

1998-1999 Physics Olympiad Preparation Program

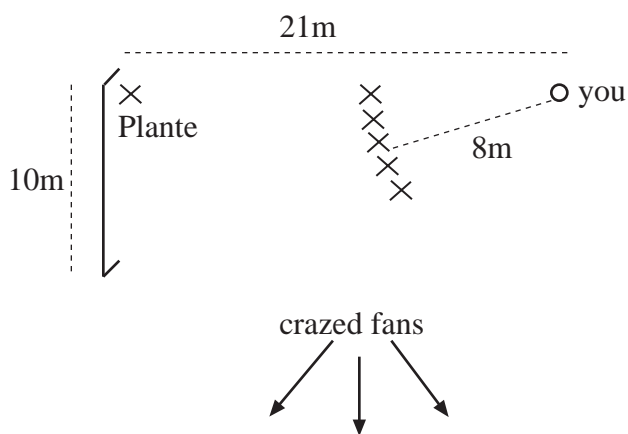
— University of Toronto —

Problem Set 2: Mechanics

Due December 11, 1998

1) Football fizix

In the dying minutes of the World Cup finals, your team is awarded a direct free kick near your opponents' goal (see figure). The defense has set up an impenetrable wall of defenders 8 m from the ball such that the only way to score is to kick it over their heads and into the 3m-high net. Also facing you is Pele Plante, the superb goalie that has kept your team off the score board all game. You want to keep the ball as far away from him as possible. What initial velocity (speed and direction) do you give the ball? Remember the highlight tapes you viewed before the game indicated that the defense can jump to a head height of 2.5 m and your hardest kick corresponds to an initial ball speed of 30 m s^{-1} . (Ignore air effects for the purpose of this question.)



Also remember that if you do not score, your team loses the Cup *and* Plante takes over your Nikke promotional contract. The pressure is on...

BONUS: Describe how your answer would change when you also consider the effects of air on the ball's trajectory. [James]

2) Chow, baby

Cat food is what Kit buys at the corner store — in 175 g flat cans. She knows from sad experience that the plastic carry-bags that the store provides will hold well enough for 15 cans, but will tear through right away if she puts in 16 cans.

Coming home with 15 cans in one plastic carry-bag, Kit realizes she has a problem. Between her and her apartment lies The Elevator. Kit knows from physics class that the acceleration in the elevator will add to the acceleration due to gravity, and that the cans will weigh more as the elevator accelerates (though their mass remains constant).

This elevator starts with constant acceleration over two floors between the ground floor and the second floor, and then rises at a constant speed of three seconds per floor. All floors are 4 m apart. Will the bag split open — should Kit have double-bagged?

What should be the maximum number of cans that Kit can load up with? [Robin]

3) Sink, sank, sunk

At some point in the movie “Titanic” it is said that the pressure at its sunken depth is 5500 pounds/inch². Given this, find the depth at which the *Titanic* supposedly came to rest. The ‘real’ answer is 12600 feet, density of water is 10^3 kg m^{-3} (ignore air pressure). Comment on the results, i.e., truth in film-making.

At another point in the movie someone estimates that the *Titanic* will sink in a time of roughly half an hour. Approximating the *Titanic* as a rectangular box of cross-sectional area $A = 882 \times 34 \text{ feet}^2$, mass $M = 40$ tons (loaded), total height 80 m, and a horizontal hole at the bottom of a side of the box of area $A' = 100 \times 0.5 \text{ m}^2$, estimate the time it takes sink — that is, when the top of the box is even with the water line. Ignore air.

[HINT: assume the flow stays practically at equilibrium as water goes in; you *may* use Bernoulli’s equation, but if you do, you have to prove it.] [Peter]

4) When bowling, always signal your lane-change...

Consider the problem of throwing a bowling ball with an initial spin, so that its path curves. Take the x-axis as the centre-line down the bowling alley, and y as the horizontal sideways direction. Say that when the ball is released, it has an initial spin ω with the top of the ball moving sideways (i.e., $\vec{\omega}$ points along the x-axis), and a velocity \vec{v} at an angle θ to the centre-line.

Find the path of the bowling ball, by giving the x-coördinate and y-coördinate as time goes by, i.e., $x(t)$ and $y(t)$, where t is time (this is called ‘parametric form’). Assume the point of contact between the ball and the floor is very tiny. The moment of inertia of the ball will be that of a sphere.

Plot some nice-looking representative paths ($\omega = 0$, $\omega > 0$, $\omega < 0$) [Peter]

5) A matter of some gravity

i) The moon rose in the east today at 7pm. If the lunar cycle (number of days between full moons) is 29 days, when will the moon rise tomorrow? (One other subtly ‘hintful’ piece of information: if the earth were moving more *slowly*, in orbit around the sun, the lunar cycle would be shorter.)

ii) The spaceship *Galileo’s* mission is to study Jupiter and its surrounding moons. To get the vessel into a Jovian orbit, NASA had the vessel fly-by both Venus and Earth to

increase its speed. This type of trajectory which provides acceleration without the burning of expensive fuel is called a *sling-shot maneuver*. Consider one of these interactions in which the Earth's large velocity was used to accelerate the vessel. What is the maximum speed that the vessel could have attained after this interaction, if its speed before the encounter was 5 km s^{-1} ?

