2000-2001 Physics Olympiad Preparation Program

— University of Toronto —

Solution Set 1: General

1) The path of least resistance (...and highest voltage)

If two identical batteries are connected to each other in series, the equivalent resistance is the sum of their individual resistances. The voltage across them would also be the sum of the individual electromotive forces.

When they are connected in parallel, the total resistance is half the individual resistance and the voltage would be the same as individual voltages.

If we connect all the batteries in series, we could get a high voltage but at the same time the total resistance is big and lowers the current. We could reduce the resistance by connecting all in parallel but then the total voltage is small.

The optimum case is to set the batteries in n rows and m columns. If the voltage of each battery is V and its resistance is r then the voltage in each row is mV and the resistance is mr. So we have n equivalent batteries each having voltage mV and internal resistance mr. Therefore the total votage is mV and the total resistance is mr/n. Then the current that passes through a resistor R is equal to

$$I = \frac{mV}{R + \frac{mr}{n}} = \frac{V}{\frac{R}{m} + \frac{r}{n}}$$

We have to maximize the above equation with the condition $N=nm$. Therefore we should minimize the denominator, which is the sum of two terms having the product of Rr/mn or, equivalently, Rr/N. We know that the sum of two numbers which have a constant product is minimized when they are equal.

Therefore $R/m = r/n$. It means $mn= 32$ and $m/n=2$. So $m=8$ and $n=4$. *[Yaser]*

2) The last straw...

a) By creating a partial vacuum in your mouth, atmospheric pressure forces the liquid up the straw. The difference in pressures will work only to a certain height h until the weight of the water in the straw will balance the differential-pressure force:

$$P/P_{atm} = 0.5, \quad P = 0.5P_{atm} = 5.05 \times 10^4 \text{Pa}$$
$$P_{atm} = P + \rho g h$$
$$h = (P_{atm} - P) / \rho g$$
$$= (1.01 \times 10^5 \text{Pa} - 5.01 \times 10^4 \text{Pa}) / \{(1000 \text{kg/m}^3)(9.8 \text{m/s}^2)\}$$

h = 5.15 m

Experimental Results

Note: Inhaling is different than sucking   By sucking, resting, then continuing to suck, your tongue acting as a piston can produce a vacuum effect capable of raising the liquid in the tubing higher than a height your lungs are capable of sustaining. I did the experiment both ways.

Experiment by inhalation

At the hardware store I picked up 3 1.5 meter tubes with diameters 1.58 cm, 1.90 cm, 2.54 cm. The tubing was taped along side my washing machine so as to get the tubing as straight as possible. One end of the tubing was submerged in a bucket of water . The other end was brought to my mouth. I exhaled, and then I inhaled as much as possible. As the water level rose in the tubing, it became more and more difficult to breathe in . This was not the result of me running out of lung capacity, because I was able to continue breathing in if I took my mouth away from the tubing.

Once the water level stopped rising I quickly plugged the hole of the tubing with my tongue. The level of the water in the tubing was then recorded.   My lungs, I felt, where getting tired after a few attempts so I rested every 3 to 4 attempts. 5 attempts for each tube were performed. Below are the results.

Diameter of tubing (cm):	1.58	1.9	2.54
Attempt 1	112	99	106
Attempt 2	105	114	105
Attempt 3	112	115	111
Attempt 4	106	108	117
Attempt 5	100	109	103
Average	**107 ± 2**	**109 ± 3**	**108 ± 2**

Table 1. The heights of water within a tube achieved by inhaling on different-sized tubing.

b) This result would suggest that the you could *not* raise the water level within the tubing to 5m simply by inhalation.

The results above show that the different average heights for each tube-diameter are within each other's error. The diameter of the tube had *no bearing* on the height attainable, as one expects. The weight of the water within a larger tube will exert greater force against the atmosphere, but this force is distributed over a greater surface area due to the increased tube diameter. The resulting pressure exerted by the weight of the water, Force/Area, is therefore not affected.

These results suggest that your lungs are only capable of producing a partial vacuum of 0.895 atmospheres — 0.105 ATM below atmospheric:

$$1.08 = (P - P_{atm}) \;/\; \rho g = (1.01 \times 10^5 Pa\,(1 - C) \;/\; \{(1000 kg/m^3)(9.8 m/s^2)\}$$

solving for C,

$$C = 0.895$$

Why does this differ from our earlier calculations? Because experiment shows that a typical person's lungs cannot deliver 0.5 atmospheres of partial vacuum!

