# 1998-1999 Physics Olympiad Preparation Program

— University of Toronto —

*Problem Set 4: Optics and Waves*

*Due February 12, 1999*

**1) The land of the midnight sun (is where, exactly?)**

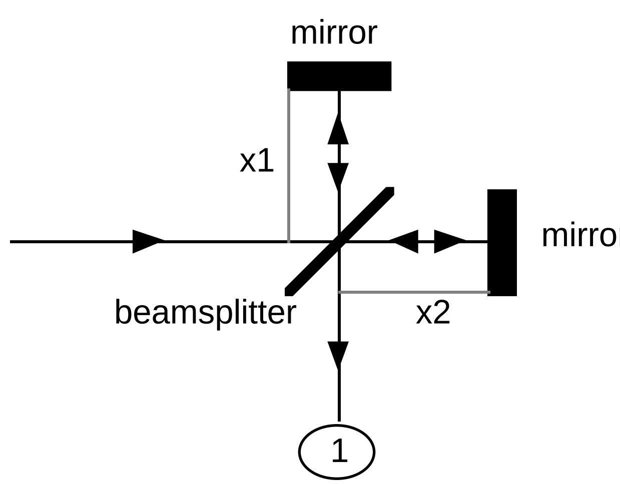
While travelling through the Yukon this past summer, we travelled across the Arctic Circle (latitude 66.7 degrees). If the earth was a simple sphere (like a billard ball) circling the sun while rotating on its own axis, a line at this latitude would experience exactly one day a year of unending daylight. North of this line would experience more than one day of continuous daylight and south of this line would have a sunrise and sunset every day of the year.

a) Using this information, find the tilt of our sphere's axis relative to the plane it sweeps out as it circles the sun.

b) In reality, it is possible to see the midnight sun at points *south* of the Arctic Circle. In each of the following cases, calculate how much farther south than the Arctic Circle you could position yourself and still be able to observe the midnight sun, by taking into consideration:

i) you are standing on a hill (height: 300 m, rising from a perfectly spherical Earth).
ii) the Earth has an atmosphere. For the purpose of this problem, assume that the atmosphere is 3 km high and at standard pressure and 0° C, and that the Earth is a perfect sphere. *[James]*

**2) Making light of photons**




Since light is a wave, two identical beams can add together constructively or destructively depending on their relative phases. Interferometers, whether large such as found in a satellite array suspended high above the Earth or else small like the one on my laboratory table, make use of this fact to measure very small distances. Consider my simple interferometer: the beamsplitter reflects 50% of the incident intensity and transmits the rest. All reflections involve a 180° phase change.

i) What is the electric field at the output (1) as a function of the relative displacement ($\Delta x = x1 - x2$). Consider the input field to be monochromatic and to have an electric field of $E = E_0 \cos(2\pi\ \nu(x/c - t))$ where $E_o$ is the amplitude, t is time, ν is frequency (Greek 'nu'), and c is the speed of light.

ii) You will notice for certain values of Δx, there is <u>zero</u> electric field at (1). Where has the energy from the light gone?!

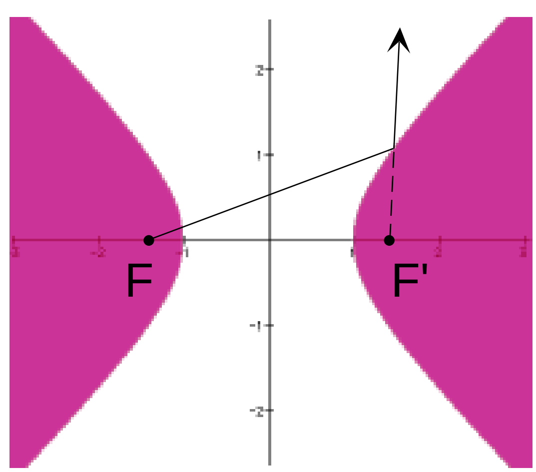
iii) Put a screen at (1). For a beam from my HeNe laser (frequency ν = c/632nm), what is the smallest Δx you could clearly distinguish?

