1996-1997 Physics Olympiad Preparation Program

— University of Toronto —

Problem Set 4: Optics and Waves

Due February 14♥, 1997

1) Jesse's 'anti-music'

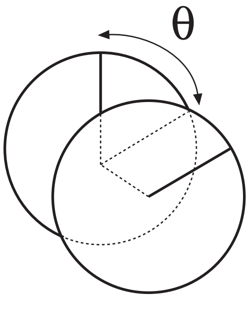
Jesse has a problem with his roommate Bongo. Bongo's stereo is way TOO LOUD! In fact, Bongo has already blown his right speaker, and is pumping his Too-Much Music over the left one. So Jesse decides to fight sound with sound. He places an identical speaker of his own, on the left channel of the *same* station, face-to-face with Bongo's and 6.0 m away. Jesse then sits with his one (remaining) good ear 1.5 m from his own speaker, on a direct line between the two. For the following, you can assume that Jesse and Bongo are pumping out exactly the same signal, in amplitude and in phase, and that Jesse's head doesn't interfere with the sound. The speed of sound in his apartment is 331.45 m s^{-1}.

a) At what frequency(ies) in the music would Jesse hear minimum sound at his position?

b) Are there points along the line joining the speakers at which Jesse cannot hear sound of *any* frequency?

c) What ways might Jesse set up his speaker, better to cancel Bongo's racket? *[Chairul]*

2) A new twist on light



Like any vector, the electric field of linearly polarized light can be considered as the sum of two components. A linear polarizer allows through only the component that is aligned to its 'axis', and absorbs the other component. This results in Malus' law, which states that the light transmitted by a polarizer is: $I = I_o \cos^2\theta$, where I is the transmitted light intensity, I_o is the incident light intensity, and θ is the angle between the axis of the polarizer and the input $\vec{E}$ field direction.

a) Why does Malus' law involve $\cos^2\theta$, not $\cos\theta$?

b) Kimberly wanted to verify Malus' Law. She shone a flashlight on a photodetector and measured 1.0 mA of current generated by the detector due to the light intensity. She inserted a linear polarizer between the flashlight and the detector and measured the current. What was this value?

c) She inserted a second polarizer and rotated it to minimize the current generated by the photodetector. What was this current, and what was the angle between the polarizers' axes?

d) On a whim, she placed a third polarizer after the first two and aligned it with its axis 45° to the first one. Without changing the orientation of the first two polarizers, she measured the current. What current did she measure? She exchanged positions between the second and third polarizers. What was the new current generated?

e) As a result of her findings in section (d), she decided to extend her studies. With the first and last polarizers aligned perpendicularly, she inserted two polarizers aligned at 30° and 60° to the first one. What current did she measure? Write down the general expression for the amount of current generated for a total of N polarizers, all aligned at $90°/(N\text{-}1)$ to each other. For really big N, what do you expect this value to be?

f) For large N, Kimberly effectively rotated the polarization of the beam by 90°. What problems would you face if you attempted part e) in real life? What is a much easier way to produce vertically polarized light from a horizontally polarized beam, using only a few standard mirrors? *[James]*

3) People see through Claire

Claire (*not* her real name) plans to smuggle a tiny glowing radioactive pellet out of Ukraine. She safely implants the pellet in the centre of a solid transparent cube of index of refraction $n = 1.75$, but realizes the glow of the pellet will give her away. She decides to paint the cube black, but knows that Customs officers will be suspicious of such an obvious concealment.. Therefore she wants to paint only some parts of the surface.

a) What is the least surface she must paint — and in what shape?

b) Unfortunately, at Customs she accidentally drops the cube in a glass of water and the inspectors catch her. What is the smallest surface she should have painted? (The index of refraction of water is 1.33.) *[Chairul]*

4) Nothin' but blue sky...

A plane wave of light has a wavelength λ. It scatters off a small sphere of radius $a << \lambda$. The light scattering off of the sphere causes the charges in it to oscillate and thus induces an electric dipole moment p in the sphere. p increases as a function of the volume of the sphere, $p \propto a^3$. Oscillating dipoles radiate electromagnetic waves, so that the intensity of scattered radiation I_{scatt} from such a sphere scales as the square of p: $I_{scatt} \propto p^2$.

If we know the intensity of the incident wave, I_o, we should be able to calculate the intensity of the scattered wave I_{scatt} as a function of the distance R from the sphere. In particular, we want to know how the scattered intensity varies with the wavelength of the incident radiation. Note that the three length-dependent variables in the problem are λ, R, and a.

a) By conservation of energy, the amount of radiation flowing through any spherical shell at any distance R from the sphere must be a constant. How does I_{scatt} scale with the distance R from the scattering sphere?

