1997-1998 Physics Olympiad Preparation Program

— University of Toronto —

Problem Set 1: General

Due November 3, 1997

1) Connect the dots!

Who said that physics wasn't all fun and games? See the figure on a separate page at the end of this set: your mission, should you decide to accept it, is to connect the dots using the components given, to simultaneously achieve the desired voltage *and* current. Note that you may not need to use *all* the dots (or components) and you may want to attach more than two components to the same dot. The symbols 'A' and 'V' correspond to ideal ammeters and voltmeters. *[James]*

2) Out with the bad air, in with the good...

In homes which meet the R-2000 standard of energy conservation, there are virtually no drafts which let in (cold) fresh air, and there might arise a risk of pollution-buildup from building materials, from cooking and cleaning, or from the occupants themselves. So for such houses, designers may include a heat-recovery ventilation system — basically a kind of heat-exchanger in which stale air exits the house but first passes much of its heat on to the incoming cooler air. Because the outgoing and incoming air streams are kept separate as they pass each other, but still allowed to exchange heat, fresh air enters the house fairly warm and stale air exits the house fairly cool. In fact, the efficiency is surprising.

It is a bit too complicated for POPTOR to calculate efficiency for a heat-recovery ventilation system, but consider this problem:

Say you have 1L of hot water (say, dyed red), of temperature 100ºC, and 1L of water (say, dyed blue), of temperature 0° C, presenting the warm and cool air above. Say also that you have many specially-insulated containers, any sizes you wish. These containers prevent heat loss to the air, but exchange heat perfectly when two (or more) touch each other. The question is like warming the incoming air in the house: without mixing the two together, to what temperature can you heat the initially cold (blue) water, and to what temperature can you cool the initially hot (red) water, by arrangements of letting them swap heat? *[Peter]*

[Canada Mortgage and Housing Corporation (CMHC) is a remarkable Canadian gold-mine of housing research and public information of every imaginable sort, including energy-efficient R-2000 houses, heat recovery ventilators, indoor pollution, etc. To order publications, check out: www.cmhc-schl.gc.ca/InfoCMHC/contact.html#Publications]

[Wonder what a heat-exchanger looks like? There are some photographs of heat-exchangers for industrial processes at: www.souheat.com/gallery.htm]

3) Waves à la mode

If you have ever sung in the shower, or maybe a marble washroom stall, you may have noticed that certain frequencies really pick up: they resonate within the space. For sound, the air molecules moving cannot move into the wall, so the walls are 'nodes' or zero-points for the molecular motion. Only certain-frequency waves naturally meet this condition, so they can be resonant within the space, and their reinforced amplitudes can be larger than that for other frequencies. These are 'standing waves,' and sometimes called 'modes.'

In making a laser, several items are brought together: a *laser cavity* (consisting of two mirrors facing one another) and an optical amplifying *laser medium* are the bare minimum. The laser cavity provides 'feedback' — a way to recirculate the light many times through the amplifying medium to make it more intense. Metallic mirrors also set a condition on the electric field in light: the transverse electric field must vanish at the surface of a conductor (see question below). Therefore, standing electromagnetic waves are formed in the laser cavity, which is for that reason sometimes called a 'laser resonator'.

a) Luciano Pavarotten is an operatic shower-singer, with a three-octave range starting at C below middle C (middle C is 256 Hz). You may not know that a note an octave above another has twice the frequency. If his shower stall is 1.5 m × 1 m × 2 m, how many standing wave modes in the shower can his voice excite? (Alas, only the standing waves are excited by his voice) How many different frequencies is this? You should ignore the issue of how his (large) body interferes with the sound in the shower.

b) Krystal has a Nd:glass ('neodymium glass') laser, which typically operates at about 1 micrometer (1 μm = 10^{-6} m) wavelength. It can amplify any wavelength within ±5 nanometers (5 nm = 5×10^{-9} m) of this wavelength. How many standing-wave modes of light are amplified in a laser resonator which is 80 cm long? *[Robin]*

4) Opposites attract, but a narcissist always loves himself...

