1994-1995 Physics Olympiad Preparation Program

– University of Toronto –

Problem Set 2: Mechanics

Due November 28, 1994

1. A crude estimate of the depth of a well can be obtained by dropping a stone down it and listening for the splash, then using $s = gt^2/2$. You are asked to make first-order estimates of the corrections due to

   (a) the finite speed of sound ($v_s = 340\,\mathrm{m/s}$), and

   (b) air drag.

   Which of these is the bigger correction for $s = 30\,\mathrm{m}$ and a stone of radius $r = 2\,\mathrm{cm}$ and density $3 \times 10^3\,\mathrm{kg/m^3}$? Would your conclusion be significantly different if a table tennis ball were dropped in the same way ($r = 2\,\mathrm{cm}$, $m = 1.7\,\mathrm{g}$)? You may assume that the splash would still be audible!

   Note: the drag for a sphere of radius a depends on the dimensionless Reynolds number $R = \rho va/\eta$, where ρ is the fluid density and η is its viscosity. For $R < 10$, Stokes' law $F = 6\pi r\eta v$ is valid, but for $R > 100$ the drag force is given by $F = C_D \rho\pi r^2 v^2$; the variation of C_D with $\log_{10} R$ is given in the diagram. For air at 20 ℃, $\rho = 1.3\,\mathrm{kg/m^3}$ and $\eta = 1.8 \times 10^{-5}\,\mathrm{kg/ms}$.

2. Your phys-ed teacher is a student of the physics of human motion, and asks you the following questions:

   (a) When told that the world record for the pole vault was about 5.5 metres, the fast rising athlete Rod Fibreglass told the press, "Give me a pole long enough, and I will raise the record to 9 metres." Could he manage it? How high might he raise it if he tried hard?

   (b) I obtained the following data one Sunday:

      i. There are 238 steps from the basement to the 10^{th} floor of our building.

      ii. After measuring four steps, I believe the step height is $18.2 \pm 0.3\,\mathrm{cm}$.

      iii. I jogged up the steps in 1 minute and 40 seconds, give or take 2 seconds. Well, actually started off jogging and ended up walking up with a not-so-steady pace.

      iv. My mass is $82.07 \pm 0.2\,\mathrm{kg}$. The uncertainty is due to the fact that I drank some water, ate a banana, sweated and urinated in the time between doing the climb and measuring my mass.

      v. The value of g measured by some geophysicists at the base of the north stairwell is $9.804253\,\mathrm{m/s^2}$. They don't give an uncertainty estimate. Note that they say that g varies with height as $\Delta g/g = -2\Delta R/R$, where[1] $R = 6371\,\mathrm{km}$.

   How much (useful) work did I do? What was the average power output? Calculate the uncertainties in these values as well.

   (c) It is proposed to make a human-powered helicopter with a rotor 10 m in diameter. Assuming that the rotor blows a cylindrical column of air uniformly downwards, the cylinder diameter being the same as the rotor diameter, and that the mass of the pilot plus machine is 200 kg, calculate the minimum mechanical power (in Watts) that is necessary for the pilot to generate to remain airborne. Is the system practicable? Compare with my power output above. The density of air is $1.23\,\mathrm{kg/m^3}$.

3. NASA has just contracted you to make the following calculations, you lucky dog. As with many NASA contracts, you are being paid a handsome sum with bonuses for correctness. No $10,000 toilet seats, please.

[1] I don't know why this number, posted in our undergraduate labs, disagrees with others quoted here (6.38×10^3 km), except to say that the Earth isn't a sphere.

(a) A small moon of mass m and radius a orbits a planet of mass M while keeping the same face towards the planet. Show that if the moon approaches the planet closer than $r_c = a(3M/m)^{1/3}$, then loose rocks lying on the surface of the moon will be lifted off.

(b) It is well known that in an orbiting space vehicle the occupants can drift around freely in "zero-gravity" conditions. Assume that you are in a (strong) spaceship, 100 m long and fairly narrow, which is in a circular orbit of 1000 km radius around a neutron star, with its long axis always pointing towards the centre of the star. There is an inspection tunnel running down the axis of the ship.

   i. What would happen if an astronaut attempted to float down it?

   ii. Calculate likely values of the acceleration observed, assuming that the mass of the star is 3×10^5 Earth masses and that the radius of the Earth is 6×10^6 m.

   iii. Is an orbit such that the axis of the craft always points towards the star a stable condition? Explain briefly.

(c) Two stars in a binary system have a separation $2r$ and equal masses m, and move in circular orbits about their centre of mass. One star explodes by expelling a small fraction of its mass very rapidly, and immediately after its recoil speed is v_f. What is the largest value of v_f for which the two stars will remain gravitationally bound?

4. So, Minnesota, you think you know your billiards? Try to make this trick shot. A billiard ball of radius a rests on a table. It is hit with a cue in such a way that it starts out with speed u_0 and backspin ω_0 about a horizontal axis perpendicular to the direction of motion. How does the subsequent motion depend on the ratio $u_0/a\omega_0$? Discuss.

5. It is sometimes stated that it would be possible to construct a spaceship using a photon drive, which would be able to travel away from Earth with a speed close to that of light. Assuming that the fuel consists of equal masses of protons and antiprotons, does one get a greater final velocity

   (a) by allowing half the fuel to annihilate and using the energy released to eject the remaining half, or

   (b) by allowing all the fuel to annihilate and ejecting photons?

   You may use the relativistic relationship between energy and momentum for a particle of rest mass m_0, $E^2 = (pc)^2 + (m_0c^2)^2$ and for a massless photon, $E = pc$, where E is energy, p is momentum, and c is the speed of light.

6. You are on board the USS Enterprise when an artificial structure known as a "ringworld" is discovered circling a star similar to Sol. It is a large circular ribbon-shaped object with the following characteristics. The flat side is illuminated by the star; its radius is that of Earth's orbit, $R_{SE} = 1.50 \times 10^{11}$ m. The width is 1.00×10^9 m and the thickness is 100 km. The edges of the interior "floor" are lined with walls which have a triangular cross-section, are 1000 km high and have a base width of 100 km. This keeps in an earth-like atmosphere. Data is off-duty now so Captain Picard asks you the following.

   (a) If the floor is to have the same "gravity" as the earth, what rotational period should it have? Before calculating, give some limits on the period based on simple facts about the Earth.

   (b) Calculate the tensile strength needed for the material in order for it to remain intact. You may assume the density of the material is that of aluminum, $2.70 \times 10^3\,\mathrm{kg/m^3}$. For comparison, aluminum's tensile strength is about $2 \times 10^8\,\mathrm{N/m^2}$.

   (c) Remember, there is a star in the centre of the ring. Is the spinning ring stable with respect to the star? That is, if we were to push the ring a bit, would it return to its original position (like a marble in a bowl) or would it move further out of position (like a marble on a beach ball)? You may answer qualitatively.