1996-1997 Physics Olympiad Preparation Program

— University of Toronto —

Problem Set 6: AC Circuits and Electronics

Due March 28, 1997

1) What's a gigawatt? Thirty of 'em??

The province of Ontario requires something like 3×10^{10} watts of electrical power (a plausible guess?: a kilowatt or so for every person, and then twice as much again for all of industry), at 110 volts AC (*r.m.s.*). A heavy power cable might be an inch in diameter.

Say the whole province was supplied by a single cable a metre in diameter — what would happen? Take it that the cable has a resistance of 0.15 $\mu\Omega\ \text{m}^{-1}$. Calculate:

a) the power lost per metre from 'I^2R losses,'

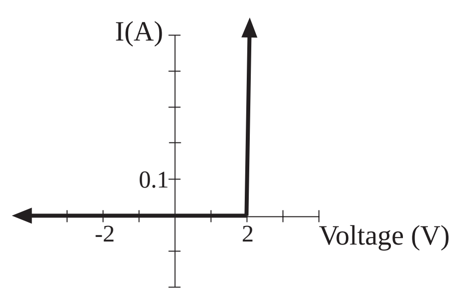
b) the length of cable over which the whole 3×10^{10} watts would be lost, and

c) how hot the cable will get, if it loses all this power by blackbody radiation.

Your answer ought to be preposterous — why is our electrical power system not ridiculous in this way? *[Robin]*

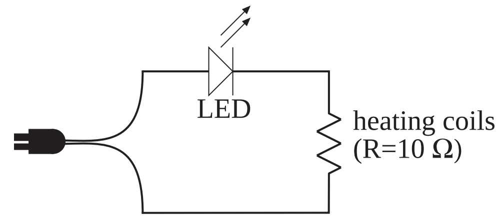
2) Waffling on the issues... the issue of waffles

A diode is an electronic device that allows current to flow only one way. An ideal diode has zero resistance for positive voltage and huge resistance for negative voltage. A slightly more realistic diode has a turn-on voltage, as shown in the figure at right. One typical use of a diode is as a light source. These light-emitting diodes (LED) produce visible light when a voltage greater than the turn-on voltage is applied across the device.



a) After many serious burns, Anton *(the Waffle Czar)* Duzzleeater pledged to improve current waffle-maker technology. He decided to incorporate an LED as an indicator that would light up whenever the waffle iron is plugged in. To begin, Anton decided to characterize his voltage source — normal household AC — so he measured the voltage as a function of time. Graph what Anton measured (2 cycles is sufficient). Don't forget to indicate your axis and scales.

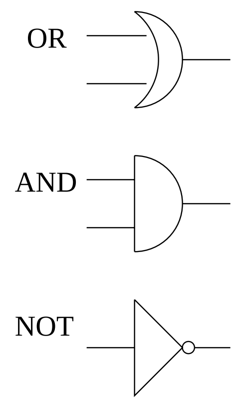
b) Without bothering to think about it, Anton wired the LED in series with the waffle heating element (schematic at right). If the diode had a similar $I–V$ curve to the one shown in figure 1, what would be the voltage drop across it as a function of time? Does the LED emit light? When?



c) Unfortunately for Anton as soon as he plugged the circuit in, the diode flashed a brief burst of light and began smoking. The diode couldn't take the excessive current. Draw a more intelligent circuit that would avoid this problem. *[James]*

3) 2B ∨ (~2B) ← Q! *[...to be or not to be, that is the question!]*

Wendy, a Physics Olympiad student, is given a bunch of digital gates. Unfortunately, she only has three types of gates: *AND*, *OR*, and *NOT*.



Her supervisor said to her that she can make any combination. Lo and behold, she finds

((NOT A) AND B) OR ((NOT B) AND A)

circuit is a very useful combination. Draw the circuit for that combination. Make a Boolean table for that circuit. Can you name that simple circuit? *[Chairul]*

4) Connect the dots!

Who said that physics wasn't all fun and games? See the figure on a separate page: the challenge is to connect the dots using the components given, to simultaneously achieve the desired voltage and current. Note that you may not need to use all the dots (or components) and you may want to attach more than two components to the same dot. The symbols 'A' and 'V' correspond to ideal ammeters and voltmeters. *[James]*

5) Impeding one's reactance

In a DC circuit, the total resistance is a sum of the individual resistances in the circuit, the nature of the sum depending on whether the resistors are in series or in parallel. In an AC circuit, the impedance, Z, plays the role of resistance and sums in the same way as resistance, however it has both real and imaginary parts. For ideal resistors, inductors and capacitors, the contributions to the total impedance are as follows:

resistors:	$Z_R = R$
inductive reactance:	$Z_L = i\omega L$
capacitive reactance:	$Z_C = (i\omega C)^{-1}$

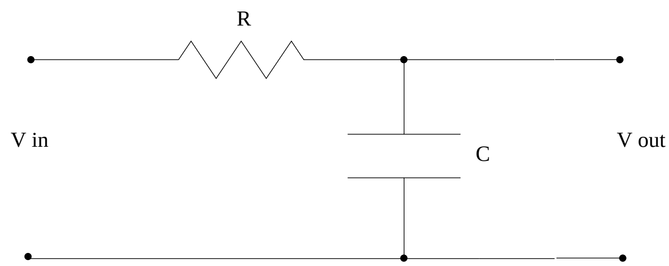
Here, $\omega = 2\pi f$, where f is the frequency of the input AC signal.

a) What is the total impedance of the circuit in the figure below?

b) We are interested in only the amplitude of V_{out}, which is given by

$$|V_{out}| = (V_{out} \bullet V_{out}^*)^{1/2}$$

where V_{out}^* is the complex conjugate of V_{out}. What is $|V_{out}|$ for this circuit? Sketch $|V_{out}| / |V_{in}|$ as a function of $\log_{10} \omega$.

