1997-1998 Physics Olympiad Preparation Program

— University of Toronto —

Problem Set 6: AC Circuits and Electronics

Due April 1, 1998

1) Misha's Mom's Medallions

Misha decided to make a big surprise for his mom's birthday: silver-plate her favourite old medallion, which had become really dark and unattractive. He took it out of the cupboard and ran to an electroplating-shop to get the job done as soon as possible. The medallion had an area $A = 57\ \text{cm}^2$ and should get $h = 50\ \mu\text{m}$ of silver. The electroplating tank used at the shop contains a solution with silver salt as electrolyte, and runs a current $I = 1.8$ A. How much will Misha pay to the technician, if the charge is $10/hour independent of silver used, to silverplate his mom's 1976 Olympic Bronze Medal?

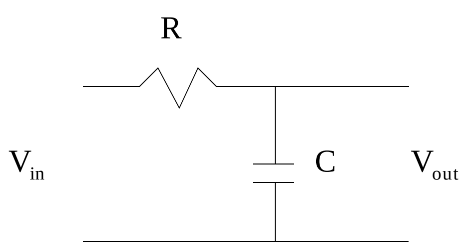
Molar mass of silver $m_0 = 0.108$ kg mole^{-1}; density of silver $d = 10.5 \times 10^3$ kg m^{-3}; the Faraday constant $F = 9.65 \times 10^4$ C mole^{-1}. *[Lev]*

2) Tesla Yes! Edison No!

An alternating current generator supplies a factory with electrical power. A cable has a resistance of 0.15 $\mu\Omega$ m^{-1}. The factory should receive 110 V and 500 A (r.m.s.) at a distance of 10 km from the generator. Find the power loss in the transmission lines and the required output voltage of the generator. Now, to get a reasonable result, let's use step-up and step-down transformers, so that the power transmission takes place at 55 kV. Calculate the power loss in this case. *[Lev]*

3) Analog differentiation and integration

a) The circuit below is known as a *differentiator*. It functions only for (periodic) input signal frequencies $\omega << 1/(RC)$.



Verify that this is true (find V_{out} as a function of V_{in}), and plot the output signal for an input of:

i) a sine curve
ii) a square wave
iii) a triangular wave

b) Design a circuit that integrates the input wave. Prove that it works. For what frequencies is it valid? Show the output wave for:

i) a sine curve
ii) a square wave
iii) a triangular wave *[Peter]*

4) What goes around, comes around

Your pal Frido runs up to you "I'm never going to need to buy electricity from Hydro again! I just got an I.C.D. — an Infinite Current Device!" Being the skeptic that you are, you ask him to explain. He shows you a strange-looking box with several sockets protruding from it. "Take a look at this. When you short out this battery and remove it from the I.C.D., current runs through the circuit forever! It's even AC current, just like you get from a wall plug." He shows you the ammeter that indicates a steady current of 1.0 A AC. Upon closer inspection of the interior of the box, it appears it contains only a capacitor and an inductor in series with the battery terminals. You think back to the wise words of advice you received from your physics instructor...

"...the voltage across a capacitor is proportional to the total charge on it. Current is the rate of charge flow. So the rate of voltage increase is proportional to I(t)."

And inductors, what did she say about them again? Think, think!....

"....Listen up! What I am about to tell you will one day help you avoid a horrible scam that would leave you destitute. An inductor acts like a simple coil of wire which induces a magnetic field. According to Lenz's law, it resists changes in the current running through it. Thus, the voltage across it is proportional to the rate of current change."

a) Now it is up to you to use these words of advice. With the battery in series with the capacitor and inductor, what is the steady-state voltage across the capacitor and inductor, and what is the current as a function of time? (This one's a no-brainer.)

