1997-1998 Physics Olympiad Preparation Program

— University of Toronto —

Problem Set 2: Mechanics

Due December 5, 1997

1) The Pluto Problem

As everybody knows, there are nine major planets in the solar system. Which is the farthest from the sun? Neptune. Thought it was Pluto? Not since 1969!

1969 was a 'remarkable' year for Pluto, the year when following its elliptical orbit led it inside the orbit of Neptune, pushing Neptune to the ninth position. Pluto's orbital period is 248 years and its orbit is a little eccentric, with major axis $a = 5.9 \times 10^9$ km and minor axis $b = 5.73 \times 10^9$ km. Consequently, its distance from the sun changes from $r_{min} = 4.4 \times 10^9$ km to $r_{max} = 7.4 \times 10^9$ km. Neptune, on the other hand, has an almost circular orbit with the radius $R = 4.5 \times 10^9$ km. As a result, for part of Pluto's orbit, spanning an angle $\theta = 100°$, Pluto is closer to the sun than Neptune.

When will Pluto go back to being the farthest planet from the sun? *[Lev]*

2) Free falling, at $4.50 a throw

For a few designers, there is good money in designing first-rate amusement park rides. At the same time, there might be huge lawsuits for any ride-designer who failed physics. For this bungee-jump, you can be the first one to try the ride:

a) Say that you are jumping from a height H = 25 m (6-storey building). Don't worry, you will be tied to a bungee-cord (elastic rope) whose other end is fixed to the jump-off platform. The length and elasticity for this ride are chosen so that you will not hit the ground (but only nearly!), so your speed at ground level will be exactly zero. After you quit bouncing up and down, you'll find yourself hanging at a height h = 10 m above the ground. Find your maximum downward velocity, assuming that air resistance is negligible. What is the answer if the height of your starting point is H = 50 m?

b) Show that if your speed is zero not at ground level but at a height $h_1 = 5$ m, there will be a slight increase in your maximum downward velocity.

c) Estimate the value of the elasticity constant (spring constant) k of the rubber rope designed for a person of m = 100 kg, both for H = 25 m and H = 50 m (people of less mass will be able to use this rope too). *[Lev]*

3) 'Bob's your uncle', or sometimes he's a *simple harmonic oscillator (SHO)*

Probably since the start of the legend that Archimedes streaked an ancient Greek city yelling 'Eureka! I have found it!,' teachers of physics have assigned questions relating to buoyancy. In this more-interesting-still question, there is a fellow floating upright in the ocean (call him Bob), whom we will roughly approximate by a weighted cylinder of circular cross-sectional area a, and height h. (sorry, Bob!). In our case, we'll ignore drag.

a) *Bob's equilibrium:* If the cylinder sticks into the water a certain distance, what is the buoyant force? Find Y, the equilibrium floating position of the Bob-cylinder, measured as the submerged length of cylinder. Use ρ_{water} for the density of water.

IF WE PUSH the cylinder a little deeper than its equilibrium position, the buoyant force upward is greater than the (constant) force of gravity downward on the cylinder, so there is a net force opposing the change. If we lift the cylinder out, the buoyant force is decreased, and the net force again opposes the change. This type of force is called a *restoring force*, since it acts to restore the cylinder to its original position.

b) *Bob's restoring force:* Find the value of the restoring force in terms of y, the small displacement from equilibrium. Define y to be positive when the cylinder is pushed down a little from equilibrium.

c) Letting go of the cylinder, this restoring force causes an acceleration a:

$$F = ma = m\frac{d}{dt}\left(\frac{dy}{dt}\right)$$

Find the relationship between Bob's displacement y and his acceleration d^2y/dt^2. What is the constant of proportionality?

IN GENERAL, for *any* such force of the form $F=-ky$, we can write

$$m\frac{d}{dt}\left(\frac{dy}{dt}\right) = -ky$$

and $y = y_o \cos(\omega t+\phi)$ is a solution of, where y_o, ω, and ϕ are constants.

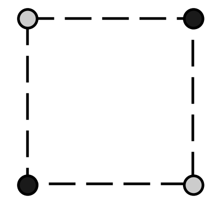
d) *Bob's oscillations:* What must be the value of ω to make $y = y_o \cos(\omega t+\phi)$ a good solution? What about y_o? What about ϕ? What is the value of ω for Bob?

e) Would the period be different if Bob were not in the ocean, but in a tank of cross-sectional area A, twice the area a of the cylinder? If so, write an expression for the period of oscillation for such a case, still ignoring drag. *[Nipun]*

4) Full of fury, and signifying nothing...

Quando M. McAnic finally finished his correspondence course in charged-particle design, and set about creating his first minimalist installations.

a) For his first effort, he set four particles — 2 electrons (e^-) and 2 positrons (e^+) — at the corners of a square of length a on each side. Like particles are placed diagonally opposite, as shown in the figure at right. *If* the electrons and positrons (the antimatter version of an electron) come into contact, they will annihilate each other and gamma ray photons will be emitted. Is Quando such a Nihilist?



b) For his second effort, Quando replaced the electrons (e^-) with protons (p^+). Find the ratio between the final speed of the protons and that of the positrons. *[Peter]*

5) Sikorsky meets Newton

Helicopters are fascinating things, having the ability to move forward, backward, to the sides, up and down, spin around and hover motionless. All this is achieved by varying the pitch, or angle of attack, of the helicopter's blades, and their rotational speed, using the *cyclic* and *collective* controls.

