1994-1995 Physics Olympiad Preparation Program

– University of Toronto –

Problem Set 4: Waves and Optics

Due February 6th, 1995

1. One dark and stormy night you find yourself lost in the forest when you come upon a small hut. Entering it you see a crooked old woman in the corner hunched over a crystal ball. You are about to make a hasty exit when you hear the howl of wolves outside. Taking another look at the gypsy you decide to take your chances with the wolves, but the door is jammed shut. Resigned to a bad situation you approach her slowly, wondering just what is the focal length of that nifty crystal ball...

   (a) If the crystal ball is 20 cm in diameter with $n = 1.5$, the gypsy lady is 1.2 m from the centre of the ball, where is the image of the gypsy in focus as you walk towards her?

   (b) Describe the image of the crooked old lady, and be more quantitative than just telling me she is hideous.

   (c) The old lady moves the crystal ball closer to her wrinkled old face. At some point you can no longer get an image of her, which at this point is no small blessing. At what object distance will there be no image of the gypsy formed?

2. If any of you people end up going into fysix, you will study about Young's two slit experiment until you are just about ready to yak. Just to magnify the extent of any future yakking, here we go... The idea of the experiment is to take a wave and pass it through two slits, and then observe an interference pattern on a screen (see figure #1). The figure uses a laser but the experiment can be done with any wave-like entity such as electrons or water waves.

   (a) What pattern is observed on the screen if the laser has a wavelength λ? The answer should be expressed as the intensity of the interference pattern at the screen in terms of the variables x, h, D and λ, where x is the distance from the middle of the screen. You will need to make two assumptions to solve this problem, namely $D >> x$ and $x >> h$. Start with an electric field at the two slits that looks like

   $$E = E_0 e^{i\phi}$$

   where $i = \sqrt{-1}$ and $\phi = \text{pathlength}/\lambda$ is the phase of the wave at the slits.

   I will give some quick hints on how to handle complex numbers. We introduce complex numbers in problems like this to make the math easier to handle. At the end of the problem we get rid of the complex parts of the equations to give us a real answer. Some rules when handling complex numbers:

   i. $e^{i\phi} = Cos(\phi) + iSin(\phi)$

   ii. $e^{i\phi} \times e^{i\delta} = e^{i(\phi+\delta)}$

   iii. Intensity $= E \times E^*$

   iv. $E^* = E_0 e^{-i\phi}$ is the complex conjugate

   (b) In the diagram the laser source is centred with respect to the two slits. What changes in the pattern would occur if the laser was shifted such that the waves going into the two holes were π radians out of phase?

   (c) This theoretical pattern goes on forever in both directions along the screen, whereas the experimental pattern fades. What causes this fading of the experimental interference pattern?

3. You are conducting a feasibility study into the SDI (Strategic Defense Initiative) missile defense system. You are to consider the viability of a ground based laser defense against ICBM's, which will use powerful CO_2 lasers to bore holes in the incoming missiles, thus destroying them. The lasers have an output power of 50 W in a beam diameter of 1 mm. The laser beam is fired at the missile when it is 10 km away and the beam loses 3% of its intensity for every kilometer travelled. The outer skin of the missile is aluminum that is 3 cm thick. When the laser fires the skin temperature of the missile is $-50\,^{\circ}C$ and must be heated to its boiling point at $2500\,^{\circ}C$. The density of aluminum is $2.34\,\mathrm{g/cm^3}$ and heat capacity is $0.9\,\mathrm{J/g\,^{\circ}C}$.

   (a) How long will it take the laser to burn through the outer skin of the missile, thus destroying it? Assume that all of the laser power that reaches the missile goes into heating the 1 mm diameter spot.

   (b) What angular accuracy must the aiming system of the laser have in order for the laser not to drift away from the target spot?

   (c) Suggest three ways that one could use to protect a missile from this laser defense system.

4. We have all seen those really cool holographic images of our favourite cartoon characters on the cereal boxes, not that I still watch cartoons anymore, well occasionally I see the odd one, actually on Saturday... anyway, back to the problem. Being a physics nerd, and proud of it by the way, I could never just enjoy those pictures without trying to figure out how they work. I will now inflict my geeky inquisitiveness on all of you, heh heh heh.

   (a) Holograms are 3-D images that change depending on the viewer's perspective. How is the dimensionality of the image captured on the holographic plate? How is this different than normal 2-D film?

   (b) Sketch an experimental apparatus that could be used to produce a hologram. Be sure to explain what is going on.

   (c) When normal film is exposed to light and developed, a negative is produced. It is called a negative because the dark and light areas are interchanged by the developing process. When light is shone through the negative onto photographic paper a positive image is produced, which then ends up in your photo album. In holography it makes no difference whether the holographic plate ends up being a negative or positive. Why is this?

5. Now for a really dull question, for which I humbly apologize; it won't happen again. There are two thin lenses of focal lengths f_1 and f_2 separated by a distance d. In general, the focal length of a system of lenses will be different depending on which side the image is on. For systems with more than one lens we define a back focal length and a front focal length, which are just the two focal lengths for these two possibilities.

   (a) Find the back focal length and the front focal length of this system of lenses.

   (b) Something nifty happens when $d = f_1 + f_2$. What is it?

   (c) Find the focal length of the system for $d = 0$. Does it matter which side of the system the image is on in this limit?

6. Now for a few small questions that will really get you wondering why you are doing these Olympics in the first place. Just give qualitative answers.

   (a) I'm sure most of you have been on trips up into northern Ontario and seen the Northern Lights first hand. What causes the Aurora? Why are they strongest in Northern Canada?

   (b) We were all kids at one time or another (except maybe Pekka) and probably sung Twinkle Twinkle Little Star. What makes stars twinkle? Do the planets and moon twinkle also?

(c) If you are walking on the beach one day and find a seashell, and just happen to hold it up to your ear, you will be able to hear the roar of the ocean. What causes this?

(d) If lightning strikes a tree, it can either be left unharmed or be completely shattered. Why will some trees be destroyed and others be fine? It turns out that Oak trees are preferentially destroyed over other species. Why?