

# 1999-2000 Physics Olympiad Preparation Program

— University of Toronto —

## *Problem Set 5: Electricity and Magnetism*

*Due March 17, 2000 (revised date)*

### **1) Vile away the hours**

To complete his giant, super-evil-destroyer ray gun the great, evil scientist Dr. Vile requires a copper sphere with a charge of exactly  $+3 \mu\text{C}$ . He has a copper sphere of mass 2 g.

a) What fraction of the electrons must be removed from his copper sphere to give it a charge of  $3 \mu\text{C}$ ?

Having removed the correct number of electrons Dr. Vile places the  $3 \mu\text{C}$  copper sphere near two other spheres of charges  $+7 \mu\text{C}$  and  $+5 \mu\text{C}$ . The three spheres are arranged in an equilateral triangle formation, with sides of 9 cm.

b) What is the resultant force (and its direction) on the  $3 \mu\text{C}$  copper sphere? [*Carrie*]

### **2) Capacity for thought**

Two parallel-plate capacitors with equal capacitance  $C$  are charged to voltage  $V$ . The distance between the plates is  $d$  and their area is  $A$ . There is no dielectric between the plates.

Connect the two capacitors by two wires so as to make a circuit. Obviously, there is no current in the circuit. Now, imagine that the plates of one capacitor move apart with velocity  $v$  and the plates of the other capacitor move toward each other with the same velocity  $v$ . At this time, there must be some current in the circuit. Find its value. [*Yaser*]

### **3) Dielectric City**

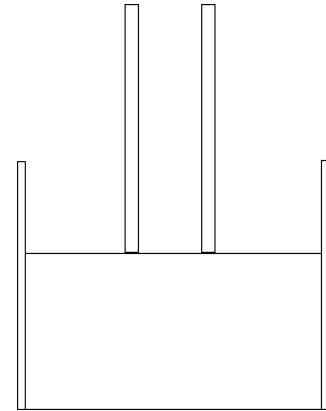
Consider two parallel-plate capacitors  $C_1$  and  $C_2$  which carry corresponding charges  $Q_1$  and  $Q_2$ .

a) What is the total energy of the two capacitors, taken as a system?

Now, we connect the two positive plates to each other and two negative plates to each other.

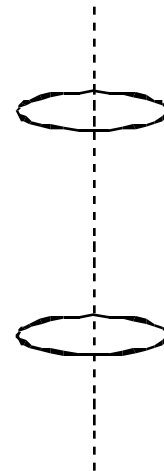
b) What will be the new energy of the system?

- c) What happened to the energy of the system when the plates were connected?
- d) What is the energy difference between an empty capacitor and a capacitor full of water, if each carry the same charge?
- e) What will happen if I put a charged-up capacitor on the surface of water, as shown in the figure? Why? How high will the column of water be when it rises into the space between capacitor plates?..... [Amir]



#### 4) Super loopy

Two identical superconducting circular loops are positioned coaxially (a line passes through both their centres, perpendicular to each) and far apart. The loops have self-inductance  $L$ . Equal currents  $I$  are passing through each of these loops in the same direction. Now, bring the loops together.



- a) What is the current in each loop?
- b) Find the difference between initial and final energy of the system? [Yaser]

#### 5) Keeping it together...

A narrow beam of electrons, of radius  $r$  and all moving at the same velocity  $v$  much less than speed of light ( $c$ ), produces a charge current  $I$ . We assume that the beam has cylindrical symmetry.

- a) What is the electric field at the edge of the beam?
- b) What is the magnetic field at the edge of the beam?
- c) What is the radial (diverging) velocity of electrons at the border of the beam, after the beam has traveled a longitudinal distance 100 times  $r$ ?
- d) What is the diverging angle for  $I = 1$  mA and  $r = 1$  cm and  $v = 1000$  m/s?

[BONUS: Since the B field depends on electron speed, can you find the speed for which the force due to B-field entirely *cancel*s the force due to E-field? Hint: think 'frame of reference'] [Amir]

## 6) A feel for fields

a) Prove that it's *impossible* to have a magnetic field which increases along the  $z$ -axis but which has only a  $z$ -component. This field must have a radial component! By choosing a short cylinder, show that for a cylindrically symmetric B-field:

$$B_r = \frac{r}{2} \frac{dB_z}{dz}$$

A loop with electrical resistance  $R$  is falling in a cylindrically symmetric magnetic field. The centre of the loop as it falls is exactly aligned with the axis of the cylindrically symmetric field, and perpendicular to it. Along this axis, the field  $z$ -component is changing as  $dB_z/dz$ . The radius of the loop is  $r$  and its mass is  $m$ .

b) Write the equation of motion governing the fall of the loop, and sketch the graph of its velocity *vs.* time.

c) What is the terminal velocity of the loop? [*Yaser*]

[Hint: you may want to use the fact that magnetic field lines are always closed loops — they cannot start or end at a point, as E-fields do...]

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