1997-1998 Physics Olympiad Preparation Program

— University of Toronto —

Problem Set 5: Electricity and Magnetism

Due March 6, 1998

1) The earth as cosmic doorknob

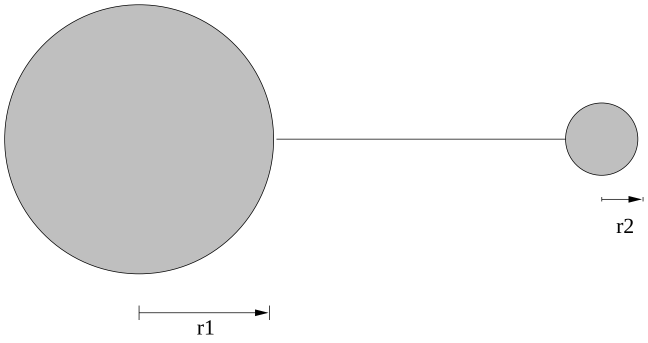
Cosmic rays are subatomic particles with very high energies arriving from deep space. About 87% of cosmic rays are protons (hydrogen nuclei) with an average energy of several *billion* electron volts (GeV) per proton. The flow of protons reaching the earth is approximately 1 particle per 1 cm^2 per 1 second. Estimate the length of time period after which the charge accumulated by the earth will prevent the protons from reaching it. Compare this time interval with the age of earth, which is estimated as 5 billion years. If it is smaller, then how do cosmic rays still reach the earth? *[Lev & Dick]*

2) C.C.? No! No!

Christine Coulomb was a good physics student, but not a very nice person. After learning about conductors and dielectrics in physics class, she decided to build a machine that would zap people who came close to it (like a Van de Graaff generator).

a) Christine built a spherical conductor of radius 2m on which she could place electrostatic charge. If she wanted to zap people at a distance of 5cm, how much charge would she need to put on the conductor? The maximum potential gradient that may exist in air before a spark discharge occurs is 0.8 kV m^{-1}.

b) Would a spark be more likely to emanate from a sharp point or from a conductor with a large radius of curvature? To answer this, consider a conductor made up of a large sphere of radius r_1, and a smaller sphere of radius r_2, the two spheres being connected by a conducting wire, as shown below. If a charge Q is placed on the conductor, a charge q_1 will distribute itself on the larger sphere, and a charge q_2 on the smaller sphere, with $Q = q_1 + q_2$. For simplicity, we can assume to a rough approximation that the electric fields of the spheres are as though the two spheres didn't influence one another, but, being a single conductor, the two spheres are at the same electric potential. What is



the electric field at the surface of each sphere? What is the ratio of the electric field strengths of the spheres in terms of their radii?

Thus, what can you conclude about whether Christine should make her zapper 'pointy' or not? *[Nipun]*

3) Goats & Sheep — the Hall effect in plasma

Practically all of the observed mass of the universe is not solid, not liquid, and not exactly gaseous, but in the form of *plasma*. Plasma is any ionized matter, whether ionized by heating (as in the sun), by electric discharge (as inside a fluorescent light tube), or by dissociation in solution (as in blood).

In an oxy-acetylene torch, oxygen and acetylene gas are mixed at a few p.s.i. pressure and then ejected as cool gases from the torch nozzle. If the torch is already lit, these cool gases expand and slow down until they reach the flame-front, the edge of a small fireball which is burning upstream against the flowing gases. The flame front is stable, and located at the place where its upstream burn-propagation speed matches the speed of the flowing gases.

Consider a portion of the flame where the hot gases are moving at 3 m s^{-1}, roughly at atmospheric pressure, and say that about 1% of the molecules have one electron removed (this is a guess). If the torch is in the midst of a uniform 0.3 Tesla magnetic field, what happens to the ionized gases? You should assume that the density of ions and electrons stays uniform. *[Robin]*

4) Shepherding fields

The phenomenon know as the magnetic 'bottle' plays an important role in such diverse areas of science from cooling atoms to almost absolute zero to the gorgeous aurora borealis displays in the northern sky. How do charged particles end up trapped in a magnetic field? Let's see: a moving charged particle in a magnetic field experiences a force $\vec{F} = q\vec{v} \times \vec{B}$.

a) For a uniform magnetic field, what is the motion of the charged particle for arbitrary initial velocity? (I suggest that you take 4 cases: zero velocity; non-zero velocity parallel to $\vec{B}$; non-zero velocity perpendicular to $\vec{B}$; non-zero velocity in both directions). What is the work done on the particle by the magnetic field?

b) Things get more interesting when we allow $\vec{B}$ to be spatially inhomogeneous. For instance, let the z component of $\vec{B}$ be $B_z(z) = B_0 + b_0 z^2$ where b_0 is much smaller than B_0. At $z = 0$ the x and y component of $\vec{B}$ are zero. Even for inhomogeneous $\vec{B}$, both the kinetic energy of the particle and the magnetic flux through a loop traced out by the particle are conserved. (*Magnetic flux* is defined as $\vec{B} \cdot \vec{A}$, where $\vec{A}$ is the area enclosed by the moving particle.) Ignore the effects of the other components of $\vec{B}$ (since $b_0 \ll B_0$). For a particle starting at $z = 0$, with a non-zero velocity parallel and perpendicular to $\vec{B}(0)$, what is the $v_z(z)$, in terms of the initial velocity of the particle, B_0 and b_0?

