of the sphere ($r < R$). Note that this is a vector formula.

- (a) Consider two overlapping spheres of radii R_1 and R_2 with uniform charge densities ρ and $-\rho$ and at a distance d from each other ($d < R_1 + R_2$). Show that the electric field in the region of overlap is uniform and calculate its magnitude and direction.
- (b) If the two spheres have the same radii R and the distance between them is much smaller than the radius ($d \ll R$), you can consider the whole thing as one sphere with a surface charge density σ .
 - i) Find σ on the surface in terms of the angle that it makes with the line connecting the two centres. What is the maximum surface charge density σ_0 ? What is the electric field inside the sphere in terms of σ_0 ?
 - ii) What is the electric potential, V , on the surface of the sphere in terms of the angle that it makes with the line connecting the two centres? (The potential is defined to be zero at infinite distance.)

[Yaser]

4) Does it (anti)matter?

When I was a kid watching Star Trek (during its original run), I sometimes wondered if antimatter falls up. According to General Relativity it should fall down at the same rate as ordinary matter, but some exotic theories predict it will fall with a different acceleration. Experiments trying to observe the gravitational force on antimatter usually use conductors to shield the antiparticles from external electromagnetic forces. Imagine a positron inside a solid piece of copper on the surface of the earth. What is the vertical acceleration of the positron?

Hints: Assume General Relativity is correct. For the purposes of this question, a positron is just like an electron, but with opposite charge. You can ignore the fact that the positron will quickly annihilate with an electron, and assume it can move freely through the copper just like conduction electrons. Before thinking about positrons, you may first want to think about the net force on the free conduction electrons inside the copper.

[David]

5) Refrigerator Art

Experimentally and quantitatively determine the arrangement of north and south poles on a flat, flexible refrigerator magnet. Describe what you did and what your conclusions are.

Hints: We are talking about the flexible magnetic cards which usually have advertising on one side and a dark smooth surface on the back. We don't care which poles are north and which poles are south, but what is the arrangement of the poles. I figured it out just by playing around with two similar fridge magnets and making a measurement or two with a ruler, but there may be other methods.

[David]

POPBits™ – Possibly useful bits of information

Constants and units^{3,4}

Boltzmann constant	k	$(1.3806503 \pm 0.0000024) \times 10^{-23}$ J/K
Earth equatorial radius	R_{\oplus}	6.378140×10^6 m
elementary (<i>i.e.</i> electron) charge	e	$(1.602176462 \pm 0.000000063) \times 10^{-19}$ C
electron (or positron) inertial mass	m_e	(510998.902 ± 0.021) eV/ c^2
electron volt	eV	$(1.602176462 \pm 0.000000063) \times 10^{-19}$ J
Newtonian gravitational constant	G_N	$(6.673 \pm 0.010) \times 10^{-11}$ m ³ /kg/s ²
permittivity of free space	ϵ_0	$8.854187817 \times 10^{-12}$ F/m
permeability of free space	μ_0	$4\pi \times 10^{-7}$ N/A ²
speed of light in vacuum	c	299 792 458 m/s
standard acceleration of gravity at the earth's surface	g	9.80665 m/s ²
Stephan-Boltzmann radiation constant	σ	$(5.670400 \pm 0.000040) \times 10^{-8}$ W/m ² /K ⁴

Interesting (?) Integrals (a is a constant)

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) \qquad \int \cos(ax) dx = \frac{1}{a} \sin(ax)$$
$$\int \sin^2(ax) dx = \frac{x}{2} - \frac{1}{4a} \sin(2ax) \qquad \int \cos^2(ax) dx = \frac{x}{2} + \frac{1}{4a} \sin(2ax)$$

Great excuses for a party

Galileo's birthday (1564)
Discovery of radioactivity (1896)

February 14
March 1

³ <http://physics.nist.gov/cuu/Constants/index.html>

⁴ http://pdg.lbl.gov/2000/contents_sports.html