iv) If you travelled back a century, you would find that many believed that light moved in a medium, just as water waves travel in water or sound waves travel in air. This supposed medium was dubbed the 'ether'. If the ether were also moving, this could change the effective speed of light, by a version of the Doppler shift. Pretend that you are a scientist at this time and you want to establish the *existence* of the ether. How might you use your interferometer to measure the speed of the ether relative to your lab? What limit you can place on the ether's speed? (For simplicity, assume you still had a monochromatic source of the same wavelength as in part iii). *[James]*

**3) Imagining imaging**

When you look at yourself in a mirror, you see yourself behind the mirror by the same distance as you are in fact in front of it. Your mirror image might even appear far enough from you to be in the next room! No light really passes from the place your image seems to be, so the image is called 'virtual'. On the other hand, when you take a photograph, you need real photons to hit the film; this kind of image is called 'real'.

i) Prove that any point *A* a distance *d* from a flat mirror has a virtual image *A'* which is the same distance *d* on the other side of the mirror. What is the magnification of this virtual image? [Hint: magnification *M* can be defined as the apparent size of an image divided by the size of the original object; it can also be defined using the angle between two rays of light leaving a point on the object, and the corresponding angle the rays seem to form when traced back to the virtual image.]




ii) Any hyperbola has two *foci*, which we can label F and F'. Show that one is the virtual image of the other, reflected in the hyperbola. Can you find a well-defined magnification?

*[Robin]*

**4) Correct time and temperature, at the tone…**

Consider a pendulum clock:

i) If the pendulum arm is made of a material with a linear coefficient of thermal expansion $\alpha$ , determine an expression for the ratio of the final period over the initial period, $T'/T$ , if the change in temperature is $\Delta T$ .

ii) If the suspension system is brass and the change in temperature is 20° C how much time does the clock gain/lose in one day if $T$ = 1.000 s?

A nickel-steel guitar string, initially of length $lo$ , is stretched to a length $L$ such that it has a frequency $f$ for its fundamental frequency.

iii) If the temperature is increased by ΔT, determine an expression for the new fundamental frequency in terms of the old one, i.e., $f'/f$ .

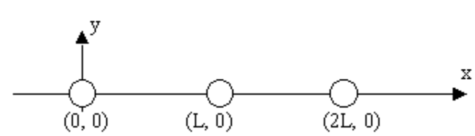
Assume Young's modulus, *Y*, and the linear expansion coefficient, $\alpha$ , to be constant.

iv) If the initial frequency is 440 Hz ('A' above middle 'C'), how 'out of tune' will the guitar be, due to an increase in temperature of 20° C? Take $lo$ = 0.795 m and $L$ = 0.8 m. *[Simal]*

**5) Toys, Toys, Toys**

What are the physics rules for oscillating molecules? In this question we will consider different physical models of a linear molecule (like $O_2$, $N_2$, $CO_2$, or some long-chain polymers) which has just been hit by a single-atom molecule (like He or Ne). What we'll change is the model of the force between atoms, and what we'll examine is their motion after the collision.

Model a linear molecule as consisting of evenly spaced particles (atoms, balls or whatever) of mass *m* each. All collisions are perfectly elastic, etc. Initially, the first (left-most) particle of the chain is hit by an object of mass *m*, and velocity ***V*** [ to the right].




Consider first a chain consisting of just 1 particle (a *monomer*) — i.e., we have an ordinary 2-particle collision. What is the speed and position, as a function of time, of the 1-atom chain afterwards?

<u>Part I</u> – stringy bonds

Consider now a 2-atom chain (a *dimer*) having an inter-particle spacing *L*. The particles are connected by an initially extended ideal (massless, non-dissipative and non-stretchable) string. The left ball [at (0,0) ] is hit by the incoming object. Find the motion of the centre of mass of the lattice.

Generalize to an n-particle chain (a *polymer*); keep it a linear (straight-line) molecule!