c) From your answer to part (b), what is the resulting motion of the charged particle along the z axis? (You needn't solve any differential equation; your result should be familiar to you. If you need to, draw the diagram of v vs. x) Sketch what the total motion looks like in real space.

d) At what value of z does $v_z(z)$ go to zero? Where has the corresponding kinetic energy gone? *[James]*

You can find in info on magnetic bottles, solar wind, aurorae, the sun's core, and other plasma physics via the link: http://fusedweb.pppl.gov/CPEP/Chart_Pages/5.Plasma4StateMatter.html

5) Science on a shoestring

Given an ordinary bar magnet, chemistry stand and clamp, compass, weighing scale, stopwatch, ruler and a nice piece of string, describe how you might find the horizontal component of Earth's magnetic field.

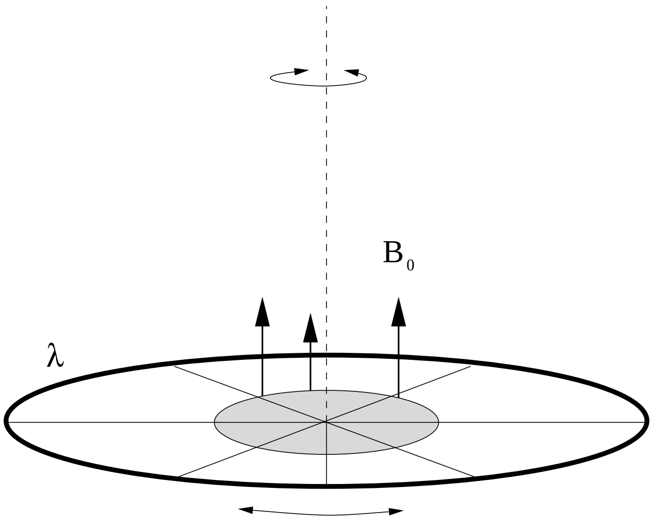
HINTS & INFO:

a) The magnetic field due to a dipole of length d (which a bar magnet is) anywhere on the axis of the dipole a distance $r >> d$ away is given by $B = (2m)/(4\pi r^2)$, where m is the magnetic moment.

b) A dipole immersed in a magnetic field B experiences a torque $T = m B \sin(\theta)$, where θ is the angle between the external field and the axis of the magnet (i.e., line connecting its two poles together). *[Peter]*

6) How much do you charge for a free ride?

Suppose you wanted to run a merry-go-round using only a magnetic field. Here's how you might do it. Suspend a non-conductive wheel so that it is free to rotate about its axis, and 'spray' a charge of λ [Cm^{-1}] all around the circumference of the wheel (see figure at right). Then, put a electromagnet coil of wire under the wheel, centred on the rotation axis, and having a coil radius a. Electric current flowing through the coil means it initially produces a magnetic field B_0 through the wheel out to a radius a, with the field



pointing in the $+z$ direction. We then turn the magnetic field *off* by cutting the current to the coil.

Faraday's law of induction says that a time-varying magnetic field produces an electric field. In fact, it may be written as:

$$\oint \vec{E}\cdot d\vec{l} = -\int \frac{d\vec{B}}{dt}\cdot d\vec{s}\text{, or in our case simply } \oint E\cdot dl = -\int \frac{dB}{dt}\cdot ds$$

or:

> *"The integral of the electric field around a closed loop = the time rate of change of the magnetic flux through the surface bounded by the loop."*

This is of the same form as Ampère's law, which can be written as,

> *"The integral of the magnetic field around a closed loop = the current through the surface bounded by the loop."*

a) For a current flowing in a straight wire, in what direction relative to the wire is the magnetic field?

b) For a time-varying magnetic field in the z direction, what is the direction of the induced electric field? Using this information and remembering Lenz's law, describe what happens to the wheel once the magnetic field is turned off if the wheel carries a positive charge density. (Hint: the wheel will rotate, but we want you to explain why and in what direction.)

c) What is $\int E\, dl$ around the circumference of the wheel in terms of the magnetic field and the radius of the coil?

d) The torque on a tiny segment of the wheel is $d\vec{N}=(\vec{r}\times\vec{F}) = r\lambda E\, dl$, where dl is the length of a tiny piece of the wheel. What is the angular momentum of the wheel in terms of B_o once the field has been switched off.

e) If you turn the field off faster or slower, how does the *final* angular momentum of the wheel change?

<u>BONUS</u>

f) What is the angular momentum of the system before the magnet is shut off? What is the angular momentum afterwards? Since angular momentum is conserved, like linear momentum, explain the details of how that happens here. *[Nipun*]