Galileo (mass: 1000 kg) actually did this type of maneuver *twice* with Earth on its way to Jupiter. How many such sling-shots would have to occur for the length of our year to decrease by one second?. [James]

[Check out www.jpl.nasa.gov/galileo/index.html for some great pictures and information on this billion dollar space mission]

6) Balloon tug-o-war experiment

Here's a neat experiment you are welcome to do with a friend (put both your names on, for POPTOR marking). It's not too hard, but it's full of physics. You will need:

- 2 good-quality round balloons
- 1 thick drinking straw
- some scotch tape

Have your friend blow up one balloon about $1/4$ full and the other about $1/2$ full. Pinch the neck of each balloon so that no air escapes. Carefully insert one end of the straw into the neck of each balloon and tape it air-tight, while still pinching. It will look like a balloon bar-bell for weightlifting.

Before actually trying it, what do you *think* will happen when your friend stops pinching the balloons so that air can flow between the balloons? Argue for your predictions on the basis of what physics you can figure out or guess. *Then*, hold the straw gently and have your friend release the balloons. Were your predictions right? Try squeezing the balloons, one at a time, and see what happens.

To examine this interesting phenomenon further, do the following measurement: Try to determine the *air pressure* inside the balloon, as a function of the *size* of the balloon.

You will need:

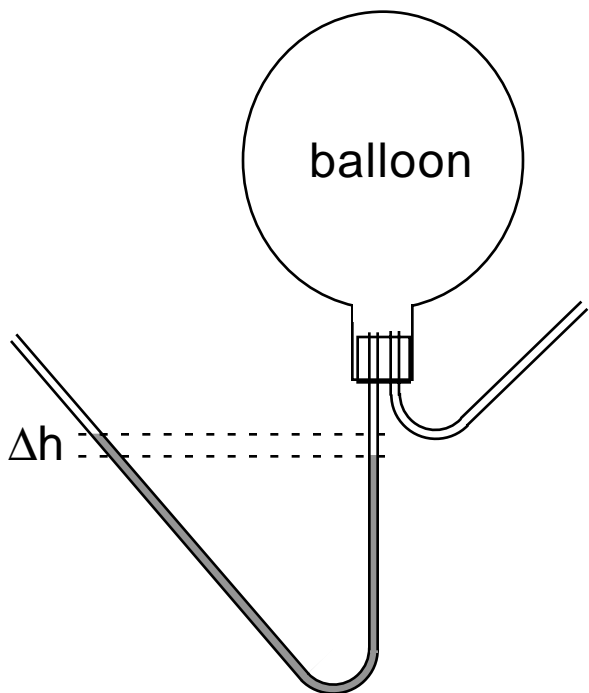
- clear flexible tubing about 3 to 4 metres in length (see INFOBITS, below)
- 1 clear flexible tube about $1/2$ metre in length
- 1 two-hole stopper with holes to fit the tubing snugly
- some duct tape
- 2 metre sticks or long rulers

You might want also to use (optional)

- 1 500 ml (or larger) kitchen measuring cup, or beaker with volume markings
- two large soup cans
- 1 large bucket or plastic laundry tub (a nearby sink or bathtub will do fine)

To set up:

1. Seal one end of each piece of tubing into the stopper
2. Make the long tube into a rough 'U' shape, and tape each side onto a meter stick or long ruler. Make the end farthest from the stopper to be sloping, as pictured. Be sure your setup is stable.
3. Half fill the long tube with water. This will be a *manometer* for you to measure pressure with. You may need more adjustments once you begin.
4. Now fit the balloon onto the stopper. Gently fill up the balloon, blowing air into it through the short tube. If you notice air leaking anywhere you might use some Vaseline or duct tape to seal it up.



With the right amount of water in the long tube, you can measure the *difference* in height of the water on each side of the 'U'. With the ruler or metre stick at a sloping angle you can measure along the tube, and then use trigonometry to figure out the height. What is the relationship between air pressure in the balloon and difference of water levels in the manometer?

Now make a series of measurements of air pressure, for the balloon inflated to different sizes (including *not* inflated). How can you best measure the size of the balloon: Cast a shadow onto paper, and measure the shadow-size? Use a tape measure to find the circumference? Use the measuring cup and tub to trap air released from the balloon, and read its volume? We're interested in *your* best ideas — change anything you want.

Finally make a graph of your data with height *vs.* balloon size, and connect the points with a smooth line. Be sure to label your axes with dimensions. From your results, can you explain the outcome of the first 2 balloon experiments?

[Bonus-mark 'sequel' experiments: It would be interesting to see if the pressure in the balloon, as a function of size, varied *differently* depending on whether you were inflating or deflating the balloon. Do you get a different answer if you measure volume of air and if you measure diameter of balloon? Would it make a difference if you filled the balloon not with air but something incompressible, like water? (How could you change the manometer in order to use water in the balloon?)] [Simal and Robin]

INFOBITS™ — Useful Bits of POPTOR Information

Mass of Earth: $6. \times 10^{24}$ kg

Mass of sun: 2×10^{30} kg

Distance between Earth and Sun: 1.5×10^{11} m

$G = 6.7 \times 10^{-11}$ N m² kg⁻²

Where to get parts for experiments:

there is plastic tubing at Canadian Tire, in the Plumbing section, for 17¢ per foot. DO NOT attempt to use *glass* tubing, unless you have teacher supervision. It is pretty easy to break it and have it go right through your hand, if your technique is wrong — especially when putting it into rubber stoppers.

- polyethylene, 0.25" O.D. (outer diameter); translucent, not quite clear (stock no. 7 78380 022022 1; SKU: 039166056347)
- vinyl, 3/16" O.D.; clear, but soft, and hard to stuff into a rubber stopper (stock no. 7 78380 02032 0; SKU: 039166056224)

2-hole rubber stoppers: from chemistry class, we thought

CHECK THE POPTOR WEB PAGE for other hints, and any corrections we might post:

www.physics.utoronto.ca/~poptor