1996-1997 Physics Olympiad Preparation Program

— University of Toronto —

Problem Set 5: Electricity and Magnetism

Due March 7, 1997

1) Gauss visits Planesville...

The (very obscure) town of Planesville, Manitoba is much like any small Canadian town, except that the town is two-dimensional — it is lacking the third dimension that we experience everywhere else on earth. We want to see how Gauss's law might be different in Planesville. For an arbitrary charge distribution, Gauss's law takes the form of an integral, but recall that for a point charge Q, we can draw a gaussian sphere at radius R from the charge and use the radial symmetry to write Gauss's law as:

$$E_{sphere} \bullet \text{(area of sphere)} = Q/\varepsilon_o$$

a) If Coulomb's law applies in Planesville in the same form it does everywhere else, would Gauss's law still apply in its usual form? (Hint: Derive the form of Gauss' law in a 2-D world for a point charge.)

b) How could we 'fix' Coulomb's law so that Gauss's law would apply in Planesville? Write the expression for the new force law.

c) For the force law of part (b), consider a solid disc of charge of radius 10 cm and charge density 0.5 mC m^{-2}. What is the electric field of such a disc in Planesville as a function of the radial distance from the centre of the disc? Sketch a graph of this field as a function of the radial distance from the centre of the disc. *[Nipun]*

2) It's not just a good idea... it's Ohm's law!

Ohm's law for conductors can be written in the form $V = IR$ or as $\vec{J} = \sigma \vec{E}$, where $\vec{J}$ is the current per unit cross-sectional area flowing in a wire and σ is the electrical conductivity of the material. For our purposes, the important thing is that the current is proportional to the electric field $\vec{E}$, so a constant electric field produces a constant current. Note also that the current in a wire is proportional to the average velocity with which the charges are moving.

a) A single point charge Q is moving freely under the influence of a constant electric field $\vec{E}$. Is such a charge moving with a constant velocity, or is it accelerating? Does this charge obey Ohm's law?

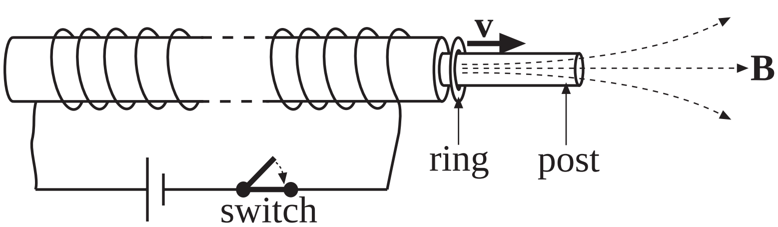
b) Now, instead of free point charges, we consider electrons flowing through a wire. Such electrons undergo occasional collisions with ions making up the metal. On average, an electron travels a distance *d* before colliding with an ion and stopping altogether. For a constant applied field $\vec{E}$, what is the average time between collisions? What is the average velocity of an electron? Does such a result agree with Ohm's law?

c) In part (b), we neglected the fact that the electrons (at room temperature) have a lot of thermal energy. Therefore, they are constantly jiggling about in random directions with a thermal velocity v_t. If this thermal velocity determines the time between electron-ion collisions (because v_t is much greater than the electron velocity due to any applied field), what is the new time between collisions? If we then apply an electric field $\vec{E}$ to the wire, what is the average electron velocity? Does this result agree with Ohm's law?

d) If the model of conduction in part (c) was accurate, what would happen to the current at very high and very low temperatures for a constant applied field $\vec{E}$? *[Nipun]*

3) Total *Recoil*

After seeing a recent Schwarzenegger film *ERASER*, Noah was impressed by Arnie's nasty rail-gun type weapon. The hand-held gun fired metal projectiles at near-light speed with rather destructive results. Noah decided to try to build one himself. He recalled a physics demonstration, in which a metal ring is placed on a metal post, attached to the top of a solenoid. The solenoid consists of a wire wrapped many times around a cylindrical metal core. When a sudden voltage was applied across the ends of the wire, the metal ring was launched away from the solenoid. Noah decided to make use of this effect to build his own rail gun.



Noah's first design contained the following components: solenoid (core wrapped with 2000 turns of wire, length = 50 cm, $\mu = 500\ \mu_o$), metal post (length = 5 cm, $\mu = 500\ \mu_o$), metal ring (inner diameter = 1 cm, outer diameter = 1.1 cm, thickness = 0.5 cm, resistance = $10^{-3}\ \Omega$, mass = 0.5 g).

a) With the metal ring anchored to the post so it could not slide, Noah turned on the voltage source and found that the current did not reach its maximum value immediately. He measured the current (in amperes) as a function of time and determined that it satisfied the following equation:

$$I = 1.0\ (1 - exp(-t\ /\ T))$$

with T = 0.04 s. What caused this time lag?

b) From a physics text, Noah determined that the magnitude of the magnetic field at the end of a solenoid is:

$$B = \mu\, N\, I \;/ 2l$$

where N is the number of turns of the solenoid, l is the length of the solenoid, and I is the current in the wire. What value of magnetic field does Noah generate, as a function of time?

c) Recalling Faraday's law of induction, what is the magnitude and direction of the current induced in the anchored metal ring as a function of time? You can assume that the current response of the ring is instantaneous.

d) If the solenoid creates a uniform magnetic field, pointing exactly parallel to the metal ring's axis, what is the Lorentz force ($\vec{F} = q\ \vec{v} \times \vec{B}$, where $\vec{v}$ = velocity of particle of charge q) the ring feels as a function of time and in what direction? If the ring weren't anchored to the post, what would be its resulting motion?

e) The magnetic field is of course *not* uniformly pointing parallel to the solenoid axis but starts to diverge as it exits the solenoid (as shown in the diagram). For simplicity, assume that at the fixed position of the ring, the field lines leave the post at 15° to its surface. What is the new net force on the anchored metal ring, as a function of time?

f) Noah turned off the voltage supply and removed the anchors so that the metal ring could slide freely on the post. He closed the switch again and the ring was projected forward. To get an approximate value for the resultant speed, approximate the force applied to the metal ring for arbitrary time t to be the same as calculated in part e) at time $t = T$. Use this constant force to get a rough idea of the velocity of the ring as it left the metal post.

