1998-1999 Physics Olympiad Preparation Program

— University of Toronto —

Problem Set 5: Electricity and Magnetism

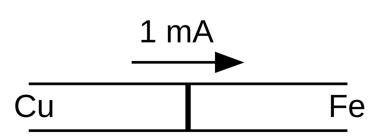
Due March 5, 1999

1) Bully for you!

i) Back in kindergarten, the big bully challenges you to a game of 'No-mercy pink leg wrestling'. You decide that this endeavour is not in your best interest so you challenge him back to a 'real tough' game of point-charge placement. "Huh?" says the bully so you explain: "Pretend you are given a set of electrically charged balls, and you must arrange them to be free, but static — so that they do not fly together or apart." "Hey, what about gravity or friction or redistribution of the charges on the balls?" the bully demands. Unfortunately, you seem to have challenged the only bully in advanced-placement physics in the entire kindergarten class. "Hmm, ignore them and pretend it's a perfect insulator, so the charges cannot move" you respond, hoping that if this doesn't go well, the recess bell will ring before leg-wrestling becomes an option again. "Okay, but I go first" says the bully. "Three balls: two with charge +4q, the other with charge –q. Hurry up twerp!" What is your answer?

ii) You got him on that one but now it is your turn. You ask him "Three balls, all with charge q, placed on the points of an equilateral triangle. Two other balls with identical charges on them, but you get to pick the amount of charge. What is the charge and in what arrangement?" He screws up his face to think, exclaiming "There better be a real answer for this, tweek!" You assure him there is one. Better think quick, what is it? *[James]*

2) Charged with resisting…



A current of 1 mA flows through a conductor made of two wires, one copper and the other iron. The cross-sections are identical, and the wires are butt-welded as shown in the figure. What electric charge naturally accumulates at the boundary between the two metals? How many elementary charges does that correspond to? [Hint: Gauss's law] *[Gnädig/Honyek]*

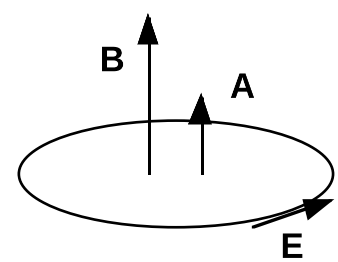
3) 'Ascending and Descending Voltages': a circular argument

"Psst!", a man in a dark trench coat whispers to you, "Psst buddy, ya interested in an almost-new physics equation?" Being the foolhardy type, you decide to check out his wares. “Okay buddy, I’ll even throw in one for free to show you that I’m legit. Say ya got a wire carrying a current. That current induces a magnetic field. Take your right hand like this ya see. Point your thumb out in the direction of the current and the magnetic field curls around the wire like your fingers do.”

You are not impressed. Heck you learned the right-hand rule in Grade 3!

“Okay, waitasecond. I got a real good one. Real hot!” Whipping out a small backboard from within the long folds of his coat, he draws the following diagram while saying, ” Say ya got a closed circular circuit, with a magnetic field going through it. If that field changes in time, there is going to be an electric field induced in the circuit, according to:

$$\vec{E} = \frac{d\phi}{dt} \quad \textit{where} \quad \phi = \vec{B} \cdot \vec{A}$$



here, **E** is the induced electric field around the circuit, **B** is the magnetic field, and **A** is the area enclosed by the circuit (with its unit vector pointing perpendicular to the circle, in the direction defined by the right hand rule). So waddya think of them apples?”

Wow! This is cool stuff, but something about it sounds fishy. “Okay, I’ll show you. Take a look a this.” He motions you into the dimly-lit alley behind him. Sure enough, your new friend has an electromagnet stashed back there. Above the end of the magnet is a circular loop of five 100 ohm light bulbs all wired in series. Even though there is no battery in the circuit, the bulbs are all glowing weakly. You whip out your magnetometer, and verify that yes, there is a uniform magnetic field running perpendicular to the plane of the circuit (area=0.001 m^2) and it seems to be ramping up in time: $B(t) = B_0 \bullet t$.

a) Using your friend’s equation, what do you conjecture is the total current in the circuit? And what is the voltage across one of the bulbs?

b) *Darn*, you think, *I wish I had brought my voltmeter*. So you say to your new friend:

“Let me get this straight. There is current running through each bulb. That means that voltage must be decreasing across the bulb. Therefore as you go around the circuit the voltage keeps dropping after each bulb. You end up back at where you started but now it seems you are at a lower voltage. Doesn’t this remind you of a print by some Dutch guy named M.C. Eaker, Esther, or something like that?”