Consider a grounded cylindrical conductor, of great length compared to its radius r. In orbit around this fat wire is a small charged particle of mass m and charge q. The distance from the point to the surface of the cylinder is $d << r$. Why should a charged particle orbit a grounded conductor? Explain the phenomenon, and find the period of the orbit. Neglect gravity, ignore air particles, and pay no attention to the man behind the curtain. *[Peter]*

<u>HINT:</u> For this quite-neat question, you may need to know these two things that you may *not* already know:

a) A perfect electrical conductor has *no electrical field inside*.

Not obvious? Say that there were a field inside: then the free charges in the conductor would see that field (and so a force), and would begin to move — any positive charges in one direction and any negative ones in the exactly opposite direction. If the conductor is of finite size, the charges can't keep on forever, but when would this whole thing stop? It would stop when the conductor ran out of free charges to move (phenomenally unlikely, for real-sized fields) or until the separated + and – charges made their own field which exactly cancelled the original field. The net result is zero DC field.

b) Therefore the surface of a perfect electrical conductor is a *surface of constant electric potential*.

If the field everywhere in the conductor is zero, then a small 'test' charge could be moved anywhere inside without seeing a force. Therefore the potential energy of the test charge is the same anywhere in the conductor— no force, no work, no potential energy change. So a charge a tiny distance inside the surface of the conductor is everywhere at the same potential energy: this means, then, that the surface of a conductor is an electrostatic equipotential.

5) Fermi Questions

The famous and fabulous Italian physicist Enrico Fermi was known for a curious type of question he sometimes posed his students. One of them was 'how many piano tuners are there in Chicago?'. The students were expected to work out the answer without resorting to any reference materials. The questions were an exercise in a very important problem-solving talent: developing the judgement that lets one make reasonable assumptions. With some notion of how many people are in Chicago, one might estimate roughly what percentage would have pianos, how much a piano-tuning costs, or how long it takes, etc. Here are some 'Fermi questions' to try. The idea is to use only what is already in your head, and to make plausible guesses for the rest. For each, give in brief point form the steps you followed in your reasoning. Estimates within a factor of 3 might be considered good, for some of these.

a) about how many piano tuners are there in Toronto? (Don't just check the Yellow Pages)

b) Claudio Chiapucci's bicycle has reflectors attached to the spokes of his wheels. He is fussy, and starts out each ride with the wheels set so each reflector is in the 12 o'clock position, at the top of the wheel, but they never stay that way. Roughly how long does he ride before they end up opposite each other, e.g., 10 o'clock and 4 o'clock?

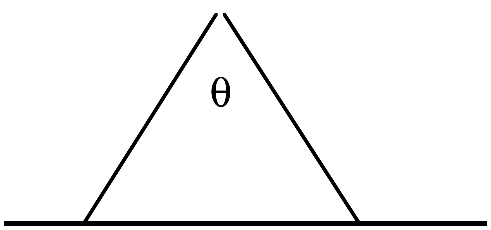
c) Jacques Villeneuve went shopping with his car one recent Saturday, but found his favorite small lot at Eaton's was full. He parked at the top of the lot, and from there he could see, and quickly get to, about fifty parking spaces. On average, about how long would he have to wait for someone to leave?

d) About how many pounds of cigarette-butts are dropped on sidewalks in Canada each year?

e) Half-time during an exciting Grey Cup football game: trips to the kitchen and washroom. How much, roughly, are the sudden increases in electrical power and water demand? *[Robin]*

6) 'Stacking the Deck — Friction in the House of Cards,' *by Kitty Kelly*

PART I – Consider the problem of making a structure out of cards. It's pretty difficult. Step One: consider the simplest problem of leaning two cards against each other on a flat surface. Let θ be the angle between the cards (we're assuming symmetrical stacking), μ_1 be the static coefficient of friction between a card and the smooth table-top, and μ_2 the coefficient between the two cards. I'm using a brand new Bicycle deck, so the cards are slippery and $\mu_2 < \mu_1$.



