1999-2000 Physics Olympiad Preparation Program

— University of Toronto —

Problem Set 1: General

Due November 12, 1999

1) Pumpkin paradox

To attract more business Farmer Joe decides to build a gigantic 1 tonne pumpkin on stilts near the highway. He first builds a small scale-model pumpkin weighing 0.1 kg, supported by 4 wooden poles. He plans to use all of the same materials for his full scale 'highway monstrosity'.

a) In order to have the same strength, relative to the pumpkins' weights, how many times thicker (*i.e.,* what diameter) should the poles be for the full-size model?

b) To paint the small model he uses 10 mL of orange paint. How much paint will he need for the 1 tonne pumpkin?

c) To lift his 1 tonne pumpkin off the ground Farmer Joe uses a see-saw, which when balanced has a vertical height of about 2 metres. He plans to use 450 kg of bricks (all the bricks he has) as a counter-weight, on one side of the see-saw. What should be the ratio of the distances (pumpkin to the pivot point: bricks to pivot point) in order to lift the pumpkin through at least 2 m? *[Carrie]*

2) Cubic quandry

a) How much time does it take to increase the temperature of a perfectly black cube, of dimension L on each side, by 1°C, using a lamp of power Q (J s^{-1}) located at distance L from one of the corners, along the direction of an edge (at point O on the figure). Assume that the lamp is emitting uniformly and the cube absorbs all wavelengths of light perfectly, but loses no energy by radiation or conduction elsewhere.

b) Really, what happens, is that the black cube is also re-emitting energy as a *blackbody radiator*. Now, adjust your answer and tell exactly how much time will it take.

c) Can you calculate the final temperature of the cube after a long time? *[Amir]*

3) Poles apart, telling

a) Recently, at work, we wanted to use a small magnet in an experiment. We had a lot of people waiting, and we needed to know very quickly which pole of the magnet was the North pole and which one was the South. We couldn't figure out how to do that:

someone suggested putting it against a known N pole of another magnet to tell S (attracting) from N (repelling), but we had no labelled magnets to compare with. Anton in our group took one look at a nearby computer monitor and then quickly found our answer for us. How did he do it?

b) When I put the North pole of a small but very strong 0.1 Tesla magnet next to my computer monitor, with the B field horizontal and in the plane of my screen, the whole picture shifts a little. My monitor is about 50 cm deep (*i.e.,* the length of the cathode ray tube (CRT)) and the screen follows the outline of a sphere. The high voltage that accelerates electrons to the screen is 15 kV. Given this, how far — and in what direction — will the picture travel? *[Peter]*

CAUTION: a magnet next to a monitor can <u>magnetize</u> the monitor and distort the picture. If you want to try this, get permission beforehand – you probably should be OK if the monitor has a <u>de-gaussing</u> feature.

4) Physics Rocks!

Known foremost for its quartzite ridges, Killarney Provincial Park is also known for its 'crystal clear' lakes — Lake Nellie is among then with up to 28 m visibility! While exploring Lake Nellie on a weekend canoe trip Jen and her friends Brian, Leslie and Maria discover a rocky outcrop perfect for cliff jumping.

Peering down into the water Brian is skeptical, "It only looks a couple of meters deep. It has to be at least 5 meters deep for me to jump. I don't think it's safe."

Jen pipes up, "Objects underwater always appear closer than they really are. Give me a minute and I'll reassure you."

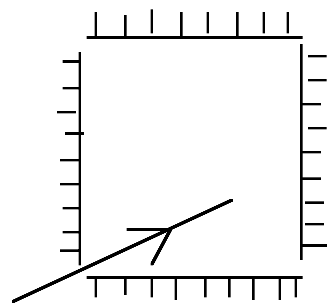
Jen picks up a round stone with a diameter of about 5 cm. She ponders the equation of motion for the stone in the water and (since she's not in the mood to do any differential equations) determines the height from which the stone should be dropped so that it hits the water and thereafter moves at constant velocity ('terminal' velocity).

a) What is this height? (assume that the drag coefficient for a spherical object, the size of the stone, is approximately 0.4 and that the density of quartz is 2600 kg m^{-3}; also ignore any effects from the stone splashing into the water)