Experiment by sucking on tube

It would seem that the vacuum your mouth is capable of producing relies on another mechanism besides lungs capabilities. As mentioned earlier your tongue acting as a piston may be able to produce a greater partial vacuum in your mouth. To test this idea we repeated the experiment by sucking. (Unfortunately we initially misguided you by suggesting a short length of the tubing to use — sorry about that!)

What I did was I found a tube longer than 5 m and repeated the experiment, in a stairwell this time. With a bucket of water on the ground floor I placed the tubing in the bucket and walked up a few flights of stairs. I then began to suck on the tubing, pausing occasionally to catch my breath. By placing my thumb over the opening of the tubing I maintained the water level in the tubing while resting. I continued this pattern of sucking and resting until I could no longer raise the water level. Measurements are shown below.

1.3 cm diameter tubing	Height in meters
Attempt 1	4.5
Attempt 2	4.2

The results are close to those calculated earlier! With each suck I was able to draw the water higher and higher. Not being restricted to one breath I was able to take advantage of the mechanisms my tongue is responsible for. There finally reached a point where the partial vacuum, created in my mouth, could no longer raise the water level beyond the 4.5 meter. This is the height that determines the vacuum capabilities of my mouth. Lungs & diaphragm muscles, in fact, are not nearly as sturdy as mouth muscles, though a little baby crying might leave you wondering... *[Sal]*

3) Fermi, fer you

Folks don't have to get the following answers – it only matters that you reason well, and work with whatever scraps of information you already know. It isn't OK to look up significant answers for things. You're supposed to work like on a desert island, with what's in their heads. Clever resourcefulness, inventiveness but mostly reasoning from common ideas and facts is what gets marks here.

a) *How many piano tuners do you figure there are in Toronto?*

- about 3.5 million people in Toronto — people: 3.5 M
- about 20% of households have a piano — pianos: 700 k
- about 10% of these get their pianos tuned regularly, say 2x per year, others maybe once every two years, (2 x 70 k + 0.5 x 630 k) = — tuning vists/year: 455 k
- say ~1.5 hours to tune a piano + 0.5 hours driving time ⇒ in an 8-hour day maybe 4 visits. If a tuner works 280 days/year — tuning visits/year/worker 1,120

Total tuners in whole Toronto area: **about 400**

Concert pianos probably are the specialty of a small handful of tuners; likewise some people have relatives who make a hobby of it, or have their pianos tuned by people 'off the books'. So I think this number is probably good within a factor of two.

How else can one get an idea? Try the Yellow Pages, which lists the fairly large tuning services, some of which employ more than one person; then figure that a certain number don't pay for ads, or list in regional Yellow Pages, and it probably agrees with this number.

b) *I saw lots of meteors streak across the sky this summer — they moved through an arc of about 45 degrees of the sky in a little less than a second. How fast do meteors go?*

Well, angular speed is only good if we can guess the height of the meteors. That can't be any higher than the height of the atmosphere, which isn't of course a sharp edge. I think the International Space Station orbits at something like 240 miles, if I recollect right. Also that the Russian space station Mir is going to be in trouble when it drops to about 120 miles. Now, a person can still breathe on top of Everest at 10,000 m, about the same height as airliners fly through the air, so there's a reasonable amount of air at 6 miles up (maybe roughly 50-70% of an atmosphere?) A meteor would burn in a lot less air than that. I think the space shuttle is in a fireball, with protection from its heat-tiles, at 40 miles up, so I think meteor showers must show up at heights of roughly 20-40 miles up.

In that case, 45 degrees per second is 2 π (30) / 8 miles per second, about **25 miles per second**. [is that reasonable? I remember that the space shuttle orbits at about 90 minutes per orbit, and the earth radius is about 8,000 miles, so about 25,000 miles in 90 minutes, or 17.000 mph, or almost 5 miles per second, so this seems possible].

