2002-2003 Physics Olympiad Preparation Program - University of Toronto - Solution Set 1: General

1. The equations of kinematics in one dimension are

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2 \ , \tag{1}$$

$$v_x^2 = v_{x0}^2 + 2a_x(x - x_0) \ , \tag{2}$$

where x_0 is the initial position, t is the time, a_x is the acceleration, and v_{x0} is the initial velocity. The ball starts at $(x_0, y_0) = (0, 10)$ m. It hits the ground at (x_1, y_1) and arrives at $(x_2, y_2) = (15, 0)$ m, the position of AM. At (x_1, y_1), we have

$$x_1 = x_0 + v_{x0}t_1 + \frac{1}{2}a_x t_1^2 \ , \tag{3}$$

$$y_1 = y_0 + v_{y0}t_1 + \frac{1}{2}a_y t_1^2 \ . \tag{4}$$

The acceleration in the x-direction (y-direction) is $a_x = 0$ ($a_y = -g \approx -10$ m/s^2). Rearranging Eq. 3 yields $t_1 = \frac{x_1 - x_0}{v_{x0}}$. This can be substituted into Eq. 4 to give

$$(y_0 - y_1)v_{x0}^2 + (x_1 - x_0)v_{y0}v_{x0} - \frac{1}{2}(x_1 - x_0)^2 g = 0 \ . \tag{5}$$

There is a similar equation involving (x_1, y_1), (x_2, y_2), v_{x1}, and v_{y1}. Since $a_x = 0$, $v_{x1} = v_{x0}$. To find v_{y1}, solve $v_{y1}^2 = v_{y0}^2 - 2g(y_1 - y_0)$. Choose a particular x_1, and y_1, and then compute v_{x0} and v_{y0}. For example, if AE (Albert Einstein) throws the ball with an initial velocity $(v_{x0}, v_{y0}) \approx (3.29, 6.39)$ m/s, it reaches AM (Albert Michelson) after one bounce at $(x_1, y_1) = (6, 5)$ m. You should check that the ball clears the raised ledge following its impact at (x_1, y_1).

2. Superpose the waves from the violins by adding them

$$F(t) = f_1(t) + f_2(t) = A sin(2\pi\nu t + \phi_1) + A sin(2\pi\nu t + \phi_2) \tag{6}$$

The loudness or intensity of a wave is proportional to the square of its wave function. The intensity for one violin is

$$\text{Intensity} = C(f_1(t))^2 = CA^2 sin^2(2\pi\nu t + \phi_1) \ , \tag{7}$$

where C is a constant and for two violins

$$\begin{aligned} \text{Intensity} &= C(f_1(t) + f_2(t))^2 = CA^2[sin^2(2\pi\nu t + \phi_1) + sin^2(2\pi\nu t + \phi_2) \\ &+ 2sin(2\pi\nu t + \phi_1)sin(2\pi\nu t + \phi_2)] \end{aligned} \tag{8}$$

Consider two cases. If the phase factors do not vary randomly, and in particular $\phi_1 = \phi_2$, then at certain times the intensity reaches $I = 4CA^2$. This is four times as loud as one violin. If the phase factors vary in a completely random way, then the last term in Eq. 8 is on average 0. The loudness is on average $I = 2CA^2$. This is twice as loud as one violin.

3. The energy flux density emitted by the Sun is $J_S = \sigma T_S^4$. This gives the energy per unit time flowing through unit area. The total energy per unit time going through the surface of the Sun is the energy flux density times the surface area of the Sun, namely, $(4\pi R_S^2)J_S$. If there is no energy lost as this radiation propagates between the Sun and Venus, then the same amount of energy per unit time must reach the orbit of Venus. In equilibrium, the radiation that Venus intercepts equals the amount it emits.

$$(4\pi R_S^2)\sigma T_S^4 \frac{\pi R_V^2}{4\pi R_{SV}^2} = (4\pi R_V^2)\sigma T_V^4 \ , \tag{9}$$

$$T_V^4 = \frac{R_S^2}{4R_{SV}^2} T_S^4 \ , \tag{10}$$

$$T_V = \sqrt{\frac{R_S}{2R_{SV}}} T_S \ . \tag{11}$$

Substituting for R_S, R_{SV}, and T_S gives $T_V \approx 330$ Kelvin. This is lower than the measured temperature of Venus of about 700 Kelvin. The atmosphere of Venus contains a large amount of carbon dioxide. This is a "greenhouse" gas. It traps the radiation re-emitted by the planet and leads to a higher surface temperature. Could this effect happen on Earth?

4. You can charge an object such as a comb or balloon by brushing or rubbing it through your hair. Bringing a charged object O close to the can redistributes the charges in the can. If object O is negatively charged, then it repels the negatively charged electrons in the can. There is an excess of positive charge on the surface of the can close to where O is located. There is an attractive electric force between the can and O. Friction between the can and the table produces a turning force or torque about the can's central axis (centre of mass or CM). Depending on the placement of O, the electric force could also produce a torque about the CM. The torque results in the can's rolling motion. The can's motion can be halted by bringing O to the side of the can that is opposite to its direction of motion.

5. Each bulb acts as a resistor with resistance r. The electromotive force of the battery is V. When all of the bulbs are in place, there is a current $i_1 = \frac{V}{3r}$ ($i_2 = \frac{V}{2r}$) through the branch with ABC (DE). The brightness of A, B, and C are equal. They will be less bright than that of D and E, since less current is going through the ABC branch. If bulb A is removed, then no current flows through the ABC branch of the circuit. The brightness of B and C are zero. The brightness of D and E are the same and unchanged from the situation in which all of the bulbs were connected. If bulb A is replaced by two new bulbs, then bulbs A1, A2, B, and C will be equally bright. The current flowing through the ABC branch is $i_1 = \frac{V}{4r}$. The bulbs A1, A2, B, and C will be dimmer than before when ABCDE were connected. Bulbs D and E will have equal brightness and maintain the same level in all of the above situations.