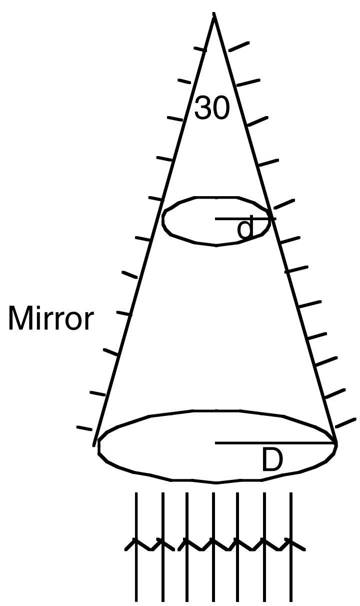
She drops the stone and times how long it takes for the stone to hit the bottom of the lake after splashing. The bright, white quartz stone is easy to spot moving through the clear waters of Lake Nellie. She finds it takes 6.4 seconds for the stone to reach the bottom. To be safe she rounds down to 6 seconds even.

b) Is the water deep enough for Brian to jump? *[Carrie]*

5) Light diversions



a) There are four plane mirrors making an square. Four small holes are made in each corners of the square. Suppose that a ray of light enters from one of the holes. What should be the direction of the ray (the angle which the ray makes with mirrors) in order that the ray finally goes out from one of the holes after any number of reflections in the square?

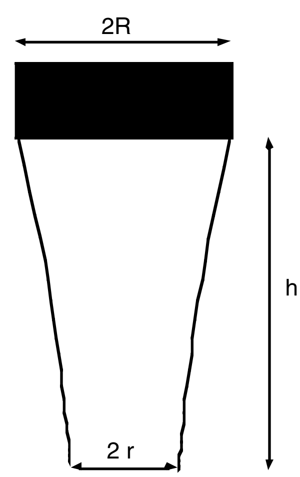


b) Winston has a 30° cone, reflective on the sides and open at the bottom. He wants to cut the point off the cone so as to make something which can concentrate light by piping it from one end to the other. By cutting the cone tip perpendicular to the axis of the cone, Winston can make a circular output-hole of radius *d*.

Light rays parallel to the axis enter the cone from the lower end, which is a circle with a radius *D*. It can be easily shown that if *d* is practically zero size then all the rays entering the cone will bounce around and after some reflections go out again from the lower face. Obviously, if *d* is nearly the size of *D* then all the rays will go straight through and out the upper hole.

Find the smallest *d* for Winston to cut the cone in order that *all* the rays entering the cone, parallel to the axis, leave through the upper hole. *[Yaser]*

6) Physicists' pipe dreams



a) Water is pouring from a tap, through a circular hole. Sketch the shape of the surface of the falling water. Explain the shape, in general terms.

b) Precisely what should be the radius of the water flow as a function of the distance *h* from the tap? Take it that the outlet hole has radius *R*, and that the exit speed of the water is V_0.

c) What will happen in reality? Can the water column be as skinny as your formula gives? Can you figure out exactly where the stream breaks into droplets? [HINT: *surface tension* matters!]

d) FINDING OUT: Experimental facts!!!!

You can see exactly (even in your home!) what happens. Using a smooth flow out of a perfectly round hole in the bottom of a coffee-can or other large container, you can set up a pretty ideal situation. You may want to arrange to have water flowing into the can to balance the outflow. Try different setups, if you like — describe for us what you have attempted, and attach a picture of your setup if it's easy for you to do so.

How can you easily measure the width of the flow at different heights? Perhaps if you can set up a parallel light-beam (e.g., sunlight, a slide projector, or a lamp shining through a small hole), you might make an easy-to-measure *shadow* that would provide a great chance to measure what you want better than you suppose you might! Try some different ideas or ways of measuring the column width as a function of distance downwards. Can you find out the height at which the stream breaks up into droplets? Compare your experimental results with your calculations, both for width and breakup. *[Amir]*

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Drag force:

$F_{drag} = b\rho A v^2$

where b = coefficient of drag (a constant)
ρ = density of fluid
A = cross-sectional area (area of an object's shadow)
v = speed

Energy of a surface due to surface-tension:

$E_{surface\ tension} = \sigma a$

where σ = coefficient of surface tension (a constant)
a = surface-area of a liquid

$\sigma_{water} = 7.28 \times 10^{-2}\ N \bullet m^{-1}$ at 20 degrees C