Well, meteor showers come often (like the Leonid shower at just this time of year) from comet dust, as the earth moves in its orbit. So if the earth orbits through the dust, the speed above ought to be roughly the orbital speed: 93 million miles radius, in one year, and the earth is moving at almost 20 miles per second. So I think this probably is a pretty good fit, for a sight at night!

c) *How many kilograms of toothbrushes do the people of Canada throw away each year?*

If we followed the dentist's advice, it would be a new brush each 3-6 months, if I remember right. Do you do that? Let's say we buy 1.5 toothbrushes per year for each Canadian (about 30 million); it might only be one, but some people buy new toothbrushes for guests, and others treat their toothbrush like an old friend... Say 5 million are either kids or folks with dentures who soak 'em. That would then mean 37.5 million toothbrushes per year that get bought. Unless we store up toothbrushes, they all eventually get thrown out (even after being used for cleaning bathroom grout or polishing fussy silverware), so we ditch 37.5 million toothbrushes per year. How much do they each weigh? I've never weighed one, but I think it would take at least 20 toothbrushes to weight the same as a quarter-stick of butter — 0.25 lbs /(2.2 kg/lb) / 20 = 10 g. That might be too low: my toothbrush is about 20 cm long, and very roughly 1 cm across, in average width, about 20 cm^3. If it has about the same density as water, then that should be about 20 g.

So 37.5 million x 0.020 kg = **750,000 kg of toothbrushes each year**.

I wonder if any of this plastic could be made recyclable...

d) *If icebergs never broke off from the ice-caps at the poles of the earth, roughly how long would it take to tie up all the water on earth as ice at the poles?*

This is the toughest one. It's like the problem in winter-time that the air moisture in your room ends up frozen on your windows – it's a kind of dehumidifier or moisture-pump, and your room can get a bit drier. Mostly winter dryness is because the frigid outside air, even if it's at 80% relative humidity, holds little moisture; if you only warm up outside air, the *relative* humidity goes down quickly, because your warm house-air could have held a lot more water, but it still has only the little amount of water in frigid air.

Let's see: air doesn't circulate very well north and south, but it would eventually pass over the polar ice-caps. So, if it falls as snow on the ice-caps and never again leaves, eventually all moisture will get trapped in the ice-caps. People suspect this may be an issue for places like Mars. However, the polar regions actually get very little snow – you may remember that people take ice-cores in order to look back at pollen and spores that fell hundreds or thousands of years ago. It's one way that people test for global warming, because that biological record, frozen at different depths, indicates ancient temperatures. Now, to go back 1,000 years in the Antarctic ice cap, one doesn't have to drill 1 km – it's much less than 1 m of snow per year.

In fact, the poles are both desert regions, which by definition means less that 10 inches of precipitation falls (measured as water). So, say that exactly 10 inches falls each year over the area of each icecap. How big is each icecap? Smaller than the arctic/antarctic circles which are at 22.5 degrees, which is the tilt of the earth on its axis. You could figure out the area of a sphere within 20 degrees of the pole, but that's too picky for such a rough approximation. Good enough (with 20% probably!) to take the area of a circle at the radius of the earth, 8000 miles, and say at 15 degrees away from the pole. That means a radius about 15/360 th of the earth circumference, 25,000*(15/360) = about 1,000 miles. That area is then about 6 million square miles at each pole, for 12 million square miles total.

If 10 inches falls each year there and is locked up, and the total surface area of the earth is $4 \pi r^2$ = 780 million square miles, then the rest of the earth loses 10 * (12/780) = 0.15 inches per year. If 70% of the earth's surface is water, then the oceans and lakes would lose about 0.2 inches per year as an equivalent amount is locked up each year in the ice caps. If the ice-caps never calved icebergs, or evaporated or whatever, then in a lifetime the oceans would drop by about 16 inches, which would be noticeable. In about 300,000 years the oceans might drop by a mile. If the oceans are in only a few places a few miles deep, then the average depth might even be less than this – **in less than a million years the oceans might all be sitting as ice at the polar caps**.

Except for icebergs.

e) *How many atoms scuff off your sneakers onto the sidewalk with each step you take?*

Let's see: my sneakers pretty much wear through through the sole in about 18 months, if I wear them to walk on sidewalks outdoors and such. That sole is about 1 cm thick, but only about 1/3 the area of the sole has to wear down before they wear through under the balls of my feet. So, for my size-12s, that's like 1 foot long and perhaps 4 inches wide, or 25.4 cm x 10 cm for a net wear of 1 cm x (25.4 cm x 10 cm) / 3 = roughly 80 cm^3 of rubber & plastic. Plastic is a hydrocarbon, so it's mostly C and H in very roughly even proportions, with an average mass per atom of about 6.5 a.m.u. It has a density a little above water, I think, so say 1.5 g/cm^3. So that's about 120 g of rubber.

