2008-2009 Physics Olympiad Preparation Program

University of Toronto

Problem set 5: Electricity and Magnetism

Due: 2009 March 16

1. An electric dipole consists of two charges +q and –q, separated by a distance b. The dipole moment **p** of such a system has magnitude q • b and direction pointing from the negative to the positive charge.

   a) Obtain an expression for the electric field at a location y along the axis of the dipole where y >> b, in terms of y and **p**.

   b) A water molecule, because of its triangular shape in which the hydrogen atoms attach to the oxygen with an angle of 104.5°, has a dipole moment of ~6.2 x 10^{-30} C m. Calculate the field strength at a point 0.1 μm away from the centre of the water molecule along its symmetry axis on the oxygen side. Calculate the force on a free, positive (charge = +*e*) lithium ion in this location. What would be the resulting acceleration on this ion?

2. The high voltage electrode in an electrostatic accelerator can be approximated by a hollow conducting cylinder 0.5 m in diameter and 1 m long. This electrode is housed in a cylindrical steel pressure vessel which is concentric with the electrode, 1.2 m in diameter and significantly longer than the high voltage electrode.

   a) If the power supply for the accelerator maintains the high voltage electrode at 3 million volts, and the pressure vessel is well grounded, calculate the charge on this electrode, disregarding any end-of-cylinder effects.

   b) What is the capacitance of this system and the energy stored in it?

   c) The reason for enclosing the high voltage in a pressure vessel is so that an insulating gas, under pressure, can be used to minimize the occurrence of corona discharge and even sparks from the high voltage electrode. The gas frequently used is sulfur hexafluoride (SF_6), which at a pressure of 6 bar is 10 time more effective as an insulator than very dry air. Calculate the maximum electric field in the system (hint: use Gauss' Law). Assuming that 3 million volts is nearly the maximum field strength that the SF_6 can sustain in this configuration, at how high a voltage could the electrode be operated in air?

3. In a mass spectrometer, a beam of negative ions (charge -1*e*) from a sample of carbon is accelerated through a potential difference of 20 kV. The beam of ions then enters a magnetic field where the central trajectory follows a circular path through 90° with a radius of 40 cm.

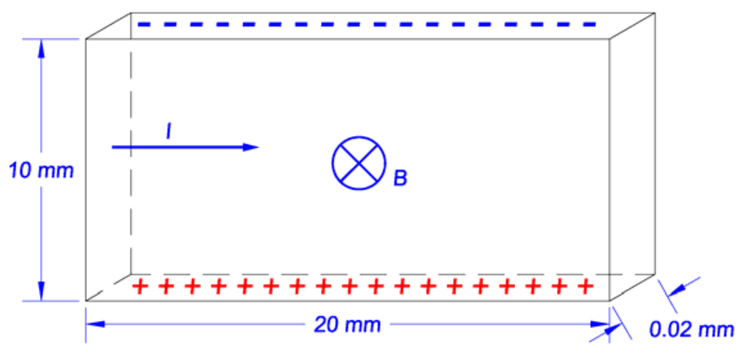
   a) Calculate the velocity of each of the isotopes of carbon that you are likely to find in the sample. What is the magnetic field necessary to bend the heaviest of these isotopes?

   b) Assuming that the pole face of the magnet provides a uniform field over a large enough area and that the pole edges are perpendicular to the central trajectory, if the field is not changed, how far away from the central trajectory does the lightest isotope emerge from the magnet?

4. When an electrical current is passed through a strip of conducting (or even somewhat resistive) material, with a magnetic field perpendicular to that strip, a voltage difference appears between one side of the strip and the other. This is known as the Hall effect and results from the Lorentz force on the charge carriers in the conductor (or resistor).

   a) You are given a thin strip of a silicon semiconductor 20 mm long by 10 mm wide by 0.02 mm thick, but do not know whether it is doped as n-type (uses electrons as charge carriers) or p-type (uses holes as carriers – i.e. positive charges). As shown in the sketch below, a current of 100 mA from left to right and a magnetic field of 0.1 T directed into the page produce a negative voltage on the upper side of the strip. What type of material is the semiconductor?

   b) From the expression for the Lorentz force, find an expression for E in the direction across the strip. Assume that there is one charge carrier per 10^8 Si atoms in the silicon strip. Using this, calculate the voltage observed. (Hint: the velocity in the Lorentz force equation is the carrier drift velocity.) Can you think of any other applications of this effect?



Infobits:

Mass of Lithium atom   6.94 amu

1 amu = 1.66×10^{-27} kg

$\varepsilon_0 = 8.854 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$ or $F\ m^{-1}$

$\mu_0 = 1.257 \times 10^{6} = 4\ \pi \times 10^{7}$ H / m

Elemental charge e = 1.602×10^{-19} C

1 eV = 1.602×10^{-19} J

For a cylindrical capacitor, $\Delta V = 2\,k\,\lambda\,\ln(r_o / r_i)$, where r_o and r_i are the radii of the outer and inner electrode, k is Coulomb's law constant, $8.988 \times 10^9\ N\ m^2\ C^{-2}$ and λ is the linear charge density on the cylinder.

Lorentz force: $\mathbf{F} = q\ \mathbf{v} \times \mathbf{B}$

Energy stored in a capacitor $= Q^2 / 2C$

Current density $\mathbf{J} = n\ q\ \mathbf{v}_d$ where n is the density of charge carriers, q is the elemental charge and $\mathbf{v}_d$ is the drift velocity of the carriers