2004-2005 Physics Olympiad Preparation Program

– University of Toronto –

Solutions. Set 1: General

Problem 1

Two identical pieces of steel wire of equal length were used to manufacture two springs. Diameter of the first spring coil was *d*, diameter of the second was *2d*. Both springs were then loaded with the equal masses. As a result, the first spring stretched out to the one tenth of its initial length.

What was the percent elongation of the second spring?

Solution

Since only one geometrical data is given, namely the diameter of two springs, let us try to use it in the relationship between the elongation, the number of spring coils, and their diameter. We should take into account that the elongation is considered to be small relatively to both initial length of the spring and the diameter of coils. Only in this case it is possible to apply the Hooke's law to solve the problem. Otherwise, it is necessary to know the specific properties of each spring under the deformation.

A stretched spring is shown in fig1.

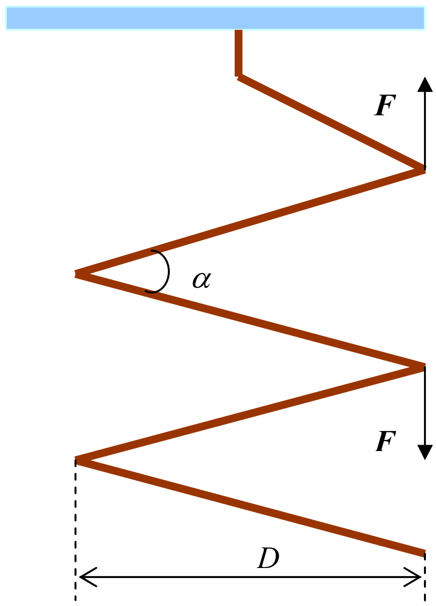

Fig.1

For any elongation Δl of the spring with n coils with diameter *D,* the following is true:

$$\Delta l = n \cdot 2D \cdot \sin\frac{\alpha}{2}$$

If an elongation is relatively small, angle is also small, and

$$\Delta l = nD\alpha$$

Angle α is proportional to the torque of force that stretches a spring. In this problem, it is a gravitational force experienced by a spring load, say ***F***:

$$\alpha = constFD$$

and

$$\Delta l = constFnD^2$$

A percent elongation is $\Delta l / l_0$, where l_0 is a length of the undeformed spring

For two different springs in our problem, values of *const* and *F* are the same. As the diameter of the first spring coils is twice less than that of the second, it has twice as much number of coils, and twice-greater initial length. Thus for two springs, we have:

$$\frac{\Delta l_1}{l_{10}} = \frac{constFn_1d^2}{l_{10}} = \frac{1}{10}; \quad \frac{\Delta l_2}{l_{20}} = \frac{constF(n_1/2)(2d)^2}{l_{10}/2} = \frac{1}{10} \cdot 4 = \frac{2}{5}.$$

Problem 2.

A container is filled with unknown gas. To heat 1 kg of the gas by one degree under the constant pressure one needs 907.8 J, while the same increase of temperature of the gas when the volume is kept constant, needs 648.4 J.
Determine what gas is in the container.

Solution

When gas is heated in a constant volume, the process is called *isochoric* or *isovolumetric*, and the input energy ΔQ is consumed only to change the internal energy of gas ΔE_i. When gas is heated under the constant pressure (*isobaric process)*, its volume increases, and the centre of mass changes its position. In this case the heating results not only in the increase of the internal energy, but also in the work $W = P\Delta V$, produced by the gas. That is why specific heat capacity is different for these two processes of heating: c_V - for the isochoric process, and c_P – for the isobaric process. The thermal energy ΔQ in general is given by:

$$\Delta Q = mc\Delta t$$

We see that actually the values of c_P and c_V were given in the problem.
Thus for the heating under the constant pressure (1) and in the constant volume (2), we can write the following expressions for the law of conservation of energy:

$$mc_p\Delta t = \Delta E_i + W \qquad (1)$$

$$mc_V\Delta t = \Delta E_i \qquad (2)$$

Internal energy E_i depends only on the kind of gas and its temperature. For this reason the value ΔE_i is the same in the observed two processes, and

$$P\Delta V = W = m\Delta t(c_P - c_V) = m\Delta T(c_P - c_V) \quad (3)$$

On the other hand, the ideal gas law for the gas with the molar mass M gives the following formula for the isobaric process:

$$P\Delta V = \frac{m}{M}R\Delta T \quad (4)$$

where R is the universal gas constant with its value of 8.314 J/(mol K).
After equating of the right sides of the formulas (3) and (4) and canceling $m\Delta T$ we have:

$$c_P - c_V = \frac{R}{M}$$

This formula permits us to calculate the molar mass for the unknown gas:

$$M = \frac{R}{c_P - c_V} = \frac{8.314}{907.8 - 648.4}\frac{kg}{mol} = 32.0 \cdot 10^{-3}\frac{kg}{mol}$$

Problem 3.

There are four thin-wire coils with resistances $10\,\Omega$; $20\,\Omega$; $30\,\Omega$; and $40\,\Omega$, that can dissipate power of not more than 2 W each.

Propose a construction of a heater with the maximum power using these coils and the source of direct current with emf of 20 V and internal resistance of 20 Ω.

Solution

First, we must find the resistance of the circuit that provides the maximum load power. Secondly, we will try to combine the given resistors to obtain the same equivalent resistance in the circuit.