At 6.5 a.m.u. average, for the atoms, that means 6.5 g per *mole of sole* (per mole of atoms in the soles of my shoes) – so I lose a little less than 20 moles of atoms from my soles in one year.

And for one step? Well, how many steps in a year? Hmmm, well I walk about 600 m to the streetcar from home, and about 500 m at the other end – 2.2 km just getting to work. My steps are about 1 m per pace (I'm tall), so that's 2,200 steps then. I walk for about 1 hour net, between classes and offices, during the day while I'm at work, and another 20 minutes at lunch. I figure I take about 1 step per second (very roughly), so that's 1 1/3 hours x 3600 seconds/hour x 1 step/second = 4,800 steps. I take my sneakers off at home. So I walk about 7,000 sneaker-steps each day, roughly, 300 days per year because I don't always wear sneakers. So a little over 3 million steps in 18 months to remove about 20 moles ($20 \times 6 \times 10^{22}$) of atoms.

So with each step I must be leaving behind roughly 4×10^{17} atoms! Yikes, one day people ought to be able to figure out where I walked! They just need to make a combination upright vacuum cleaner and mass spectrometer!

REMINDER: all these values are very rough – within a factor of 2 or 3 either way is reasonable. The important thing is the reasoning, as Sherlock Holmes might once have said. *[Robin]*

4) Number four with a bullet

The Physics – The key to this problem is realizing that the acceleration of the bullet is *not constant* while traveling though the cylinder of gas: From the drag equation $F_{\text{drag}} = -\frac{1}{2}\varepsilon\rho A v^2$ we know that bullet experiences a force that is a function of its velocity. Since F = ma and the mass of the bullet is constant, the acceleration has to be a *function* of velocity. Therefore, the following constant acceleration equations *cannot* be used:

$$x(t) = x(0) + v(0)t + \tfrac{1}{2}at^2 \qquad (1)$$
$$v(t) = v(0) + at \qquad (2)$$

including the variations on equations (1) and (2):

$$v(t)^2 = v(0)^2 + 2a\left(x(t) - x(0)\right) \qquad (3)$$
$$x(t) = x(0) + \tfrac{1}{2}\left(v(t) + v(0)\right) \qquad (4)$$
$$x(t) = x(0) + v(t)t - \tfrac{1}{2}at^2 \qquad (5)$$

In this question, only the following fundamental definitions apply:

$$v = \frac{dx}{dt} \qquad (6)$$
$$a = \frac{dv}{dt} \qquad (7)$$
$$F = ma \qquad (8)$$

The Solution – Given quantities converted to SI units where appropriate:

Variable Name	Definition	Value
v(0)	Initial velocity of bullet	250 m/s
m	Mass of Bullet	2.00×10^{-3} kg
r	Radius of bullet	4.500×10^{-3} m
L	Length of cylinder	15.0 m
R	Radius of cylinder	0.500 m
T	Initial temperature of gas	300 K
P	Pressure of argon gas in cylinder	2.027×10^{6} Pa
ε	Drag coefficient for a sphere	0.500
Mm	Molar mass of argon gas	39.948 g/mol
C	Specific heat of argon gas	312.5 J/kg/K

Quantities to be calculated:

Variable Name	Definition
v(t)	Final velocity of bullet after traversing the cylinder
t	Time it takes for the bullet to traverse the cylinder
V	Volume of cylinder
A	Maximum cross-sectional area of bullet
ΔT	Change in gas temperature
M	Mass of argon gas in cylinder

- Calculate maximum cross-sectional area of bullet:

  $A = \pi r^2 = \pi(4.50 \times 10^{-3})^2 = 6.362 \times 10^{-5}\ \text{m/s}$
- Calculate cylinder volume:

