2004-2005 Physics Olympiad Preparation Program

– University of Toronto –

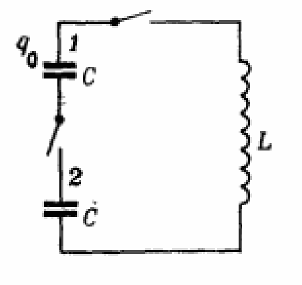
This is the last chance to try yourself in the creation of your own problem or experiment. Name your self-made problems as "My Problems", and mail them with solutions on a separate sheet of paper as an attachment to the solutions of POPTOR problems.
Authors of the best problems will be awarded regardless of their POPTOR results

Problem Set 6: AC Circuits, Particles, and General

Due April 4, 2005

Problem 1 (was created by our last year participant and awarded in the competition for the best self-made problem).

A circuit consists of two identical capacitors with capacitance C and a coil with inductance L. Initially the switches were opened and the first capacitor was charged with the charge q_0. Another capacitor was not charged. After the switches have been simultaneously closed, the system becomes an oscillatory circuit, or LC circuit, where charge and current harmonically oscillate in time.



Find all parameters of oscillations of charges and current in this circuit.
Propose a mechanical analogy for this system.

Problem 2.

The electric field vector in the electromagnetic wave of frequency $\omega = 2\text{x}10^{16}\ \text{s}^{-1}$ amplitude-modulated with frequency $\Omega = 2\text{x}10^{15}\ \text{s}^{-1}$, changes in time according to the equation:

$$E = E_0[1 + \cos(\Omega t)]\cos(\omega t)$$

where E_0 is a constant.
The wave knocks out electrons from the atoms of hydrogen with the ionization energy of W = 13.5 eV. Determine the energy of these electrons.

Problem 3.

In 1923 Arthur Holly Compton in Chicago discovered one of the most famous phenomena that proved the particle-wave dualism for all natural objects. It was found that the scattering of photons from electrons could be explained by treating photons as point-like particles with energy hf ($h = 6.63\text{x}10^{-34}$J is Planck's constant; f is a photon's frequency) and momentum hf/c (c is speed of light in free space). Such scattering may be considered a collision, in which the energy and momentum of the isolated system of the photon and the electron are conserved. The Compton shift equation gives the change in the photon's wavelength after scattering on any particle, and the new direction of photon after the scattering:

$$\lambda' - \lambda_0 = \frac{h}{mc}(1 - \cos\theta),$$

where λ' is the changed wavelength of a photon; λ_0 is the initial photon's wavelength; m is the mass of the particle; θ is the angle between the changed and the initial direction of the motion of a photon (angle of scattering). The coefficient $\frac{h}{mc}$ is called the Compton wavelength of the particle. For electron it is equal to 0.00243 nm.

Find the wavelength of the x-rays before scattering on the motionless electrons if the maximum kinetic energy of the electrons after the Compton scattering is 0.19 MeV.

Problem 4.

In 1923, in his doctoral dissertation, Louis Victor de Broglie postulated that, ***because photons have wave and particle characteristics, perhaps all forms of matter have wave as well as particle properties.*** De Broglie suggested that material particles of momentum $\boldsymbol{p}$ should also have wave properties and a wavelength of

$$\lambda = \frac{h}{p} \text{ (the de Broglie wavelength),}$$

where h is Planck's constant. The frequency of a particle is $f = E / h$. The further numerical experiments with electron diffraction proved the de Broglie hypothesis.

A beam of electrons accelerated by the potential difference $V = 25$ V is normally incident on a two-slit barrier with the slit separation of $d = 50\ \mu$ m.

Find the distance between two adjacent maximums of the diffraction pattern on the screen which is separated from the barrier by l =100 cm.

Problem 5.

The carbon dating is a method to date organic samples using the present ratio of two carbon isotopes: ^{14}C to ^{12}C in the carbon dioxide molecules. In living organisms this ratio is about $1.3\text{x}10^{-12}$. After the organism dies, it no longer absorbs ^{14}C from the atmosphere, and so the ratio of ^{14}C to ^{12}C decreases as the result of the beta decay of ^{14}C, which has a half-life of 5730 years. There are two different formulae for the same radioactive decay law:

$$N = N_0 e^{-\lambda t} \quad \text{or} \quad N = N_0 \cdot \left(\frac{1}{2}\right)^{\frac{t}{T}}$$

where N is the number of undecayed radioactive nuclei in the time interval t; N_0 is the initial number of nuclei; $\lambda = ln(2)/T$ is a decay constant; T is called a half-life, it is the time it takes half of a given number of radioactive nuclei to decay.

The decay rate R is called an activity. It is a derivative of a number of nuclei with respect to time:

$$R = \left|\frac{dN}{dt}\right| = N_0 \lambda e^{-\lambda t} = N\lambda = R_0 e^{-\lambda t}$$

R_0 is the decay rate at $t = 0$.

The technique of radioactive carbon dating has been successfully used to measure the ages of many organic relics up to 25 000 years old.

A piece of charcoal of mass 25.0 g is found in the ruins of an ancient city. The sample shows a ^{14}C activity of 250 decays/min.

How long has the tree from which this charcoal came been dead?