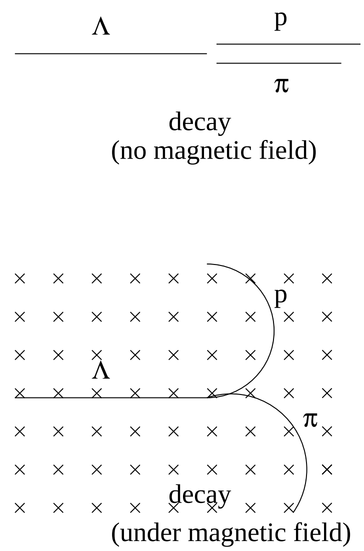
g) Noah decided to improve his design. In the movie, Arnie's gun was supposed to fire at near-light speed, but Noah didn't want to be greedy. He redesigned his device to launch the projectile with a velocity of 0.01 c (c = speed of light, 3×10^8 m•s^{-1}). Somehow Noah succeeded in building it and though it worked perfectly, Noah was seriously injured. What had he forgotten to take into consideration? *[James]*

4) The Lambda particle: outstanding in its ($\vec{B}$) field.

In the dawn of the field of particle physics, the discovery of the Λ^- particle played a significant role. It is relatively long-lived. One of the most important decay modes of this particle is the decay to proton and pion:

$$\Lambda \rightarrow p^+ + \pi^-$$

The minus sign for π denotes that the charge of this pion (π) is –e. Suppose the *free* kinematics of this decay is such that all particles before and after the decay move in the same direction (see figure). Now a magnetic field of 1 T, perpendicular to the motion of the particles, is applied to separate the moving proton and the pion. If the radius of the proton path is 3 cm, what is that of the pion? (rest-masses: $M_\Lambda = 1115$ MeV • c^{-2}, $M_p = 938$ MeV • c^{-2}, $M_\pi = 140$ MeV • c^{-2}, where c is the speed of light) This problem can be solved in a non-relativistic way (even though it will not be accurate), however, one should take into consideration the mass missing after decay, which will contribute to the kinetic energies of the proton and the pion. *[Chairul]*



5) Bohring after the truth

In the Bohr theory of the hydrogen atom an electron circles the proton in an orbit of radius 0.053 nm. The electrostatic attraction of the proton for the electron furnishes the centripetal force needed to hold the lectron in its circular orbit. Find, in this classical model:

(a) The force of electrical attraction between the proton and the electron.

(b) The speed of electron. What is this speed as a fraction of the speed of light c? *[Chairul]*

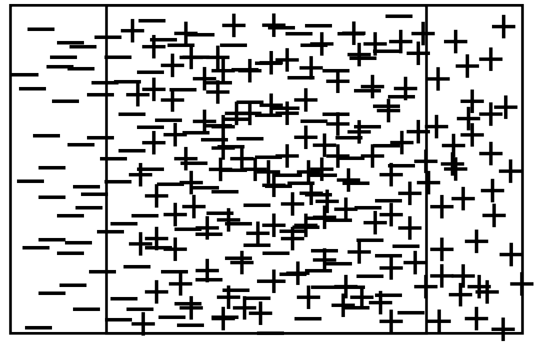
6) Plasma, plasma, on the wall

Practically all of the mass of the universe is not solid, not liquid, and not quite gaseous, but in the form of *plasma*. Plasma is any ionized matter, whether ionized by heating (as in the sun), by electric discharge (as inside a fluorescent light tube), or by dissociation in solution (as in blood).

When spacecraft re-enter the earth's atmosphere, the heat of re-entry produces an envelope or sheath of ionized air and ablating heat-shield surrounding the craft. The density of this plasma increases as the craft enters thicker atmosphere, and later dissipates as the vehicle slows down and cools. This plasma sheath can actually act like a mirror for radio-wave communications between vehicle and ground-control, reflecting the signals and blocking communication for a period during re-entry. A similar kind of reflection takes place for short-wave radio waves reflecting off the right part of the ionisphere. This happens because plasma is a good conductor and because plasma of a certain density has a characteristic resonant frequency, the *plasma frequency*.

Consider a hydrogen plasma, made of a mix of electrons and protons only, with equal density of electrons and protons. If in a region of plasma (as illustrated) we pull the electron component to the left, away from the proton component, it leaves two slabs of excess charge, like a parallel-plate capacitor.

a) Find the amount of charge in each charged slab, assuming a displacement x of the electrons relative to the protons, and a particle density of N cm^{-3} for the protons and the the electrons, each.



b) Find the force on the electrons and ions within this parallel-plate capacitor.

c) Show that this is a *restoring force* $F = -kx$, like a mass on a spring. Under the assumption that the protons have so much more mass that they hardly move, show that the force constant k leads to an oscillation frequency for the electrons sloshing back and forth given by:

$$\nu_p = \frac{1}{2\pi}\omega_p = \frac{1}{2\pi}\sqrt{\frac{Ne^2}{\varepsilon_o m_e}}$$

where N is the particle density (electrons or ions), e is the electron charge, and m the electron mass. This is the frequency of best reflection of electromagnetic waves by the plasma — any frequency lower than this is reflected.

d) Find the minimum density N of plasma needed to reflect shortwave radio at $\lambda = 30$ m. *[Robin]*