  $V = \pi L R^2 = \pi(15.0)(0.500)^2 = 11.78\ \text{m}^3$
- Calculate density of argon gas in cylinder:

  $$PV = nRT\ \ (\textit{Ideal Gas Law}) \Rightarrow \frac{n}{V} = \frac{P}{RT}$$

  $$\rho = \frac{n}{V}Mm = \frac{PMm}{RT} = \frac{(2.027 \times 10^6)(39.948)}{(8.31451)(300)} = 3.246 \times 10^4\ \text{g/m}^3 = 32.46\ \text{kg/m}^3$$
- Calculate the mass of argon gas in the cylinder:

  $M = \rho V = (32.46)(11.78) = 382.4\ \text{kg}$

Part A

- Calculate the time it takes for the bullet to transverse the cylinder:

  $$ma = F_{drag} = -\frac{1}{2}\varepsilon\rho A v^2$$

  since $a = \frac{dv}{dt}$, then

$$m\frac{dv}{dt} = -\frac{1}{2}\varepsilon\rho A v^2$$

$$\frac{dv}{dt} = -\frac{\varepsilon\rho A}{2m}v^2 = -\beta v^2$$

this is a differential equation – what function v(t) can be differentiated once to equal itself-squared, multiplied by a constant –β ?

where $\beta = \frac{\varepsilon\,\rho\,A}{2m} = \frac{(0.5)(32.46)(6.362\times10^{-5})}{2(2.00\times10^{-3})} = 0.2581\ \text{m}^{-1}$

(see _A Guide to Solving Simple Ordinary Differential Equations (ODE's)_ on the POPTOR webpage for hints etc. for this problem set to understand the following steps)

$$\frac{dv}{v^2} = -\beta\, dt$$

$$\int_{v(0)}^{v(t)} \frac{dv}{v^2} = -\beta\int_0^t dt$$

$$\frac{1}{v(t)} - \frac{1}{v(0)} = \beta t$$

$$\frac{1}{v(t)} = \beta t + \frac{1}{v(0)} = \frac{v(0)\,\beta\, t + 1}{v(0)}$$

$$\boxed{v(t) = \frac{v(0)}{v(0)\,\beta\, t + 1}}$$

equation for the velocity of the bullet as a function of time

We can't solve for time *t* since we don't know what *v(t)* is. However, we do know what the distance is so use the definition for velocity:

since $v = \frac{dx}{dt}$, then

$$\frac{dx}{dt} = \frac{v(0)}{v(0)\,\beta\, t + 1}$$

this is another differential equation

(again see _A Guide to Solving Simple Ordinary Differential Equations (ODE's)_)

$$dx = v(t)dt = \frac{v(0)}{v(0)\,\beta\, t + 1}dt$$

$$\int_0^L dx = v(0)\int_0^t \frac{1}{v(0)\beta\, t + 1}dt$$

$$L = \frac{v(0)}{v(0)\beta}\ln\left(v(0)\beta\, t + 1\right) = \frac{1}{\beta}\ln\left(v(0)\beta\, t + 1\right)$$

$$\ln\left(v(0)\beta\, t + 1\right) = L\beta$$

$v(0)\beta\, t+1=e^{L\beta}$

$$\boxed{t=\frac{e^{L\beta}-1}{v(0)\beta}}$$ equation for the time it takes the bullet to transverse the cylinder

substitute in the numbers:

$$t=\frac{e^{(0.2581)(15.0)}-1}{(250)(0.2581)}=0.726\text{ s}$$

Part B

Using the equation for velocity and the value for time found in Part A:

$$v(t)=\frac{v(0)}{v(0)\beta\, t+1}=\frac{(250)}{(250)(0.2581)(0.726)+1}=5.23\text{ m/s}$$

Part C

the change in the bullet's kinetic energy is converted to heat in the gas:

$$\Delta E_K=\Delta Q$$

$$\frac{1}{2}m\,v(t)^2-\frac{1}{2}m\,v(0)^2=\frac{1}{2}m\left(v(t)^2-v(0)^2\right)=MC\Delta T$$

$$\boxed{\Delta T=\frac{m\left(v(t)^2-v(0)^2\right)}{2MC}}$$ equation for change in temperature of argon gas

substitute in the numbers:

$$\Delta T=\frac{(2.00\times 10^{-3})((250)^2-(5.23)^2)}{2(382.4)(312.5)}=5.23\times 10^{-4}\text{ K}$$

FINAL ANSWERS

Question	Answer
Part A	$t=0.726$ s
Part B	$v(t)=5.23$ m/s
Part C	$\Delta T=5.23\times 10^{-4}$ K

[Brian]

5) Bubbling ideas

a) The temperature of the air inside the bubble is always the ambient temperature. Therefore, pV = constant, where V is the volume of the bubble. So if the volume changes by a small amount ΔV then (using the product rule) we have

$$\Delta(pV) = \Delta p \cdot V + p \cdot \Delta V = 0 \quad \Rightarrow \quad \Delta p = -p\frac{\Delta V}{V}$$

b) If we put a charge on the bubble, the bubble size increases as each part of the bubble is repelled from every other part. This will reduce the internal pressure.

