2005-2006 Physics Olympiad Preparation Program

– University of Toronto –

We continue the competition for the best problems created by our POPTOR contestants. Name your self-made problem as "My Problem", and mail this problem and its solution on a separate sheet of paper as an attachment to the solutions of the POPTOR problems. You may send us any number of problems, but only self-made and unique will be considered. Authors of the best problems will be awarded regardless of their POPTOR results.

Problem Set 2: Mechanics

Due December 5, 2005

Problem 1

A heavy disk with radius R is rolling down hanging on two non-stretched strings wound around the disk. The free ends of the strings are attached to the disk. The strings are always tense during the motion. At some instant, the angular velocity of the disk is ω, and the angle between the strings is α (fig.1.1).
Find the velocity of the disk at this moment.

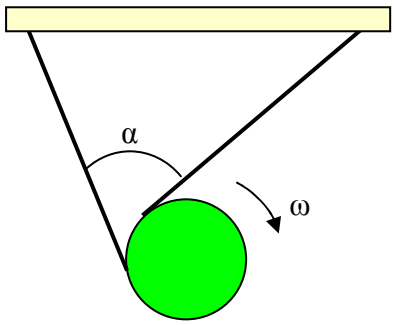


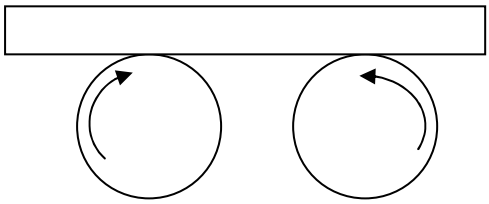
Fig.1.1

Problem 2

A spacecraft with mass m_0 is moving with constant velocity v_0 in free space. To change its direction the maneuvering jet engine is turned on, which starts to eject fuel at a constant velocity $\boldsymbol{u}$ relative to the spacecraft. During the maneuver, vectors $\boldsymbol{v_0}$ and $\boldsymbol{u}$ are always mutually perpendicular. Finally, the mass of the spacecraft becomes m.
Find the angle between the initial and final direction of motion of the spacecraft.

Problem 3

A horizontal plate with mass M lies on two rotating cylinders with equal angular speed of rotation, but opposite directions of rotation. The distance between the axes of cylinders is l. The coefficient of kinetic friction between the plate and the material of the cylinders is μ.
(a) Prove that the plate initially at the equilibrium position will start the harmonic oscillation if it is slightly displaced from the equilibrium in horizontal direction.
(b) Find the frequency of the oscillations.
(c) What is the result of the same displacement if the angular velocities have opposite directions?



Problem 4

A solid cylinder with mass M and radius R rotates around its axis of symmetry with initial angular speed ω_0. Then this rotating cylinder is carefully put with its lateral side on the horizontal plane. The coefficient of kinetic friction between the cylinder and the plane is μ.
Find:
1) the time interval in which the cylinder is rotating with slipping;
2) the total work of the force of friction.

Problem 5 (experimental)

Brief Theory.
The Hooke's law in the case of elongation of a uniform wire can be written as:

$$\frac{F}{A} = Y\frac{\Delta L}{L_i}$$

where F is an applied force; A is a cross-sectional aria of the wire; Y is the Young's modulus of the material of the wire; ΔL is the increase in the length under the force applied; L_i is the initial length of the wire. Sometimes, you can hear that the Young's modulus is stress over strain, because F/A is called the stress, and $\Delta L/L_i$ is called the strain.

Equipment:
1) A 2-m long and about 1 mm thick steel wire or any other material, which Young's modulus can be found in the available handbook; the Young's modulus for steel is $2.00\text{x}10^{11}$Pa;
2) An object of known mass (about 200g);
3) A tape measure;
4) A ruler;
5) A caliper or any other tool that permits you to measure the diameter of the wire.

Experiment:
Stretch the wire in horizontal direction and firmly attach the ends to some stable support. It can be a vertical wall or ceiling or something else.
Attach the object of a known mass to the center of the wire and carefully release it.
Measure the displacement of the object in the vertical direction. You must be sure that the distance between the ends of the wire has not changed.

Calculations:
With given L_i, Y, diameter of the wire d, and mass of the object m calculate theoretically the same value of displacement.

Compare the two values and briefly discuss the difference.