

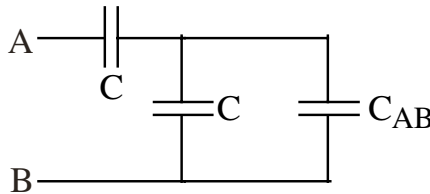
2001-2002 Physics Olympiad Preparation Program

– University of Toronto –

Solution Set 6: Electronics and Modern Physics

1) Everyone has the capacity to learn the quadratic formula!

Since this is an infinite chain, removing the two leftmost capacitors still leaves the same chain with the same total capacitance C_{AB} .



From this we see that we must have

$$\begin{aligned}\frac{1}{C_{AB}} &= \frac{1}{C} + \frac{1}{C + C_{AB}} \\ \therefore C(C + C_{AB}) &= C_{AB}(C + C_{AB} + C) \\ \therefore C_{AB}^2 + C_{AB}C - C^2 &= 0 \\ \therefore C_{AB} &= \frac{-C \pm \sqrt{C^2 + 4C^2}}{2} = C \frac{-1 \pm \sqrt{5}}{2}\end{aligned}$$

We can't have a negative capacitance, so

$$C_{AB} = \frac{\sqrt{5} - 1}{2} C$$

2) I always resist getting a shock

(a) We can estimate the magnetic field using the formula for an infinitely long solenoid:

$$\begin{aligned}B &= \mu_0 n I \\ \therefore B &\approx \mu_0 \frac{N}{L} I = 4\pi \times 10^{-7} \frac{\text{N}}{\text{A}^2} \frac{1200}{5.3\text{m}} 8000\text{A} = 2.3\text{T}\end{aligned}$$

So the magnetic field will be about 2 Tesla.

(b) Since the resistance is in parallel, no current flows in it as long as current can flow through the superconducting (and hence zero resistance) solenoid, but when the solenoid is switched off, all the current flows through the resistor and the voltage across the resistor is the voltage across the switch. The initial current is just the current originally flowing through the solenoid, and the maximum voltage is just the initial voltage:

$$V_{\max} = I_0 R$$

and the required resistance is

$$R = \frac{V_{\max}}{I_0} = \frac{80\text{V}}{8000\text{A}} = 0.01\Omega$$

The peak power dissipated is

$$P = I_0 V_{\max} = 8000 A \times 80 V = 0.64 MW$$

To be safe we should round up the size, so what we need is a 0.01Ω , 1MW resistor. This is not something you find at your local Radio Shack¹.

- (c) The resistor and the solenoid form an LR circuit with an exponential time constant, $\tau=L/R$. The inductance per unit length for an infinite solenoid is

$$\frac{L}{l} = \mu_0 n^2 \times Area$$

so the inductance of the solenoid is approximately

$$L = \mu_0 \frac{N^2}{l^2} (\pi r^2) l = \pi \mu_0 \frac{N^2 r^2}{l}$$

and the time constant is

$$\begin{aligned} \tau &= \frac{L}{R} = \pi \mu_0 \frac{N^2 r^2}{R l} \\ &= 4\pi^2 \times 10^{-7} \frac{kg \cdot m}{C^2} \frac{1200^2 (1.2m)^2}{(0.01\Omega) 5.3m} \\ &= 154s \end{aligned}$$

The current decay is exponential, so to drop to one tenth of the original voltage will take

$$\tau_{1/10} = \tau \ln 10 = (154s) 2.3 = 356s$$

i.e. about 6 minutes.

3) Cracked!

The broken wire is equivalent to two resistances and the crack is a dielectric filled parallel plate capacitor, all in series. Since they are part of the same wire, their resistivity, ρ , and cross-sectional area, A , are the same. The resistances of the two pieces of wire are

$$R_1 = \rho \frac{L_1}{A} \quad R_2 = \rho \frac{L_2}{A}$$

And the total resistance is

$$R = R_1 + R_2 = \rho \frac{L_1 + L_2}{A} = \rho \frac{L}{A}$$

where $L=10m$ is the total length of the wire.

The capacitance of the crack is

$$C = \kappa \epsilon_0 \frac{A}{d}$$

where d is the width of the crack and κ is the dielectric constant.

The RC time constant is

¹ <http://www.radioshack.ca/en/>

$$\tau = RC = \rho \frac{L}{A} \kappa \epsilon_0 \frac{A}{d} = \kappa \epsilon_0 \rho \frac{L}{d}$$

So the crack width is

$$\begin{aligned} d &= \kappa \epsilon_0 \rho \frac{L}{\tau} \\ &= 7 \cdot 8.854 \times 10^{-8} F/m \cdot 1.7 \times 10^{-8} \Omega \cdot m \frac{10m}{2 \times 10^{-9} s} \\ &= 5 \times 10^{-9} m \\ &= 5nm \end{aligned}$$

4) Gravity isn't Bohring

(a) The energy levels of an atom are determined by the quantization condition that orbital angular momentum is quantized in units of Planck's constant over 2π , *i.e.*

$$L_n = m_e v_n r_n = n \frac{h}{2\pi} = n\hbar \quad n = 1, 2, 3, \dots$$

Using a simple Bohr atomic model for our gravitational hydrogen atom, the attractive gravitational force must balance the centripetal force, *i.e.*

$$G_N \frac{m_e m_p}{r_n^2} = \frac{m_e v_n^2}{r_n}$$

Using our quantization condition and solving this gives

$$r_n = \frac{n^2 \hbar^2}{G_N m_e^2 m_p}$$

The total (potential+kinetic) energy of the electron is thus

$$\begin{aligned} E_n &= \frac{1}{2} m_e v_n^2 - G_N \frac{m_e m_p}{r_n} \\ &= \frac{1}{2} G_N \frac{m_e m_p}{r_n} - G_N \frac{m_e m_p}{r_n} \\ &= -\frac{1}{2} G_N^2 \frac{m_p^2 m_e^3}{n^2 \hbar^2} \\ &= \frac{-0.26 \times 10^{-77} eV}{n^2} \end{aligned}$$

The energy difference between the ground (n=1) and first excited state (n=2) is

$$\Delta E = -2.6 \times 10^{-79} eV \left(\frac{1}{2^2} - \frac{1}{1^2} \right) = 2 \times 10^{-78} eV$$

This is 79 orders of magnitude smaller than the corresponding energy in our universe (10.2eV).

- (b) The smallest radius of a gravitationally bound hydrogen atom is that of its $n=1$ ground state

$$r_1 = \frac{\hbar^2}{G_N m_e^2 m_p} = 1.2 \times 10^{29} m = 1.2 \times 10^{13} \text{ light years}$$

This is 3 orders of magnitude large than the universe, so a gravitationally bound hydrogen atom cannot exist in the imaginary universe.

5) Resisting the light

- (a) I used a 15W bulb, the measured resistance was about 85 ± 1 ohms.
- (b) The resistance of the bulb when operating should be $R = V^2 / \text{Power} = (120V)^2 / W = 960$ ohms.
- (c) The operating resistance is an order of magnitude higher than the measured room temperature resistance, so the the resistance of the filament must increase with temperature.