The electric field *outside* the bubble at the surface is $Q/(4\pi\varepsilon R_0^2)$ where R_0 is the radius of the bubble. The field *inside* is zero. We could say that the field exactly *on* the bubble is the average of these two fields $Q/(8\pi\varepsilon R_0^2)$. A full-theory calculation gives exactly this answer.

Therefore, the electrostatic pressure is

$$P_E = \frac{E\sigma\Delta A}{\Delta A} = \frac{Q}{8\pi\varepsilon R_0^2}\frac{Q}{4\pi R_0^2} = \frac{Q}{32\varepsilon\pi^2 R_0^4}$$

where σ is the surface charge density, E is the electric field on the bubble and ΔA is the area of a small piece on the bubble.

This pressure should be equal to the change in the internal gas pressure, following its change in size

$$P_E = -\Delta p \rightarrow \Delta V = \frac{V}{P}\frac{Q}{32\varepsilon\pi^2 R_0^4} \rightarrow \Delta R = \frac{Q}{96\varepsilon\pi^2 R_0^3 P}$$ *[Peter & Yaser]*

6) Magnus matters

a) If the tube is not spinning, it just falls straight down. If it is spinning, then it doesn't fall straight, it scoots out to one side or the other, depending on which way it was spinning. If the tube is spinning so one side moves backwards with the passing air, and the other side moves forwards against the passing air, the tube swings *away* from the side that moves forward against the passing air (greatest relative speed).

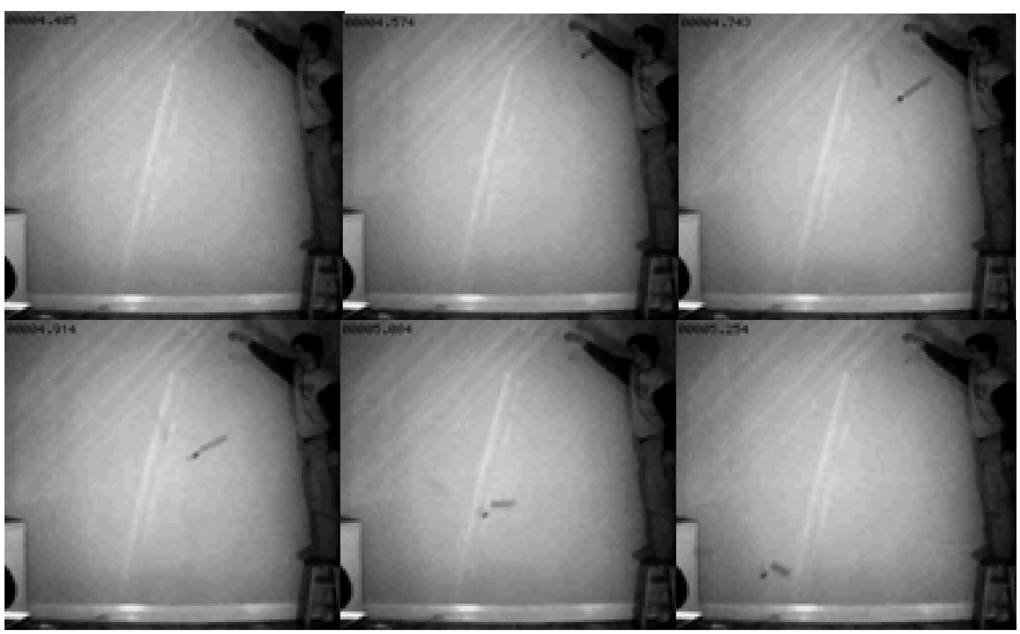
When the tube falls relatively slowly, as it does for us, the flow around the tube is pretty well-behaved — no very serious turbulence (which would make the problem tougher). We call the flow *laminar*, which means it flows in smooth layers. Now, the air is very slightly *viscous* — as syrup is more viscous than water, and water is more viscous than alcohol. That means that as the tube moves, it carries a layer of air along with it, close to its surface.