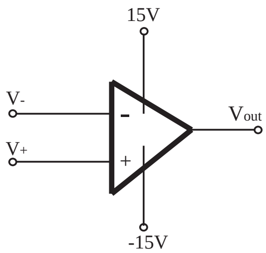
b) You short out the battery and remove it from the circuit. What is the derivative equation for current as a function of time? What equation does this remind you of? Using your experience from past problem sets, what is the current as a function of time? Think 'Bob'!

c) Energy seems to be leaving the capacitor and returning to it. Where is it going?

d) Even though he does have a steady AC current like that obtainable from a wall socket, why is Frido going to be very disappointed with his I.C.D? *[James]*

5) Operation: *'Amplifier'*

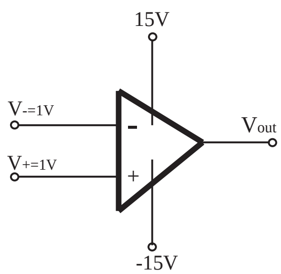
A standard component in an electronic circuit is the operational amplifier ('op-amp').



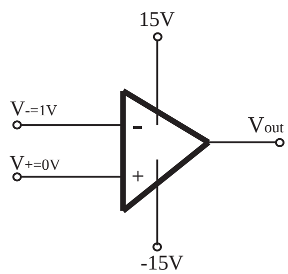
Operational amplifier

This device is a differential voltage amplifier: unless saturated, the output is proportional to the difference between the two voltage inputs. The bounds on V_{out} due to saturation are imposed by the power supply to the op-amp: the output voltage cannot exceed to supply voltage levels. In this case, the maximum V_{out} is 15 V and the minimum V_{out} is –15V. The magnitude of the gain is huge, and as you might not expect, is not a consideration in many applications of the op-amp.

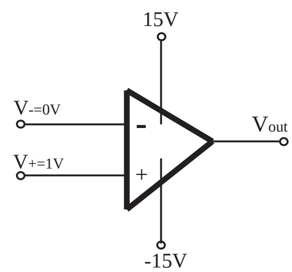
a) Let's start simple. What is V_{out} for the following op-amps?



(i)



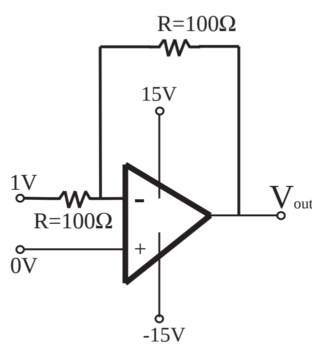
(ii)



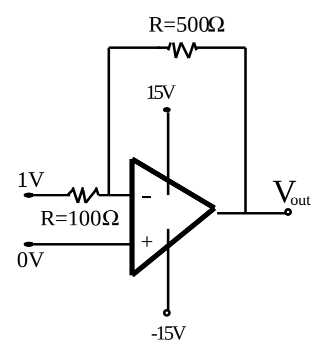
(iii)

b) One of the advantages of ideal op-amps is that they draw no current but can produce unlimited amperes (i.e., current into the V+ and V– terminals is 0, and unlimited current can be produced on the V_{out} line). An op-amp with feedback can avoid saturation. In the circuits given at right, feedback causes V+ ≈ V–.

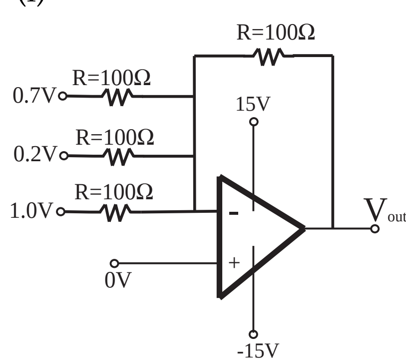
What is V_{out} for these op-amps? (Remember to conserve current flow.) *[James]*



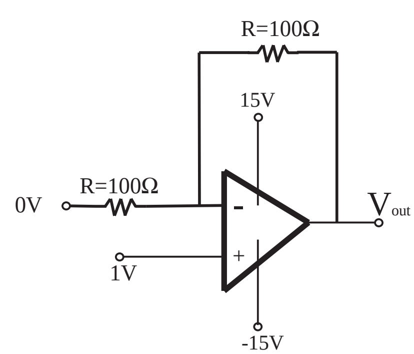
(i)



(ii)



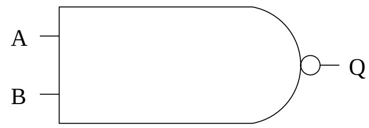
(iii)



