2007-2008 Physics Olympiad Preparation Program

University of Toronto

Problem set 2: Mechanics

Due: December 03, 2007

1. The return of the hollow sphere: Consider a vacuumed hollow sphere of radius R that has a bulb placed at its center. The bulb explodes, and the scattered fragments fly away in all directions (i.e. isotropic 3-D initial distribution) with the same initial speed v_0 (see Figure 1 for the setup). The collisions with the interior of the wall of the sphere are perfectly inelastic (the fragments get stuck in the wall). The motion is 3-dimensional.

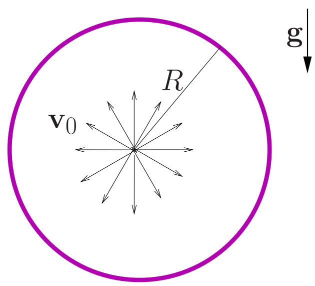

Figure 1: Explosion of a bulb. Illustration to problem #1.

What fraction η of the fragments gets trapped in the lower hemisphere? Plot η as a function of v_0.

2. 1-D jumping: Imagine a setup as depicted in Figure 2, with two identical cylinders (or "doughnut-pucks" if you wish), located initially at the heights h and $2h$, respectively, above the table top. The acceleration of the free fall is g. At $t = 0$ the cylinders are released simultaneously, and fall toward the table.

(a) Find the height the top cylinder reaches after the *second* collision with the bottom cylinder.

(b) Show that the maximum height (h_{max}) the top cylinder may reach (after an arbitrary number of collisions) is in any case not larger than $3h$. Generalize this last result for the top cylinder (the N-th) in a setup as above, but with N cylinders ($N \geq 3$).

Do you think the top cylinder can actually reach the h_{max} that you just calculated? Explain.

Assume that the friction is negligible, the motion is 1-dimensional and the rod is sufficiently long; the collisions cylinder–cylinder and cylinder–table are perfectly elastic, and the height of the cylinders is negligible.

3. A bumpy ride: A cube of mass $m = 1\,\mathrm{kg}$ is placed on a rugged table top. The static coefficient of friction table–cube is $\mu_0 = 1$, while the kinetic friction coefficient is $\mu = 0.9$. A spring of elastic constant $k = 1\,\mathrm{N/m}$ is attached to the cube. The spring is pulled horizontally in such a manner that its free end moves with constant velocity $v = 1\,\mathrm{m/s}$ (see Figure 3).

Describe the motion of the cube.

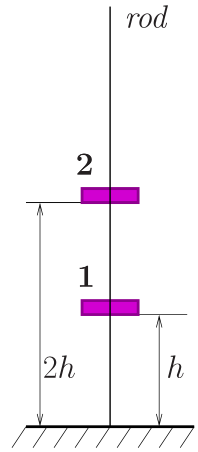

Figure 2: Jumping in 1-D. Illustration to problem #2.

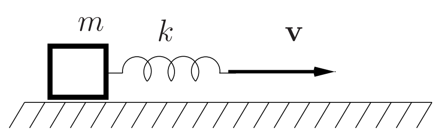

Figure 3: Illustration to problem #3.

4. **A star is born:** According to one of the cosmological theories, the stars were formed from interstellar dust by gravitational accretion. Estimate the time interval τ it takes to form a star according to this recipe.

The density of the dust is $\rho = 2 \times 10^{-17}\mathrm{kg/m^3}$, the gravitational constant is $G = 6.67 \times 10^{-11}\mathrm{Nm^2/kg^2}$. You may assume that the dust particles do not overtake one another during the accretion process, and that initially the dusty region was a homogeneous spherical cloud.

Hint: use dimensional analysis to get an idea of the functional dependence of τ on G and ρ, then use Kepler's laws to get the correct proportionality coefficient.

5. **Thinking outside the box:** An urban legend tells us that once upon a time a student named Niels Bohr was asked during an examination to describe how he would use an aneroid barometer to measure the height of a tall building.

Here are just a few of the methods he proposed for measuring the height h.

1. Tie the barometer to a rope (or a string!) and lower it from the top of the building. Measure the length of the rope (& add barometer's height) $\rightarrow h$.

2. Drop the barometer from the top of the building and clock the time it takes to reach the ground. Then use the free fall theory $\rightarrow h$.

3. Hang the barometer on a rope to form a pendulum. Measure the period of the oscillations at the ground and at the top of the building. The intensity of the gravity field g varies with

height, therefore the two periods will be different $\rightarrow h$.

4. If the sun is shining measure the length of the shadows of the building and of the barometer, measure the height of the barometer. Based on proportionality arguments $\rightarrow h$.

5. Measure the pressure p at the ground level and at the top of the building. The difference Δp allows one (surprise, surprise) to estimate h.

You may of course add to this list of exotic methods your own proposals. (I took the above ones from http://www.snopes.com/college/exam/barometer.asp). However, for the current problem let us stick with the above five methods.

Rank the *accuracy* and the *precision* of the measurements conducted with these methods. Which one would give you the overall best (worst) estimate of h? Explain.

Does your ranking change if in one case (A) you measure the height of a modest high-rise building (say, about $\sim 40\text{m}$), and in another case (B) you measure the height of a skyscraper (say, about $\sim 400\text{m}$). Explain.

Notes and hints:

1) It is assumed that you do the (hypothetical) experiment by yourself, without assistance. For example, it is you who drops the barometer *and* measures the time of the fall. Of course, this may affect the repeatability of your result and introduce supplementary errors.

2) You will have to do a bit of research on your own, to come up with reasonable specifications for the objects involved in these "thought" experiments (air, rope, barometer, meter tape, stopwatch, etc. are all to be considered *real life* objects).

3) If you wonder what an "aneroid barometer" actually is, look it up on the internet.

For a smooth start to your search visit http://www.bom.gov.au/info/aneroid/aneroid.shtml

4a) The accuracy and the precision errors are different types of errors. They both contribute to the total error of the measurement.

4b) Some suggested reading on error analysis:

A short introduction to the error analysis (designed for the first year university students) is available at:
http://www.upscale.utoronto.ca/PVB/Harrison/ErrorAnalysis/index.html

At the time of writing this problem set Wikipedia had a nice discussion on accuracy and precision:
http://en.wikipedia.org/wiki/Accuracy

A short reading on the accuracy vs. precision saga is available at:
http://www.ivstandards.com/tech/icp-ops/part14.asp

Last but not least, a classic textbook on error analysis is by John R. Taylor, *An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements*, 2nd ed. (Univ. Science Books, 1997), ISBN: 0935702423 (hardcover), ISBN: 093570275X (paper). Perhaps you don't really need this textbook to solve this problem, but it is definitely a "must read" if you plan on doing any experimental (and not only) research in the future.

Last revised: November 08, 2007