2007-2008 Physics Olympiad Preparation Program
University of Toronto

Problem set 6: AC Circuits and general
Due: March 31, 2008

The Canadian Association of Physicists (CAP) High School Exam takes place on **Friday, April 11th**.
To be selected for the National Finals, the successful participation in the CAP Exam is essential.
Please visit http://www.cap.ca/edu/HSPrize/2008HighSchool.html for details

The 2008 POPTOR Weekend takes place during the weekend of May 16 – 18, 2008

With this problem set you have the last chance to participate in the 2007-2008 competition for the best problem created by the POPTOR contestants. Name your self-made problem as "MY PROBLEM" and mail it along with its solution on a separate sheet of paper, along with your solutions of the POPTOR problems. You may send us any number of problems, but only the self-made unique ones will be considered. The authors of the best problems will be awarded during the POPTOR Weekend.

1. A voltmeter may be constructed using a parallel plane capacitor, in which one of the plates is fixed and the other one may move in the direction perpendicular to the plane of the plates. A spring of stiffness k is attached to the moving plate as shown in Figure 1. The proxy for the applied potential difference is the change in the distance between the plates. What is the maximum potential difference ΔV that can be measured with this device? The surface area of the plates is S, the initial distance between them is d (when $\Delta V = 0$), and the dielectric is dry air ($\varepsilon = \varepsilon_0$). Pick your own realistic values of the relevant parameters and provide a numerical estimate for ΔV_{max}.

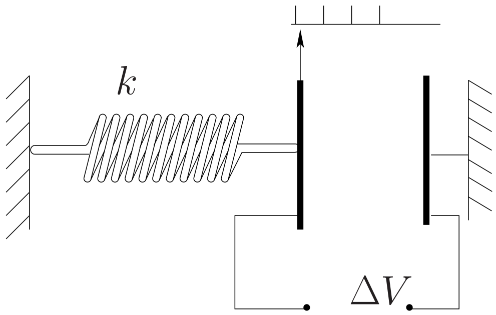

Figure 1:

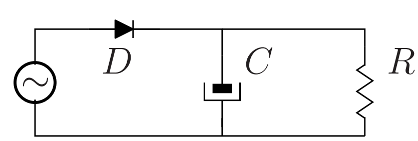

Figure 2:

2. The circuit of a half-wave rectifier (one-diode rectifier) is shown in Figure 2. To achieve a less "bumpy" voltage output, a capacitor of $C = 1000\,\mu$F is connected in parallel to the resistive load $R = 1.0\,\text{k}\Omega$. The frequency of the (sinusoidal) power source is $f = 60\,$Hz, and you may assume the diode is ideal.

(i) Determine the magnitude of the voltage pulsation coefficient $k = \Delta U/U_0$ on the resistor R. Here U_0 is the maximum amplitude of the voltage on R. Plot the voltage on the resistor, $U(t)$, for several periods of the alternating current.

(ii) To further reduce the pulsation of the output voltage an induction coil of inductance $L = 100\,\mathrm{H}$ is connected in *series* to the resistor (with the capacitor still in place). Find the coefficient k in this case. The ohmic resistance of the coil can be neglected.

By how much did k decrease in case (ii) compared to case (i)?

If you have access to an oscilloscope, you can easily check experimentally whether the formulae you derived provide a realistic estimation.

3. An electric battery may be constructed using a β-radioactive element (for example tritium, http://peswiki.com/index.php/BetaVoltaics). In this problem the "battery" consists of a metallic sphere, with a small radioactive sample plated to the end of a conducting rod (the rod is insulated from the sphere), such that the sample is positioned in the centre, as shown in Figure 3. Every second there are ν radioactive decays. The energy of the emitted electrons is uniformly distributed in the interval (W_{min}, W_{max}).

What is the EMF of the battery? What is the largest value of current one can obtain with this battery? For which values of the external resistive load the battery behaves as a *current* source (rather than voltage source)? With tritium as the radioactive material, use realistic values of the relevant parameters to provide a numerical estimate for the EMF.

4. In the circuit shown in Figure 4, how does the potential difference between the points A and B depend on the value of the variable resistor (R)?

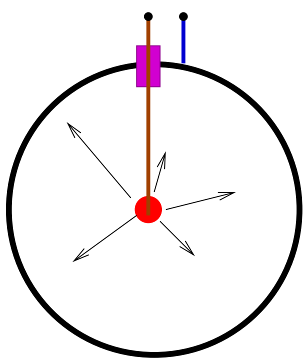

Figure 3:

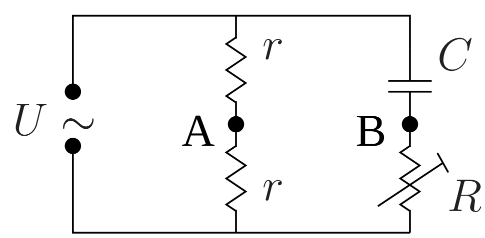

Figure 4:

5. A metallic ball of radius r is moving with constant velocity $\mathbf{v}$ in an uniform magnetic field $\mathbf{B}$. Find the points on the ball for which the potential difference is maximum, $\Delta\varphi_{max}$. Find $\Delta\varphi_{max}$. The angle between $\mathbf{B}$ and $\mathbf{v}$ is α. For $\alpha = \pi/4$ pick your own realistic values for the rest of the relevant parameters and provide a numerical estimate for $\Delta\varphi_{max}$.

Last revised: March 3, 2008