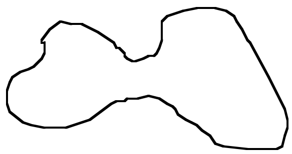
“No, no, no, buddy. You got it wrong!”. What is wrong with your logic? Draw an equivalent circuit. And what is title of the picture that you are mumbling about?

c) “Okay, now I understand”, you tell the huckster, “but there is something still wrong with your equation. If your equation was really right, those bulbs would blow up.” You got him there. Explain. *[James]*

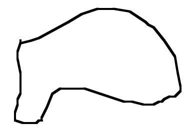
4) Flux compression — putting the squeeze on a B-field

This is an actual experimental problem in practice — a way in which very high magnetic fields can be produced, using a current-carrying circuit and high explosives.

i) Consider a circuit of total area A with some current I passing through it. This causes a uniform magnetic field B perpendicular to the plane of the device. The sides of the device are suddenly imploded so as to cause the area to shrink to A'. What is the resulting magnetic field B' ?



(total area A)



(total area A')

ii) If the object is a solenoid shrinking from a radius $10r'$ to a radius r', with I = 1A and n = number of turns per unit length = 100 / 1 cm, what is the resulting field?

[HINT: one of Maxwell's equations will be useful] *[Peter and Bryan]*

5) Scratch and dent sale on capacitors

i) What is the capacitance of the earth, taking it as a perfect sphere?

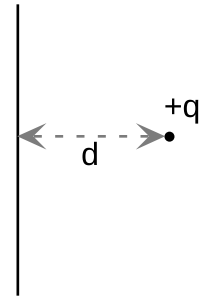
ii) Consider the earth *not* as a perfect sphere — say a comet smacks into the earth and dents it, changing its volume by 3%. By what percentage does the earth's capacitance change?

[BONUS: extra points awarded for deriving the formula for capacitance of a sphere, given in InfoBits below] *[Robin & Gnädig/Honyek]*

6) Mocking mirrors

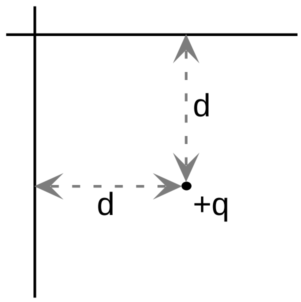
The *method of images* is very useful for solving a variety of electrostatics problems. Consider a point charge +q a distance d away from an infinite conducting plane. Because the plane is a conductor, the charges on it will move so that the potential on the surface of the plane is constant (until this happens, there is a field in the conductor; this field makes the charges move). But you could also make an infinite constant-potential surface if you had *two opposite* charges (+q and – q) a distance 2d apart – the potential on their perpendicular bisector is a constant.

This wouldn't be very important, except there is a theorem which states that the solutions to electrostatic problems are *unique:* if you find *some* electric field that satisfies the boundary conditions, it is the *only* solution. The twinned charges give the right potential for the infinite plane, so their field around +q is the same as the field of a single +q charge above a conducting plane.

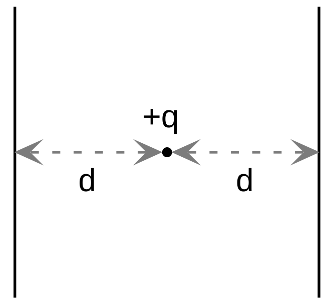


i) So, what is the field at any point y just outside our infinite conducting plane?

ii) Consider *two* infinite conducting planes, crossing at a 90° angle. A charge is placed symmetrically inside one quadrant, a distance d from either plane. What is the field just outside the conductors now? What image charges can you use to produce the same field?



iii) Consider now two infinite, conducting, parallel planes separated by a distance 2d, with a charge +q midway. Where do you have to place image charges so that the planes will be equipotential surfaces?



[HINT: in this case you can think of the planes as mirrors, the charge +q as an object, and the image charges as images of the object formed by the two mirrors] *[Peter]*

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Remember to check the POPTOR web-page for hints and any necessary corrections!

www.physics.utoronto.ca/~poptor

$C_{sphere} = \frac{R}{k}$, C = capacitance, R = radius

$k = 8.9875 \times 10^9 \; N m^2 C^{-2}$

$= 8.9875 \times 10^9 \; F m^{-1}$

$E_{capacitor} = \frac{Q^2}{2C}$, E = energy, Q = charge, C = capacitance