Describe (in words and equations) also the motion of the two balls as seen from both the centre of mass frame, and an outside (laboratory) frame of reference. Sketch the corresponding position-time graphs for each ball (use one graph for each of the two reference frames).

Part II – springy bonds

This is same as Part I, except that now the two atoms are connected by a spring (with spring constant *k*). Describe (in words) the ensuing motion from both frames (laboratory and centre of mass) of reference; find the motion of the centre of mass and sketch the position-time graph of the left ball in both frames. [HINT: consider the centre of mass frame first].

Discuss the plausibility of the models used in Part I and Part II.

BONUS: Consider that there still is a spring, as in Part II, but there is now also a *dissipation* mechanism. That is, there is some dissipative force of size –γ* (instantaneous drag force on object) = –γ(dx/dt). What is the motion of the centre of mass now? Find and draw a sketch of the position (as a function of time) of the left ball in both the centre of mass and outside frame.

[HINT: a solution to the differential equation: $md^2x/dt^2 = -kx - \gamma(dx/dt)$ is

$$\frac{2Ue^{-\sqrt{\frac{\gamma t}{m}}}\sin\left(\frac{1}{2}\sqrt{4\frac{k}{m}-\left(\frac{\gamma}{m}\right)^2}\,t\right)}{\sqrt{4\frac{k}{m}-\left(\frac{\gamma}{m}\right)^2}}$$

where the initial conditions are: x(0) = 0, dx/dt(0) = V(0) = U; also, $4k > \gamma^2$]

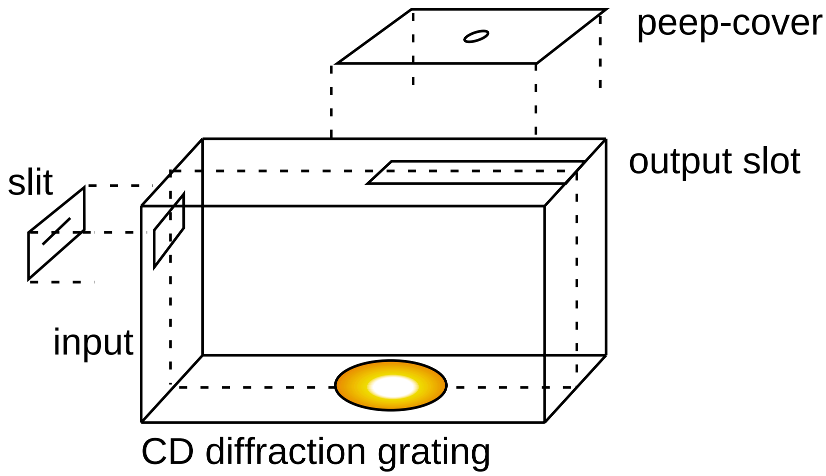
Can you suggest a realistic dissipation mechanism for molecules? *[Peter]*

**6) Making a shoebox spectrograph, using a compact disc**

Have you ever noticed the rainbow colours reflected from a compact disc? The tracks on the CD are arranged with such mathematical regularity that a CD makes a nice *diffraction grating*. This pretty easy experiment lets you make a spectrograph from a shoebox and a CD, one which is good enough to reveal the individual lines contained in the spectrum of light from a fluorescent light tube, streetlights, or other light sources you may have.

You will need:

- any compact disc (my favorites are the gold-coloured CD-R blank used for recording CD-ROMs; about $2-3. It saves my music collection, too!)
- a cardboard box the size of a box for hiking boots; a shoebox or small corrugated cardboard box might do
- aluminum foil
- some duct tape, or other reasonably opaque tape
- scissors
- a pen or marker
- a utility razor-knife, or Exacto™ knife (CAUTION: danger to fingers!)
- a ruler




To set up, read all these instructions before beginning. Refer to the illustration, too.