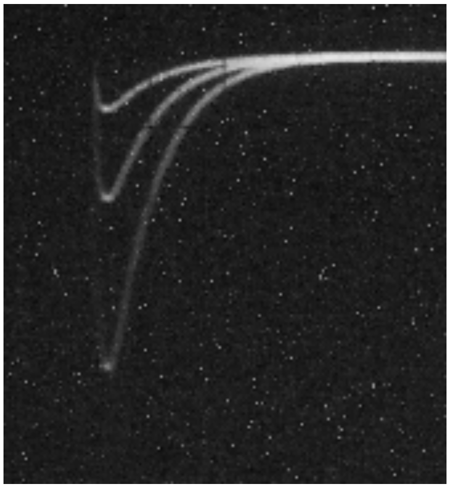
Consider the lift obtained from the blades: given that a helicopter having N rectangular blades (length a, width b, negligible height), each tilted at the same angle α to the horizontal, rotating steadily (why is this important?) at ω [rad/s], find the total lift (upward force) produced. The air density is ρ [kg/m^3]. State clearly all your approximations and explain your steps. You can model the air as made up of very small, non-interacting particles, initially at rest.

BONUS: Explain how the helicopter might move sideways <u>without</u> tilting the rotor's axis. *[Peter]*

6) Pulse-height analysis

Often, we want to know what the *distribution* of something is: how much of something in each of a number of categories. In physics, often what is wanted is an energy distribution — for a certain type of particle, how do the numbers of particles sort out by energy, say. One technique for this is to use a detector which detects particles one at a time, measuring their energy, and then to analyze the data afterwards to find the energy distribution. When the detector gives a pulse for each detected particle, and the height of the pulse is proportional to the particle's energy, the technique is called 'pulse-height analysis'.

a) The figure at left on the next page overlays three oscilloscope traces of different pulses from a detector (a photomultiplier with a NaI(Tl) scintillator), hit by gamma-ray photons emitted by a ^{137}Cs radioactive source. Opposite it is a data-set of 50 pulse-heights



measured from such pulses in an actual experiment. Plot the distribution of pulse-heights from this data, first specifying a reasonable-sized 'bin' or energy resolution for your distribution-plot. ^{137}Cs emits a gamma-ray of energy 0.662 MeV, but this gamma-ray interacts with the detector in several ways (*photoelectric effect*, *Compton effect*, and by *pair-production*), so your distribution may show more than just the single-peak spectrum of a 0.662 Mev gamma-ray.

24.5	53.0	82.0	80.5	14.0
76.5	56.2	26.7	5.0	80.2
25.8	22.3	80.9	21.5	48.3
10.8	78.6	7.5	83.0	81.1
82.0	32.3	18.5	40.0	21.0
77.0	79.6	43.0	29.5	15.5
13.0	9.0	78.5	38.0	80.3
63.4	82.2	83.6	84.0	30.5
16.5	19.2	35.0	6.0	45.3
80.0	33.5	28.0	79.3	80.7

b) Raindrops also can have a distribution, one of sizes, depending on the weather conditions, type of cloud, winds, etc. One way to measure the size of a raindrop might be to use a microphone, or even an audio speaker hooked up as a microphone. Due to drag from air resistance, raindrops reach a terminal velocity in falling. This terminal velocity depends on the raindrop size. When a raindrop hits the microphone, the larger ones hit with more momentum and a tape recorder or other detector records a bigger 'hit'. Assuming that the microphone records an impulse proportional to the momentum of a raindrop, what formula(s) will let you infer the raindrop size from the pulse height? Assume the droplet is a sphere (it isn't, quite, depending on size).

Spheres and Drag: the *Reynolds number* R_e is a dimensionless number which characterizes the flow of liquid around an object:

$$R_e \equiv \frac{\rho_f v d}{\eta},$$

where v is the fluid speed, d is the diameter of the object, η is the viscosity of the fluid and ρ_f is the fluid density.

$\eta_{air} = 1.8 \times 10^{-4}$ g cm^{-1} s^{-1} = 1.8×10^{-4} poise

$\rho_{air} = 1.2928 \times 10^{-3}$ g cm^{-3} at NTP

The drag force on a sphere is given by:

$$F_d = \frac{1}{2} \rho_f C_D v^2 S$$

where C_D is a drag coefficient which depends on the object shape, and S is the cross-sectional area of the object as presented to the flow.

For $R_e < 1000$, C_D depends on the Reynolds number; for $R_e > 1000$, $C_D \sim$ constant. Particularly, for a sphere and $R_e < 80$, $C_D \bullet R_e \sim 24$, a constant; for $R_e > 1000$, $C_D \sim 0.4$. *[Robin]*