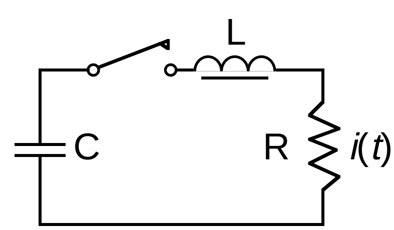


c) How would such a circuit affect a high-frequency input signal? A low-frequency signal? What do you conclude it might be used for? *[Nipun]*

6) Say '*cheese*'! (...Zap !!)

In high-power lasers pumped by a *flashlamp discharge*, large capacitors are charged to several kilovolts and then a fast, high-voltage switch closes to discharge them through the flashlamp. It is very much like an electronic camera flash, but hugely larger. These are the facts about the flashlamp circuit:

- the electric current pulse through the flashlamp must last for a certain amount of time τ in order best to 'pump' the laser
- if only a flashlamp and capacitor are connected in series, the current will oscillate back and forth through the lamp before dying away, and this can damage the lamp or cause it to explode
- the best shape results when the capacitor voltage is a simple decaying exponential
- the exponential should be the fastest-decaying one possible



To make these things possible, an inductor like a coil of heavy wire is added to the circuit, which can then be represented as shown at right: C – capacitor; L – inductor; R – resistance of flashlamp; $i(t)$ – time-dependent current through flashlamp.

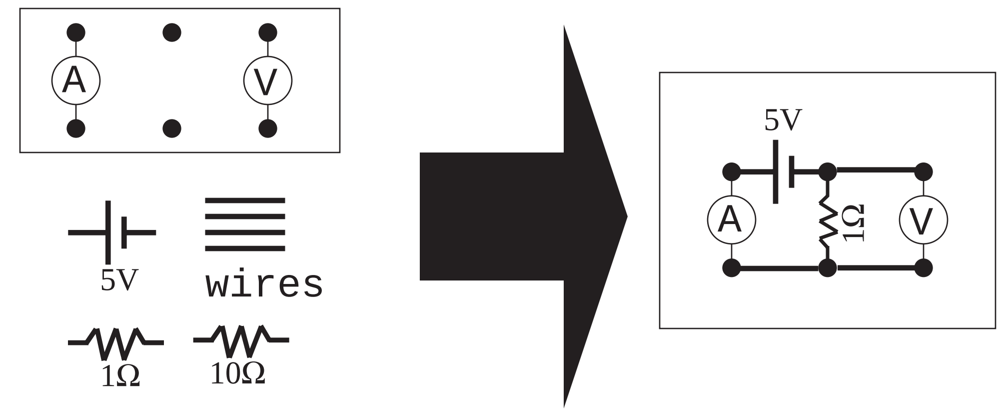
a) What is the equation for the charge q on the capacitor? What is the voltage across the capacitor in terms of the charge q on the capacitor? Show your approach.

b) Assuming the voltage is the fastest-possible decaying simple exponential, and that C and R are fixed, what must be the value of L? What is the exponential decay time, given $R = 1\Omega$, $C = 200\ \mu F$?

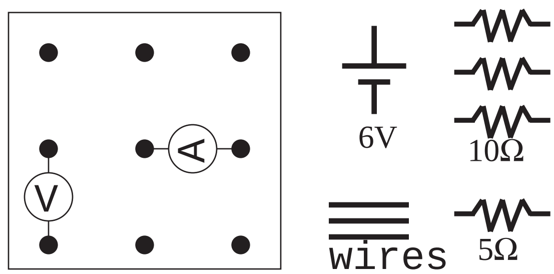
c) Find the current $i(t)$ through the resistor R. If the capacitor is charged to 2.5 kV, how much energy is discharged through the lamp? *[Robin]*

Figure for ‘Connect the Dots!’

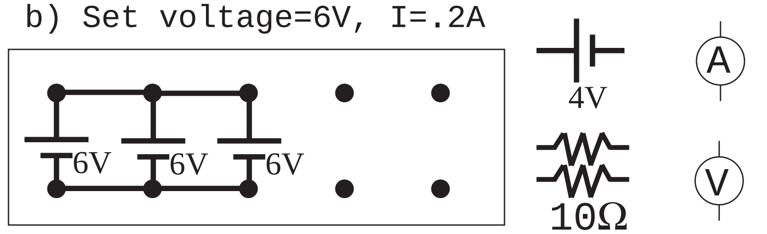
e.g.) Set voltage=5V, I=5A



a) Set voltage=4V, I=.2A



b) Set voltage=6V, I=.2A



c) Set current I=0A
(note: short-circuiting batteries is baaaaaad!)