2007-2008 Physics Olympiad Preparation Program

University of Toronto

Problem set 1: General

Due: November 06, 2007

1. Consider a hollow sphere of radius R. From the point (A) on the horizontal diameter you throw a little steel ball, aiming to get to the opposite side (B) – see Figure 1 for the setup. (N.B. you may throw the ball below (as shown, $\alpha_0 < 0$) or above ($\alpha_0 > 0$) the horizontal line AB.)
(i) The first task is to reach B without touching the sphere, in just one jump from A to B. The initial speed of the ball is v_0 and can be varied. Find the angle(s) α_0 (as a function of v_0) that gets the job done.

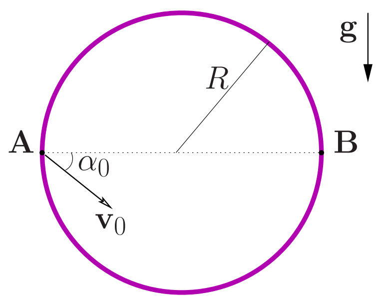

Figure 1: Illustration to problem # 1.

(ii) Take $\alpha_0 = 0$. If you want to reach B in one jump as before, what would be the value of v_0? Explain.
(iii) With $\alpha_0 = 0$, on the way to B you are now allowed to bounce off the sphere *once*. Find v_0.

In this problem assume the motion to be *two-dimensional* only, the gravity field $\mathbf{g}$ is uniform, the friction is neglected, the collisions ball – sphere are perfectly elastic, and the steel ball is modeled as a material point.

2. You are given a black box, with only 3 terminals to the outer world; let's call them A, B and C, as depicted in Figure 2. The box, you are told, contains one or more resistors and one or more

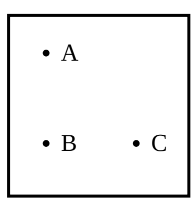

Figure 2: Black box electronics

semiconductor diodes (presumed to be ideal); your task is to figure out the inner circuit.

First, just in case, you check with the voltmeter that indeed there is no *emf* source inside the box. Second, you measure the resistance between terminal pairs A-C, C-B and B-A, respectively, using the ohmmeter section of the multimeter. Next, you perform one more set of measurements, this time testing C-A, B-C and A-B (do this by inverting the polarity of the ohmmeter test leads). The measurements are summarized in Table 1.

Set 1	$+/-$	$-/+$
A–C	8.0	∞
C–B	∞	5.5
B–A	∞	∞

Table 1: Black box measurements – part 1. The notation $+/-$ and $-/+$ means, for example for the A-C pair, that in the former case A was connected to the positive test lead, and C to the negative one, and the opposite in the latter case. The numbers are resistance values in kiloohms ($1\text{k}\Omega = 10^3\Omega$).

It doesn't look particularly encouraging, does it? Four out of six measurements are ∞!

But you don't want to give up, do you? The next thing you do is to short-circuit A and B and measure the resistance between the (AB) "terminal" and C, then short-circuit B and C and measure the resistance between the (BC) and A, and the same for the (CA) and B. The new data set is presented in Table 2, with the same convention on the $+/-$ $(-/+)$ notation as in Table 1.

Set 2	$+/-$	$-/+$
AC–B	∞	5.5
CB–A	∞	8.0
AB–C	4.8	∞

Table 2: Black box measurements – part 2. The numbers are resistance values, in $\text{k}\Omega$.

Draw a simple circuit that gives you the measurements presented in Tables 1 and 2. Provide the value(s) of the resistance(s) with two significant digits. Is your solution unique? Explain.

3. Imagine you have a direct current of constant intensity I_0 that flows through a rectilinear wire of constant cross section, as shown in Figure 3. Along the segment A-B the resistivity *increases* from ρ_A at the section A-A to ρ_B at the section B-B. What is the absolute value and the sign of the charge that forms along the segment A-B?

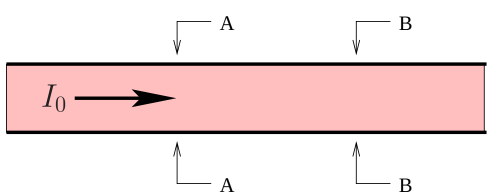

Figure 3: Illustration to problem # 3.

4. Consider a flat mirror resting on the table top, and a biconvex thin lens of focal distance $f_0 = 10\,\mathrm{cm}$ that is placed on the mirror as shown in Figure 4(a).

(i) Take a bright point-like source (e.g., a LED) and place it at distance $\ell_1 = f_0$ above the lens (on the optical axis). Where does the source's image form?

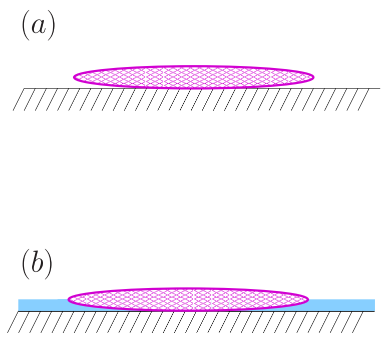

Figure 4: Illustration to problem # 4.

(ii) Add a layer of water on the mirror. The depth of the water is such that the bottom half of the lens is immersed, as depicted in Figure 4(b). If the light source is now placed at distance $\ell_2 = 15\,\mathrm{cm}$ above the center of the lens, its image forms at the same height as the source itself.

Now add water on the mirror just enough to cover the whole lens. At which height do you have to place the source such that its image coincides with the source itself?

Hint: For closely packed thin lenses their optical powers (reciprocal of the focal length) add up.

5. A planet of mass M and radius r is surrounded by an atmosphere of constant density that consists of a gas of molar mass μ. The height h of the atmosphere is much less than the radius of the planet, $h \ll r$, and the planet is far away from any stars.

Find the temperature of the atmosphere at the surface of the planet.

Last revised: November 13, 2007