When two cards are stacked together at some angle θ, there is some minimum force F_{min} required in order to collapse them. Find out whether there is some optimal angle, θ_{best} — an angle at which the cards will be most stable (i.e., the *largest* F_{min}) — and determine that angle.

($\theta = 180°$, with the cards lying flat on the ground is stable, alright, but useless — that special case does NOT count!)

PART II – Suppose that you want actually to predict the value of the angle θ_{best}, using results from PART I. How do you get μ_1 and μ_2 — just look them up in the CRC Handbook of Playing Card Constants? *Doh!* You could measure it, you know.

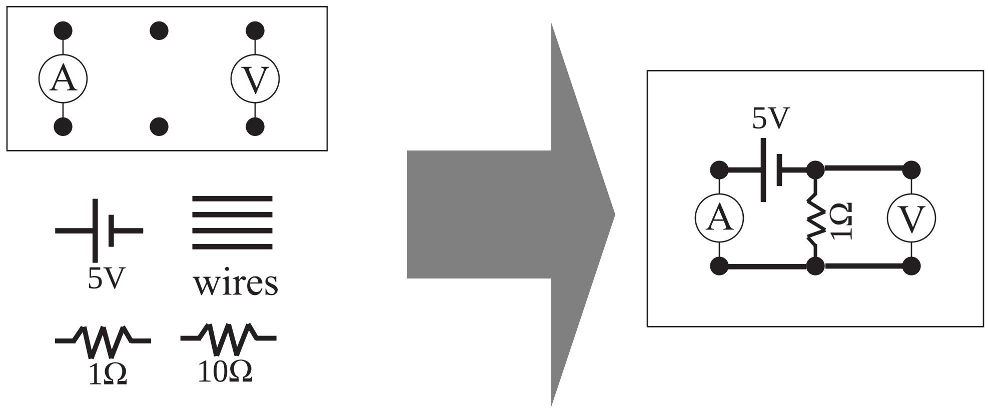
a) Find a deck of playing cards — a couple of baseball or hockey cards (or perhaps even computer diskettes) will do. For your cards, and your smooth table-top, measure μ_1 and μ_2. Describe what you do, and why, and figure out estimates of uncertainties about your measurement process. Give an error-range for your final answer; this is essential because it represents to others how closely they can trust your exact answer.

Note that everyone will (most likely) get a different answer — it's how you do it that matters for POPTOR.

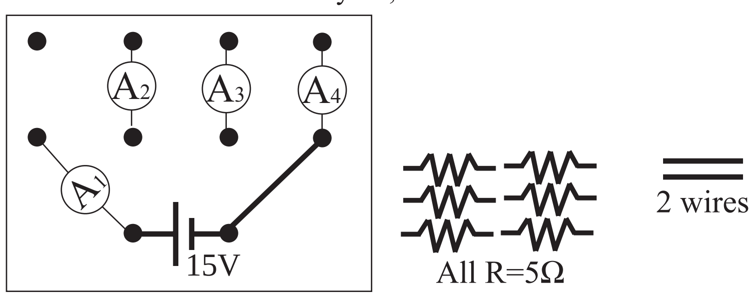
b) Now knowing μ_1 and μ_2, predict θ_{best}. Does it seem reasonable? Explain why. *[Peter]*

For Question #1:

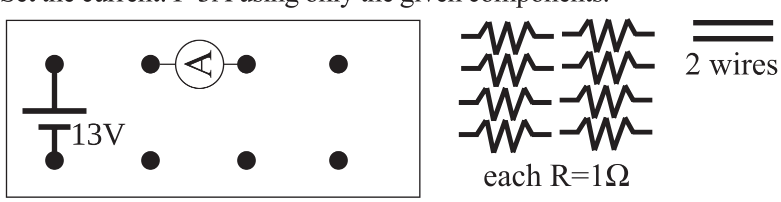
e.g.) Set the voltage: V=5V, current: I=5A using only the given components.



a) Set the current measured by A_1 to be I=2A, using only the given components.
What is the current measured by A_2, A_3 and A_4?



b) Set the current: I=3A using only the given components.



c) Set voltages: V_1=11V, V_2=7V, and V_3=5V using only the given components.