1. lay the box on its side, as sketched in the picture above. Measure and mark a mid-line around the outside of the box, to use for alignment (up, across, and down the middle of each face — see dotted line in figure). Put a similar line around the inside of the box, also at exactly the midpoint.
2. use scissors or razor-knife to cut a lengthwise slot about 1 cm wide along the top of the box, centred on the alignment line. Cut from about the midpoint lengthwise, and go all the way to the end of the topmost panel.
3. cut a hole through the box, on the small end-panel of the box farthest from the slot you cut in (2). It should be roughly 2 cm high by 3 cm wide, and positioned toward the top of the box-end. It must be centred on your alignment line.
4. cut a piece of aluminum foil about twice the size of this hole in (3). Put it on a firm pad of paper to cut it, and along the middle of this foil make a 3cm-long cut with your razor-knife (hold the knife at a shallow angle, to pull the cutting edge, and not the blade-tip, across the foil). You're making the input slit — you want the narrowest slit that will be sure to pass enough light: 0.1 mm is pretty good. When done, tape the foil over the rectangular hole in the end-face, with the slit horizontal and centred.
5. tape a compact disc, label-side down, inside the box on the side box-panel. Set it up so that the hole through the CD's middle is centred on the alignment

line you drew inside the box. Cover the front half of the disc with opaque tape, so only the half farthest from the input slit will be used. The placement of this disc depends on your placement of the input slit: a ray from the input slit to the middle of the exposed section of the CD should make an angle of about 30° from the CD surface. You may have to try moving the disc forward and back in the box to get the input and output right, relative to the disc.

The spectrograph is now complete, except for figuring out how to use it! You'll need to look down at the CD through the long slot in the top, and try different places along the slot. *But* you want the only light inside the box to have to come through the slit, so try cutting a piece of cardboard or dark cloth to cover the slot, with a 1 cm peep-hole to look through. Then you can peep at different points along the slot, and still keep stray light out by sliding the peep-hole card.

To use your spectrograph, you should point it so that light will pass *through* the slit and onto the compact disc in the bottom. When this is exactly right, you can look at different places along the output slot to find the best spread-out rainbow, from an incandescent lamp, or many spectral lines from a fluorescent light, streetlamp, or neon light.

<u>The experimental assignment</u>:

i) Write down your own description of how you made this spectrograph, and how you made it *work*.

ii) Describe what you see, for different light-sources such as fluorescent lights (different types?), incandescent lights, candle-flames, neon lights, streetlights of different types, a laser-pointer (<u>caution:</u> never stare directly into a laser pointer, or HeNe laser, or the sun. Even these simple sources can cause damage to your eyes, though you may not recognize the damage immediately).

iii) Can you use the formula above, and your own observations, to figure out how many total tracks there are on a CD? *[Robin]*

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www.physics.utoronto.ca/~poptor

index of refraction for air at standard pressure and 0 deg C: $n = 1.0003$

radius of the earth: $R = 6 \times 10^6$ m

nickel steel

Young's modulus: $Y = 18.2 \times 10^{10}$ Pa
coefficient of thermal expansion: $\alpha \approx 1.4 \times 10^{-5}\ K^{-1}$

brass

Young's modulus: $Y = 9.0 \times 10^{10}$ Pa
coefficient of thermal expansion: $\alpha \approx 2.0 \times 10^{-5}\ K^{-1}$

MINI-TUTORIAL: *The physics of diffraction-grating spectrographs:*

The input and output angles of light diffracted from a diffraction-grating depend on wavelength, and on the diffraction grating itself:

$$n\lambda = 2d \bullet (\sin\theta_i + \sin\theta_d)$$

where $\theta_i$ is the input angle, measured from the perpendicular to the surface (the surface *normal*), and $\theta_d$ is the angle of the diffracted light, measured similarly; $d$ is the spacing between grooves of the diffraction-grating, or tracks of the CD. The $n$ is an integer which gives the *order* of diffraction, which you can investigate.

The input light is scattered from very many regularly spaced grooves or tracks in the compact disc. Only at very special angles are the conditions right for constructive interference for waves coming from *all* of the grooves. At these special angles the light of a certain wavelength is bright; at other angles there is only random or destructive interference, and practically no intensity of light is sent on.