1999-2000 Physics Olympiad Preparation Program

— University of Toronto —

Problem Set 2: Mechanics

Due December 17, 1999 (revised date)

1) Going for a spin

Cally is playing on the whirly-ride (like a merry-go-round, but non-motorised) in her neighbourhood park. To make it go faster she pushes against the ground using one of her feet.

a) If she accelerates the whirly-ride from rest to a speed of 15 r.p.m. in 3 seconds, what is the corresponding angular acceleration?

b) By what fraction does the angular velocity change when her 20 kg kid brother Matt leaps radially onto the edge of the whirly-ride? Assume the initial moment of inertia (just Cally and the whirly-ride) is 2000 kg m^2 and that the whirly-ride has diameter 5 m.

c) Since one of her all-time-favorite past times is tormenting her little brother, Cally decides to speed up the merry-go-round in an effort to fling her little bother from the ride. Assume that Matt is sitting at a radius of 4.5 m, that the coefficient of static friction between him and the floor of the whirly-ride is 0.7 and that he is naively not holding on! At what speed will Cally's little brother start to slip? . *[Carrie]*

2) Bubble bonanza

a) What is the presure inside a soap bubble of radius 'r' if the surface tension of the soap solution is σ?

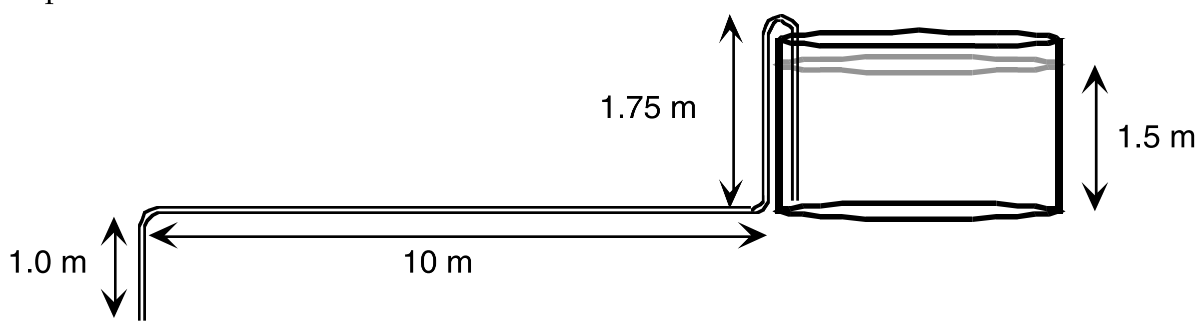
b) What will happen if two bubbles are brought to touch each other? (assume that one of them is bigger than the other, and there is still a bubble-film between them

c) What will be the end result if the wall between the the two touching bubbles breaks, and the two bubbles merge?

d) What if those two bubbles were attached to opposite ends of a tube (e.g., a drinking straw) like a weightlifter's dumbell? *[Amir]*

3) Siphoning cellar

My outside water faucets in my new house don't work, and I'm reduced to running a hose from the laundry tubs out my basement window to fill my above-ground swimming pool. If I forget and leave the hose-end in the pool, then when I disconnect my hose from the tap, water from the pool will siphon back into my basement. The setup looks like this:



Water in a 2 cm-diameter hose flows with a drag force F_{drag} on the hose which is proportional to the length of the hose and to the speed of water flow. This drag results in a pressure drop along the hose. This pressure drop, per unit length, is proportional to v by the formula:

$$\nabla p = \frac{\Delta p}{\Delta x} = \frac{32U\eta}{d^2}$$

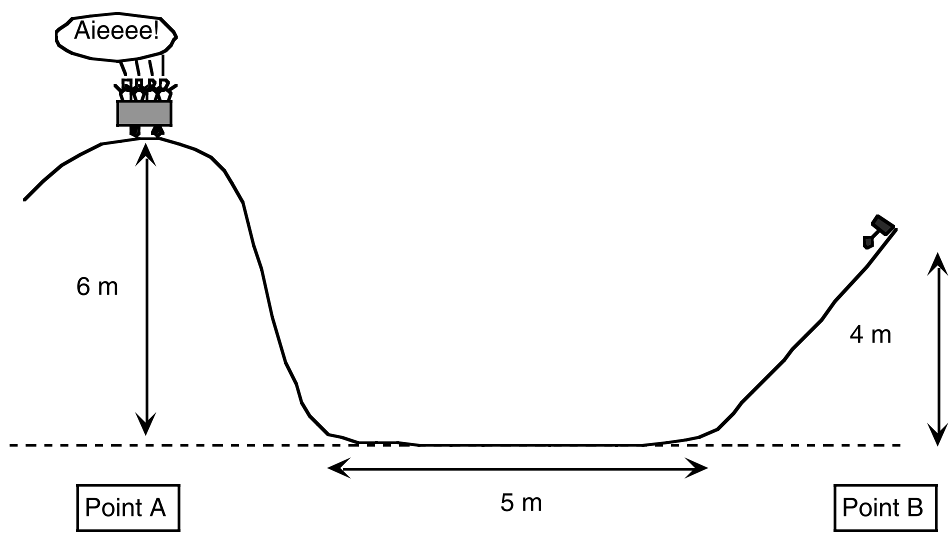
which is described more fully in INFOBITS™ below.