It also means that when the tube rotates, it carries the air with it then. Imagine the tube holding still, and the air rushing past (as in a wind tunnel). The side of the tube rotating forward *against* the oncoming air will actually slow the air slightly; the side of the tube rotation *with* the oncoming air will carry the airflow a little faster. This means the air is slower on the side of the tube that rotates against the flow, and faster on the other side. Bernouilli's principle (which really is just a conservation of energy formula for fluids!) means that the air pressure will be higher on the side moving against the airflow, and less on the other side. The net pressure difference is a force, which pushes the tube sideways — the *Magnus Force*.

Bernouilli's principle is a big part of how an airplane wing generates lift out of the forward thrust of the engines. It's also the reason why a strip of paper, held below your lower lip, will rise as you blow air across the top surface.

[EXTRAS: When the object moves very quickly (like a fast baseball, or many real airplane wings), it isn't any more exactly the Bernouilli effect, but a bit more complicated. What happens there has to do with the transition that airflow makes when it isn't smooth, but becomes turbulent and begins to detach from the flow around the tube. When the relative airspeed is higher, the flow detaches earlier in the stream around an object; where the surface moves back with the airflow, it delays this *boundary-layer separation*. The result is that the air flowing past a spinning body is tilted, in the stream past the object. So the air in the wake of the spinning tube actually kicks over to one side. The momentum change of that air results in the force pulshing sideways. (Look for a streaming-video description of this on the archive of the Science of Baseball website, from a show on the Discovery Canada: www.exn.ca)]

b) I recorded this on my computer, using a cheap CCD web-camera and shareware NIH Image 1.62 [http://rsb.info.nih.gov/nih-image], which can put a timestamp on each frame to help analyse motion. Here are a few frames in the sequence:



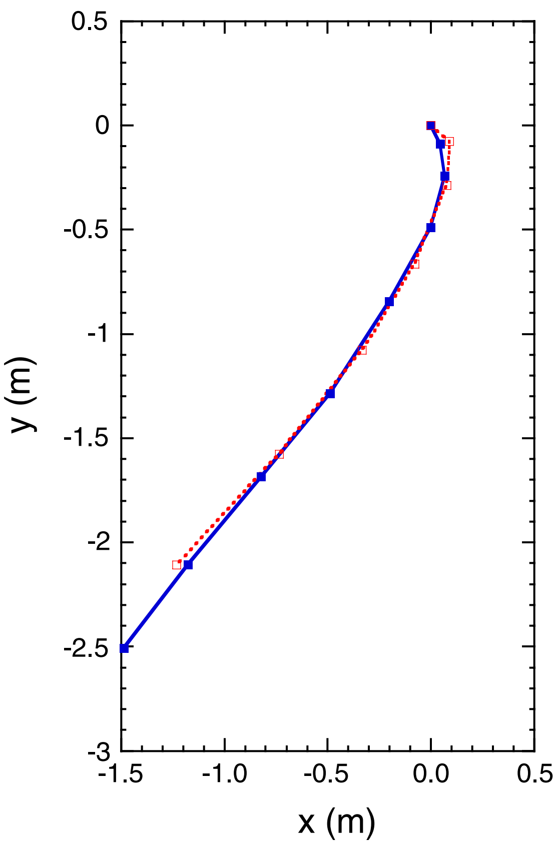
You don't have to do this, though — you can get very nearly as much by finding where it hits the ground, measured from where you let it go, and using a stopwatch to time it.

b) I measured from the video frames on my computer. If you use a video camera, you can measure from the TV screen. You might want to use a dry-erase felt pen (but remember to erase it afterwards!); but be careful about using any markers at all on a computer monitor, since they are sometimes anti-reflection coated with a material that dissolves (they used to often dissolve with ammonia-based cleaners like Windex too! I'm not sure this still is true, so check the manual for your display for warnings!)

