2007-2008 Physics Olympiad Preparation Program

University of Toronto

Problem set 4: Optics and Waves

Due: February 04, 2008

We continue the competition for the best original problems created by POPTOR contestants. Name your self-made problem as "MY PROBLEM" and mail it along with its solution on a separate sheet of paper, along with your solutions of the POPTOR problems. You may send us any number of problems, but only the self-made unique ones will be considered. The authors of the best problems will be awarded during the POPTOR Weekend.

1. On a transparent solid glass globe a spider is stalking a fly. Where on the surface of the globe should the fly locate itself, so that the spider cannot see it? Assume the radius of the globe is much larger than the size of the insects. The refraction index of glass is $n = \sqrt{2}$.

Hint: Assume for the beginning that the spider's eyes are located at some height H above the surface of the globe. Then take the limit $H/R \to 0$.

2. A laser beam propagates through a spherically symmetric media (see Figure 1). The refraction index varies with the distance to the symmetry center C by the law

$$n(r) = n_0 \frac{r}{r_0}, \tag{1}$$

where $n_0 = 1,\ r_0 = 30\,\mathrm{cm}, r_0 \le r < \infty$. The beam's trajectory lies in the plane that includes C. At distance $r_1 = 80\,\mathrm{cm}$ the beam makes and angle $\varphi = 30^\circ$ with $\vec{r}_1$, as shown in Figure 1.

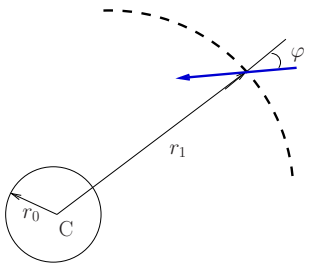

Figure 1: Illustration to problem #2.

What minimal distance does the beam reach relative to the symmetry center C?

3. Joe encountered a dust storm during his trip to the U.S. South, two years ago. He took a sample of the airborne dust, and found that the dust particles were opaque, and had a radius $r_1 = (5 \pm 1)\,\mu\mathrm{m}$. The mass of dust per cubic meter was $m_1 = (12 \pm 1)\,\mathrm{g}$. In Joe's recollection, the visibility at the time of the storm was $d_1 = (50 \pm 10)\,\mathrm{m}$.

Joe went back to the South last year, and ended up in yet another dust storm. This time he found that the radius of the dust particles was $r_2 = (10 \pm 3)\,\mu\mathrm{m}$ and the visibility $d_2 = (20 \pm 5)\,\mathrm{m}$.

However, Joe didn't have the "dust catcher" with him, therefore the mass of dust per cubic meter, m_2, could not be measured. Please, fill in the blank for Joe (i.e., determine m_2).

4. Construct the image of the square through a convex lens (see Figure 2). The bottom of the square lies on the optical axis, and the middle of the square's side is located at the focal point.

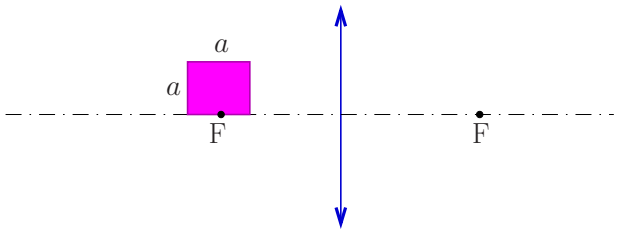

Figure 2: Illustration to problem #4.

5. To measure the speed of some small particles in suspension in a flowing stream one may use an interference scheme, as depicted in Figure 3. A parallel laser beam with $\lambda = 0.63\,\mu\mathrm{m}$ falls

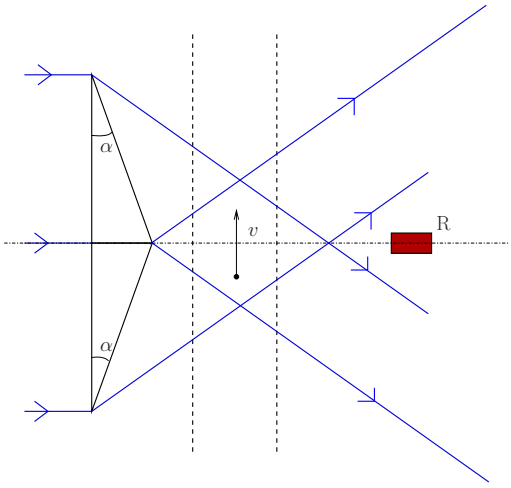

Figure 3: Illustration to problem #5.

onto two identical glass prisms, mounted as shown, base-to-base. The vortex angle of each prism is $\alpha = 5.7°$, and the refraction coefficient of the glass is $n = 1.5$. After the refraction through the prisms the beam splits into two; the refracted beams subsequently pass through the fluid of interest. The light is scattered on the particles traveling with the fluid, with the speed v. Find the speed of the particles if the current through the photodiode (R) used to record the scattered light experiences oscillations of frequency $f = 10\,\mathrm{kHz}$.