1999-2000 Physics Olympiad Preparation Program

— University of Toronto —

Problem Set 4: Optics and Waves

Due February 25, 2000 (revised date)

1) Bob-bob-bobbin' along

A *terrifically* important concept in waves and oscillations is that of the *harmonic oscillator*. We want to use some basic principles of the harmonic oscillator in later POPTOR questions, so have a look here at the basics.

The 'generic' harmonic oscillator that you'll see in university, and forever after that on into graduate school and beyond, is a mass resting on a frictionless table and attached to a spring. A property of springs, for small stretches or compressions, is that the force exerted by the spring is proportional to the amount of stretch. This is called a *linear restoring force* and the force is represented $F = -k\,x$, because the force tends always to restore the position of the mass to its original neutral position (unstretched spring).

a) Write the force equation for such a mass-on-a-spring (inertial force and restoring force). Since acceleration is the time-derivative of velocity, and velocity in turn the time-derivative of position, this actually is a 'second-order linear differential equation'.

b) You can do pretty simple derivatives. Suppose that $\sin(\omega t)$ is a solution of this particular differential equation. For that to work, what must be necessary?

c) So, what is the rate of oscillation of a 100g mass on a spring that has a spring constant of 80 N m^{-1}? *[Robin]*

2) Keeping up an image

The distance between a screen and a light source lined up on a table is 120 cm. Moving a lens between them, sharp images can be obtained at two different positions, producing two different images. The ratio of *sizes* of these two images is 1:9.

a) What is the focal length of the lens?

b) Which image is the brighter one? Determine the ratio of the brightness of these two images. *[Gnädig/Honyek]*

3) Shifty radar

Officer Smith and Officer Wesson are on patrol monitoring speeding along Hwy 401. They are using a hand-held radar gun which is set to detect the speed of approaching cars, sending out waves at a frequency of 1000 MHz. After several hours of speed monitoring Wesson turns to Smith and asks, "How does this thing work anyway?" To which Smith replies, "Well now, are you familiar with the Doppler effect?"

a) Finish Smith's explanation.

Suddenly the two officers are distracted from the conversation, coffee and donuts by the sounds of tires screeching and horns honking. They look up to see a milk truck racing down the 401. Smith gives Wesson the eye and Wesson fires the radar gun.

b) If the observed frequency difference is 330 Hz, how fast is the milk truck going? *[Carrie]*

4) To air is human

A cylinder of radius r contains n moles of a monatomic ideal gas. A tight-fitting piston of total mass M is fitted into the cylinder, sealing it. The room temperature is T. Consider that the cylinder is really well thermally insulated, so all changes are 'adiabatic'.

a) What is the new resting height h (height of the piston from the base of cylinder)?

Now, we attach or add a small mass m to the piston at rest. As you may guess, it'll start to oscillate.

b) If the piston's oscillations damp out (e.g.., due to friction) what would the final h be?

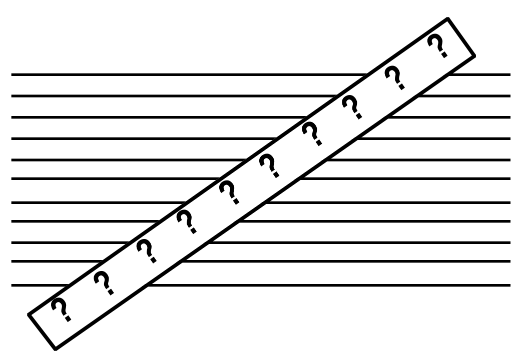
Suppose the motion is *not* damped, and that height from (b) is the equilibrium height of piston and the piston will oscillate around that equilibrium point.

c) Each oscillator can be modelled by a system of mass and spring. What is the corresponding mass and spring coefficient for this example?

d) Find the minimum amount of h in this oscillation. *[Amir]*

5) Bending the truth

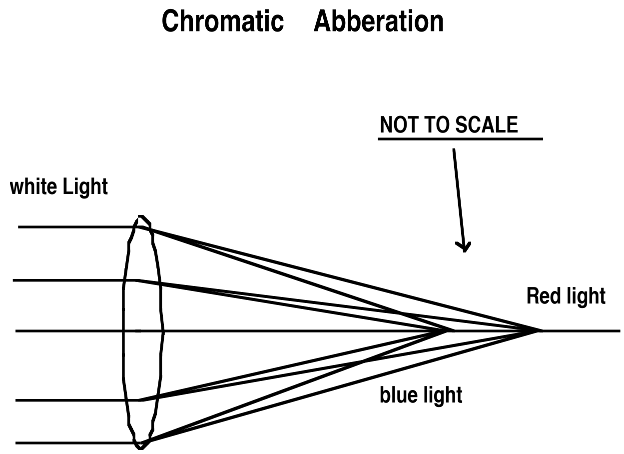
Here's an actual test used to find the index of refraction of a material: Consider a piece of paper with a drawing of a group of parallel lines. Consider also a polished cylindrical rod made of some unknown glass (for which we want to find the index of refraction). Set the rod down on the paper, at some angle θ such that $0 < \theta < \pi/2$.



a) How will the lines appear as viewed through the glass rod? Make a sketch.

b) Find the index of refraction of the glass, using details from what you see in part (a). *[Amir]*

6) Cool abrrrations!



Consider a beam of light incident on a convex lens. We expect all the parallel rays to converge at a focal point. But, in reality, the visible beam of light consists of different wavelengths from violet to red. Since the index of refraction of the lens *depends on the wavelength* we are faced with a defect called *chromatic aberration* — different wavelengths will focus at different places (see figure at right).

a) Using the lens-maker's formula, find a formula that gives the change in the focal distance of a lens due to a small change in the index of refraction.

b) One way to minimize the chromatic aberration is to put two lenses of *different* materials together, to make one new lens. Suppose one part is made of Light Crown Glass and the other from Heavy Crown Glass. Below is a table showing the index of refraction vs. wavelength for these two different kinds of glass. We are looking for a lens with f = 10 cm. What should be the focal lengths of the two Light and Heavy Crown glass lenses so as to minimize the chromatic aberration?

Index of refraction of two kinds of glass in terms of wavelength

Wavelength (nanometer)	488	520	568	632	694	890
Light Crown Glass	1.4877	1.4859	1.4836	1.4813	1.4796	1.4758
Heavy Crown Glass	1.5793	1.5766	1.5735	1.5704	1.5681	1.5634

Hint : Lens-maker formula $1/f = (n-1) * (1/R1 + 1/R2)$
f : focal length of the lens
n : index of refraction of the lens material
R1 & R2 : The radii of curvature of the two sides of the lens *[Yaser]*

Remember to check the POPTOR web-page for hints and any possible corrections!
www.physics.utoronto.ca/~poptor