In the graph on this page, each marker on the trajectory is a measurement at a different time (the blue ones are a little more than 0.1 second apart). The path *is* curved: it starts off going to the right, because of the elastic band I used to spin it, I think. Then it drops, but as it gains speed it begins to push to the left.

c) The angle of descent was about 45°, so the tube moves at the same speeds in the downwards direction and the sideways direction. It almost reaches a constant speed (*terminal velocity*) after 1 second, when it hits the floor, but doesn't quite get there. If you used a balcony in your school, you probably did better than I did. I *was* able to extrapolate that the terminal velocity would be around 3.8 m s^{-1}.

d) At terminal velocity, there is no acceleration — all forces balance. Then the 45° trajectory indicates that the net force is at 45° down and to the left, so in this case the Magnus force is about the same as gravity, at terminal velocity. People have patented 'sail'-steamships with tall rotating cylinders for sails; they could use wind-power to make the ship tack into the wind at 45° this way.



From POPBits™, at the end of question-set #1:

$$F_{drag} = \varepsilon \rho A v^2$$

The mass of my cardboard tube was 25 g, and its cross-sectional area was 125 cm^2 (see comments below). Balancing forces, and therefore setting ma = F_{drag} at terminal velocity, we can solve simply for $v_{terminal}$:

$$m\,a = \varepsilon \rho A v_{terminal}^2$$

$$0.025\text{ kg} \bullet 9.8\text{ m s}^{-2}$$

$$= 1.0 \bullet 1.2928\text{ g L}^{-1} \bullet (10^3\text{ L m}^{-3}) \bullet 0.0125\text{ m}^2 \bullet v_{terminal}^2$$

Thus $v_{terminal} = 3.9$ m s^{-1}

as compared to 3.8 m s^{-1} measured above (this worked unusually well for such a formula). The sideways Magnus force is comparable to the force of gravity, so the x-component of velocity is about the same as this value, and the final terminal velocity is the resultant, about √2 times larger.

e) BIG BONUS: You can come fairly close to the right answer with this slightly hand-waving reasoning (you can also have a look in The Physics of Baseball (2nd Ed.), Robert K. Adair, Harper Collins, NY, 1994, ISBN 0-06—95047-1).

The drag force is:

$$F_{drag} = \varepsilon \rho A v^2$$

which can be described as a pressure-differential ΔP (front – rear) multiplied by the area presented to the wind or rushing air. Since this pressure-differential depends on the relative speed, if an object spins there will be a difference in the force from side-to-side across the ball. Then

$$F_{Magnus} = \Delta P_{left\text{-}right} A = \varepsilon \rho A (v_{right}^2 - v_{left}^2)$$

now,

$$v_{right} = v + 2\pi r \bullet f$$

$$v_{left} = v - 2\pi r \bullet f$$

where f is the rotation frequency (revolutions per second) of the object. Substituting:

$$(v_{right}^2 - v_{left}^2) = (v + 2\pi r \bullet f)^2 - (v - 2\pi r \bullet f)^2$$

$$= 8\pi\, r\, v\, f$$

so

$$F_{Magnus} = \varepsilon \rho A (8\pi\, r\, v\, f)$$

For our case at 45° falling angle, $F_{drag} = F_{Magnus}$ so we can figure out the spinning frequency f:

$$(8\pi\, r\, v\, f) = v^2$$

which we solve to find $f = 6.3 \text{ rev s}^{-1}$

I do think the elastic band setup I used could provide this spin-rate. But, in fact, this hand-waving argument fudged a little on the appropriate use of areas and direction of airflow. Even so, any error is by a constant factor: the formula shows the appropriate dependence on speed v and spin-rate f.

Comments on my own setup

Cardboard tube:

circumference 14 cm --> diameter d = 4.5 cm
length 28 cm
cross-sectional area presented to wind (rectangle) $A = 125 \text{ cm}^2$
mass 25 g

(I didn't have a weigh-scale at home, so I took a 25 cm rod, a handle from a cat-toy, and hung a plastic grocery bag tied to a string on one end, and just a piece of string on the other. The rod had a piece of string tied around the middle to suspend the whole thing to tilt freely. I slid the string along the middle of the rod until the grocery bag and loose string balanced each other evenly. Then I taped the cardboard tube to the loose string, which tipped the balance over. I then dripped water drop-by-drop into the plastic bag until it balanced the cardboard tube again — it took 25 ml, which weighs 25 g.) *[Robin]*