2002-2003 Physics Olympiad Preparation Program - University of Toronto - Solution Set 2: Mechanics

1. The equation of motion for the cylinder along the surface of the incline is

$$Ma \;=\; Mgsin\alpha - f \;, \tag{1}$$

where a is the linear acceleration and f is the force due to friction (Figure 1). The equation for the angular motion about the cylinder's axis is

$$IA \;=\; fR \,, \tag{2}$$

where $I = \frac{1}{2}MR^2$ is the moment of inertia of the cylinder and $A = \frac{a}{R}$ is the angular acceleration. Solve Eq. 2 for f. Substitute $f = \frac{IA}{R} = \frac{Ia}{R^2}$ into Eq. 1 and solve for a.

$$Ma \;=\; Mgsin\alpha - \frac{Ia}{R^2} \rightarrow a = \frac{Mgsin\alpha}{M + \frac{I}{R^2}} = \frac{2}{3}gsin\alpha \;. \tag{3}$$

The coefficient of static friction is specified by

$$\mu_s \geq \frac{f}{N} = \frac{\frac{1}{3}Mgsin\alpha}{Mgcos\alpha} = \frac{1}{3}tan\alpha \;, \tag{4}$$

where N is the normal force. The minimum coefficient that is required is $\frac{1}{3}tan\alpha$.

2. As they travel in a circular orbit of radius r and speed v, each planet feels a centripetal force $F_c = \frac{Mv^2}{r}$ (Figure 2). The gravitational force on each planet is

$$F_G \;=\; F_{12} + F_{14} + F_{13} = 2cos(\frac{\pi}{4})\frac{GMM}{L^2} + \frac{GMM}{(2r)^2} = \frac{GM^2}{L^2}(\sqrt{2} + \frac{1}{2}) \;, \tag{5}$$

where $(2r)^2 = L^2 + L^2$ implies that $r = \frac{L}{\sqrt{2}}$. Equate the centripetal and gravitational forces to find v

$$\frac{Mv^2}{\frac{L}{\sqrt{2}}} \;=\; \frac{GM^2}{L^2}(\sqrt{2} + \frac{1}{2}) \rightarrow v^2 = \frac{GM}{L}(1 + \frac{\sqrt{2}}{4}) \;. \tag{6}$$

The speed at which the planets must move is $v = \sqrt{\frac{GM}{L}(1 + \frac{\sqrt{2}}{4})}$.

3. Due to the conservation of linear momentum, the initial and final linear centre of mass (CM) momenta of the objects are zero. Conservation of energy tells us that

$$2 \times \frac{1}{2}(2m)v^2 \;=\; 2 \times \frac{1}{2}(2m)v'^2 + \frac{1}{2}I\omega^2 \rightarrow v^2 = v'^2 + \frac{1}{8}L^2\omega^2 \;, \tag{7}$$

where v (v') is the speed before (after) the collision, $I = m(\frac{1}{2}L)^2 + m(\frac{1}{2}L)^2 = \frac{1}{2}mL^2$ is the moment of inertia of the left object (LO), and ω is the angular velocity of LO about its CM. Conservation of angular momentum requires that

$$(2m)(\frac{L}{2})v \;=\; (2m)(\frac{L}{2})v' + I\omega = mLv' + \frac{1}{2}mL^2\omega \;, \tag{8}$$

$$v' \;=\; v - \frac{1}{2}L\omega \;, \tag{9}$$

where the first two terms of Eq. 8 are angular momenta calculated with respect to the collision point of the objects and the third term of Eq. 8 is the angular momentum about the CM of LO. Substitute Eq. 9 into Eq. 7 to establish

$$v^2 = (v - \frac{1}{2}L\omega)^2 + \frac{1}{8}L^2\omega^2 \rightarrow -vL\omega + \frac{3}{8}L^2\omega^2 = 0\,. \tag{10}$$

One solution is $\omega = 0$ and $v' = v$. This implies that the objects pass through one another with no collision. Another solution is $\omega = \frac{8v}{3L}$ and $v' = -\frac{1}{3}v$. Following the collision, the objects rebound from each other. Their speeds are reduced and LO rotates about its CM.

4. Suppose the hole is rectangular in shape and has a cross-sectional area A and length z. The energy of each molecule leaving the box is $\frac{1}{2}mv^2$. If there are N molecules, then the total energy per unit volume is $\frac{N\frac{1}{2}mv^2}{Az} = \frac{\frac{1}{2}(mN)v^2}{Az} = \frac{1}{2}\rho v^2$ where $\rho = \frac{Nm}{Az}$ is the mass density of the molecules. Pressure and energy per unit volume have the same dimensions. To obtain an estimate of the speed of the molecules, equate the pressure P to the above total energy per unit volume to find $P = \frac{1}{2}\rho v^2$ and $v = \frac{2P}{\rho}$.

5. Each part of the wall of the smaller balloon B2 is more curved and under greater tension than that of the larger balloon B1. In order to support the high tension in the wall of B2, the air must exert a bigger pressure P2 than that in B1. Once the balloons are allowed to equilibrate, the larger P2 forces air from B2 into B1. So the smaller balloon decreases in size and the larger balloon becomes even bigger.

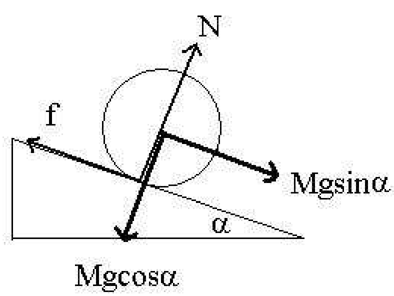

Figure 1

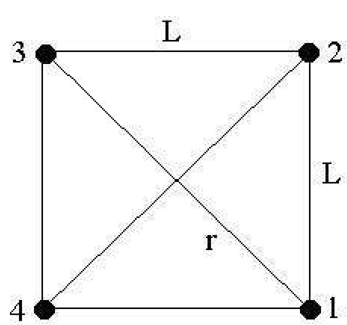

Figure 2