a) If I forget the hose when I leave for work, and it siphons for 12 hours while I'm gone, how much will the water level in the pool drop? Estimate, or calculate exactly.

b) When the last of the water drains out of the pool, then the water in the hose becomes of shorter and shorter length. This changes the drag on the water. See if you can approximately calculate the flow rate of water out of the hose as it empties. *[Robin]*

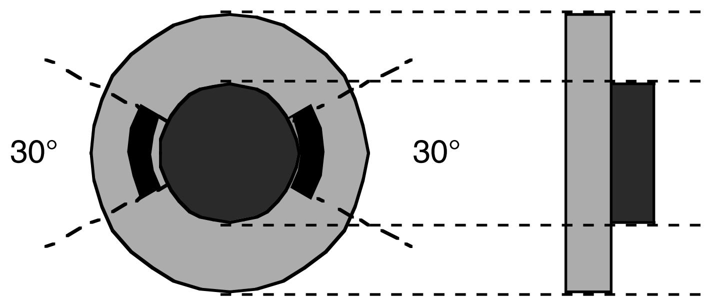
4) Gimme a brake...

The Bumble Bee ride at Super Fun Fun World consists of carts (painted yellow and black of course), which seat 4 people, that traverse a windy, hilly track. The figure below illustrates one section of the ride. On a particularly unusual Tuesday there is a massive power failure at Super Fun Fun World (which consequently isn't so fun fun). The power fails just as one of the carts nears point A with a velocity of 0.5 m/s.



a) If no emergency brakes are applied with what speed will the cart hit the bumper at point B?

Take it that you have *drum brakes*, which press on a hub which is half the wheel diameter. Each drum presses onto a 30° arc of the hub. The thickness of the hub is the same as the wheel. For steel wheels on steel rails, the coefficient of static friction is 0.78, and the coefficient of kinetic friction is 0.42. You can assume that only about 3 mm of the rail is in contact with the wheel.



b) At the bottom of the first hill the emergency brakes are applied radially to either side of the wheels. If the brake pads have a static coefficient of friction 0.85 and kinetic coefficient 0.65, what is the optimum force to apply to the brake pads such that the braking force is a maximum?

c) With the brakes applied (assume with optimal constant force) how far up the second hill will the cart go?

d) With the brakes held on, will the cart go back down the hill? *[Carrie]*

5) It's a ball? What a gas!

This question is the beginning of a kinetic model for an ideal gas. It can lead you to figure out the *ideal gas law* PV = nRT.

a) A ball with mass m and velocity v collides with a stationary wall. If friction is negligible, and bouncing is perfectly elastic, the ball bounces back with the same speed. But what happens if the wall, itself, moves toward the ball with velocity u? Consider all velocities be in the same direction, perpendicular to the wall.

b) Going through (a) you can find out that the momentum has been changed. This means that the wall did some work on the ball. Can you show explicity that the difference in kinetic energy of the ball is exactly the same as the work done by the wall?

BONUS: Can you use this model to show how much the temperature of a gas of atoms behaving like such elastic balls goes up as it is compressed — as long as no heat is allowed in or out? *[Yaser]*

6) Dimensional thinking

A cylindrical vessel of water is leaking via a small hole of radius ‘r’. We want to keep the height of the water constant. The best way is to add water at some rate R (kg per second).

a) Can you guess what should be R in terms of ρ (the density of water), H (height of the water level in the vessel), r (radius of the hole), g (acceleration due to gravity) and A (the cross-sectional area of the vessel). [HINT: Use your intuition, or common sense, and watch that you keep the correct units in each stage (i.e., acceleration has units $L\ T^{-2}$, where L is distance and T is time)]

b) Can you prove your formula with physics theorems?. *[Amir]*

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Coefficient of resistance for pipe flow:

There is a drag on the inside of a pipe, for a liquid flow through it. This drag means that there's a pressure drop as you go along the pipe — it is why small plumping-pipes lead to poor 'water pressure' for a shower or similar. The pressure gradient (change in pressure per unit length) is:

$$\nabla p = \frac{\Delta p}{\Delta x} = \frac{\lambda}{d}\frac{1}{2}\rho U^2$$

where p is pressure, ρ is fluid density, d is pipe diameter, U the average speed of liquid flow, and λ is a dimensionless constant which is equal to:

$$\lambda = \frac{64}{R_e} \quad \text{for } \textit{laminar} \text{ (smooth) flow}$$

R_e is called the *Reynold's number*. (usually a value of a few 1,000 for laminar flow) The Reynolds number for a pipe can be found from:

$$R_e = \frac{\rho\, U\, d}{\eta}$$

putting all this together, we see that:

$$\nabla p = \frac{\Delta p}{\Delta x} = \frac{32 U \eta}{d^2}$$

CHECK THE POPTOR WEB PAGE for other hints, and any corrections we might post: www.physics.utoronto.ca/~poptor