(iv)

6) Flip-flopping on a counter proposal

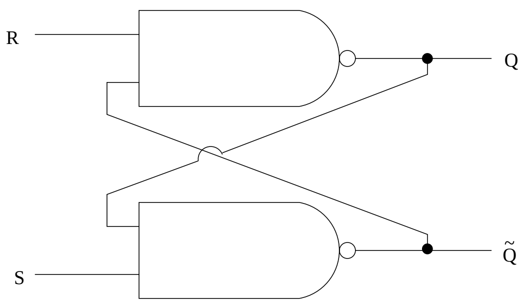
At right are the symbol and the truth table for a 'NAND gate' (NAND = 'Negated AND'; with negation being $\overline{1}$=0, $\overline{0}$=1; 1 = 'on', 0 = 'off'):



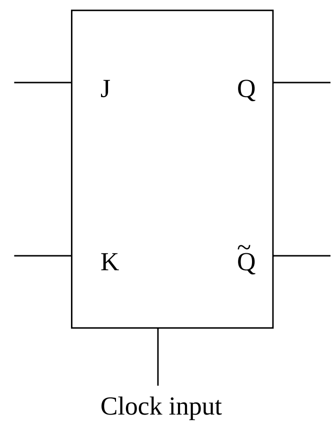
A	B	Q
0	0	1
1	0	1
0	1	1
1	1	0

a) Consider the circuit diagram below for what is called an *RS flip flop.*

If R and S are both fixed at a logical 1, what is/are the output configuration(s) for Q and $\overline{Q}$? What happens if we momentarily switch R to a logical 0 and then back to a 1?



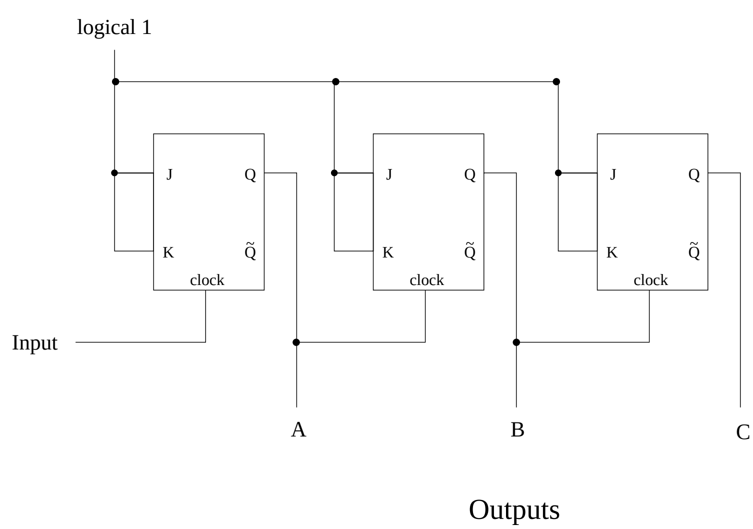
b) A *JK flip flop* consists of 2 RS flip flops and a clock pulse. We show it schematically at right.



Only when there is a pulse in the clock input are the values of J and K processed by the device (a pulse is a signal that starts at zero, goes to a logical one, and then back to zero). If a clock pulse is present, the final states of the outputs, Q_f and $\overline{Q}_f$ are dependent on J and K, and on the initial values of the output, Q_i and $\overline{Q}_i$. The truth table for this device (in the presence of the clock pulse) is as at left.

J	K	Q_f	$\overline{Q}_f$
0	0	Q_i	$\overline{Q}_i$
0	1	0	1
1	0	1	0
1	1	$\overline{Q}_i$	Q_i

Suppose now we construct the circuit shown at right.



The inputs J and K of all flip flops are kept high, and a series of pulses are entered through the clock input. Initially (with no input pulse) $Q_1 = 0$ and $\overline{Q}_1 = 1$. Make a table showing what the values at outputs A, B and C are, and of the values of each of the Q's as 1, 2, 3, and 4 pulses are input into the device. In what sense could this device be considered a digital counter? [*Nipun*]