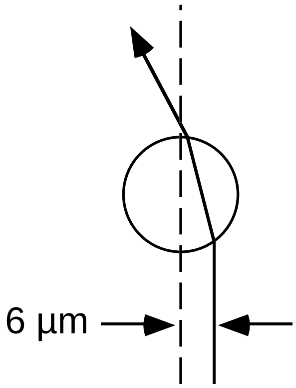
b) The ratio I_{scatt}/I_o is dimensionless. As a result, how must I_{scatt} depend on λ? Use the dependence of I_{scatt} on the other length-dependent variables (which you know from above) to obtain your answer.

c) The situation above can be used to model the sky, where visible sunlight of all wavelengths scatters off of water molecules in the air. The human eye has evolved so that it is most sensitive to that part of the spectrum for which the intensity of light given off by the sun is maximal. As a result, we are most sensitive to green and this sensitivity falls off as we move to either the red or violet ends of the spectrum. Use this knowledge and the result of part (b) to explain why the sky appears blue, rather than, say, green. *[Nipun]*

5) Just tweezing...

Focussed laser light can be used to push tiny spheres around. It is possible to coat these tiny spheres with biological antigens and make them glue themselves to particular polymer molecules, even the end of DNA strands. Then in moving the spheres with a laser, the polymer molecules can be manipulated, stretched, moved. Sometimes this is referred to as *optical tweezers*

The force doesn't come from light pressure, exactly, but through refraction. Consider a plastic sphere 25 μm in diameter and with an index of refraction $n = 1.4$. Visible laser light can be focussed to a spot a few micrometers across, through a microscope, so let's approximate the light by a single ray as illustrated, incident on the sphere at 6 μm off the normal-incidence axis.



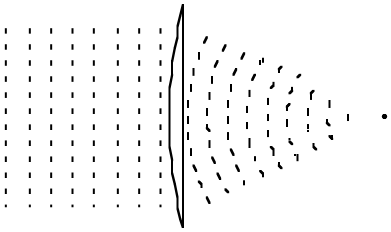
a) Find the approximate path of refraction through the sphere, and analyze the change in momentum for a photon of wavelength $\lambda = 530$ nm.

b) For a laser with power 1 W, what is the net force exerted on the sphere as it changes the momentum of all the photons in the beam? You can assume the sphere doesn't move *[Robin]*

Web reference: *http://www-leland.stanford.edu/dept/news/relaged/940509Arc4280.html*
(a sort of press-release on related work at Stanford University); perhaps also see 'Optical Levitation by Radiation Pressure,' A. Ashkin and J.M. Dziedzic, Appl. Phys. Lett. 19, p.182 (1971)

6) Photons get the bends

In one kind of description, lenses work by changing the curvature of the wavefronts of an incoming beam. When a plane wave passes through a positive lens, the flat wavefronts come out curved concave-forward. Then by Huygen's construction, it is not hard to see that the beam must collapse to a focus.



The speed of the wavefronts is the *phase velocity* $v_\phi = c/n$, where n is the index of refraction.

a) Consider a simple lens, flat on one side and with a spherical surface on the other, 50 mm in diameter, and 5 mm thick in the middle (see figure). If the glass has an index of refraction $n = 1.66$, find out how much the wavefronts in the middle of the beam are delayed in time relative to the part of the same wavefront that passes through the very edge of the lens.

b) What is the radius of curvature of the new wavefronts, and where do they come to a focus?

Something similar can happen in an ordinary flat block of glass when *very intense* light is incident on it. It turns out that the index of refraction depends also on intensity I, as:

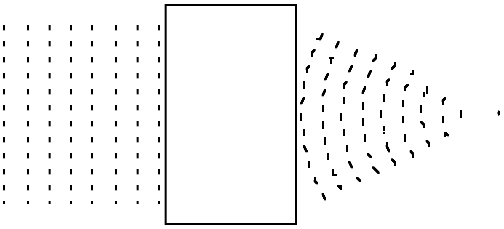
$$n = n_o + n_2 I$$

where n is the actual index of refraction, n_o is the ordinary index of refraction for low-intensity light, I is the intensity of light in W cm^{-2}, and n_2 is a constant equal to 5×10^{-15} W cm^{-2} for glass. Where light is very intense, it increases the index of refraction it encounters.

c) Consider light which has flat wavefronts as before, but is more *intense* in the middle than at the edges, with the distribution:

$$I \;\; [W\,cm^{-2}] \;=\; \begin{cases} 2 \times 10^{13} \left(-b + \sqrt{10^6 - x^2}\right) & for \;\; |x| \le 2.5 \\ 0 & else \end{cases}$$

$$where \;\; b \;\equiv\; \sqrt{10^6 - (2.5)^2} \;=\; 999.996875$$



Because the light is more intense in the middle, the wavefronts are delayed in the middle of the beam relative to the edge, and again the beam will come to a focus. For this intensity and for a block of glass 10 cm thick, find the approximate focal point of the beam. *[Robin]*