According to the Ohm's law, the current *I* in the circuit that consists of resistance R, the source with the emf ε, and internal resistance *r*, is given by

$$I = \frac{\varepsilon}{r + R}$$

The power *W* that dissipates in the resistance *R* is I^2R, or

$$W = \frac{\varepsilon^2 R}{(r+R)^2} \qquad (1)$$

When $R = 0$, $W = 0$, and when *R* is much greater than *r*, we can neglect *r* in denominator. The function *W(R)* becomes hyperbola and decreases approaching zero for great values of *R*. That is why function *W(R)* has its maximum for some specific value R_m, $0 < R_m < \infty$.

This maximum value can be determined by two different ways.

Since the slope of the tangent at the maximum point of any function is zero, and slope equals the derivative of the function at this point, we can find the derivative of the function *W(R),* equate it to zero, and from the obtained equation calculate the value of *R* corresponding to this point.

$$W'(R) = \varepsilon^2 \cdot \left[\frac{-2R}{(r+R)^3} + \frac{1}{(r+R)^2}\right] = \varepsilon^2 \cdot \frac{-2R + r + R}{(r+R)^3} = \varepsilon^2 \cdot \frac{r - R}{(r+R)^3} = 0$$

The solution is

$$r = R \qquad (2)$$

We can also solve the problem without derivatives, but again with the help of advanced mathematics. Fraction (1) has its maximum when its reciprocal has its minimum. It is easier to examine the reciprocal expression, in which we can separate terms with and without *R*.

$$\frac{(r+R)^2}{\varepsilon^2 R} = \frac{1}{\varepsilon^2} \cdot \frac{r^2 + 2rR + R^2}{R} = \frac{2r}{\varepsilon^2} + \frac{1}{\varepsilon^2}\left(R + \frac{r^2}{R}\right)$$

This expression has its minimum when the second term is minimum (the first term is a constant).

Let us compare the arithmetic mean and the geometrical mean of two numbers: *R* and $\frac{r^2}{R}$. The arithmetic mean of any two numbers is always greater than the geometrical mean of these numbers, or

$$\frac{R+\frac{r^2}{R}}{2} \geq \sqrt{R \cdot \frac{r^2}{R}}$$

$$R+\frac{r^2}{R} \geq 2r$$

$$(R+\frac{r^2}{R})_{min}=2r$$

$$(R - r)^2 = 0$$

$$R = r$$

The last solution coincides with the solution (2). As a result, we can say that the resistor with resistance r will produce the maximum power. In our problem, we must assemble the four given resistors in a circuit with equivalent resistance of $20\,\Omega$. We cannot use the single resistor with the resistance of $20\,\Omega$. In this case the load power will be equal to 5 W, whereas its power rating is only 2 W.

Combining our resistors, we can find the best circuit as in Fig.1 with equivalent resistance of $20\,\Omega$ and the power of

$$W = \left(\frac{20}{20+20}\right)^2 20 = 5W$$

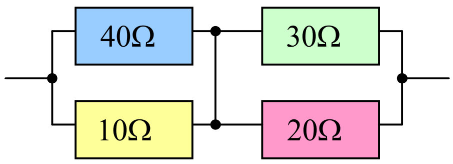

Fig.1

Examination of the power dissipated by each resistor is essential, as it must not exceed 2 W. The maximum power is dissipated by the resistor with $20\,\Omega$, and is equal to 1.8 W.

We can also examine an efficiency of our electrical device. A power yield is given by the eq.1. The output power is $\varepsilon I = \frac{\varepsilon^2}{R+r}$. Let us express the ratio $\frac{R}{r} = a$. Thus, the efficiency is

$$\eta = \frac{R}{R+r} = \frac{a}{1+a}$$

The conditions with the great R and small I provide the great efficiency. For $R \to \infty$, $\eta \to 1$. When $a = 1$, and the yield power is maximum, $\eta = 0.5$, or 50%. In this interesting case, the same power is dissipated in the source, which becomes a heater!

Problem 4.

A neon discharge lamp consists of two electrodes in an evacuated glass bulb filled with neon gas. Electrons from cathode are accelerated by voltage between electrodes. Electrons impact ions of neon and excite them. After a short period, the excited ions return to the state of equilibrium (ground state) simultaneously radiating photons, which result in the bright red fluorescence of the lamp.
In our problem the electrodes are considered to be a pair of big plates separated by the distance d = 3 mm. The ionization energy for neon atom is I = 21.5 eV. An average distance traveled by an electron between two consecutive collisions with neon atoms is l = 0.4 mm.

What is the voltage of ignition of the neon lamp?

Solution

When the voltage between the electrodes is V, the electric field in the same space is $\boldsymbol{E} = V/d$. In this field electron experiences the force $\boldsymbol{F} = \boldsymbol{E}e$ (e is the electron's charge), and moves with acceleration

$$\boldsymbol{a} = \boldsymbol{F}/m = \boldsymbol{E}e/m = Ve/(md), \quad (1)$$

where m is the electron's mass.
As a result of perfectly inelastic collision (we accept this consideration due to the ratio of the masses of two particles), electron stops, and the electron's kinetic energy $mv^2/2$ is transformed to an atom of the gas. To initiate the further ignition in the bulb this energy must exceed or at least be equal to the ionizing energy of the gas:

$$\frac{mv^2}{2} = I \quad (2)$$

From the kinematics relationship for the uniformly accelerated motion and eq.(1), we can find the maximum speed of electron between two collisions:

$$l = \frac{v^2}{2a}$$

$$v = \sqrt{2al} = \sqrt{\frac{2Vel}{md}} \quad (3)$$

To determine V, we must substitute v in the eq.(2) by its expression from the eq.3:

$$\frac{Vel}{d} = I$$

$$V = \frac{Id}{el} \approx 160V$$

Problem 5

This problem can be solved theoretically and then proved experimentally.
You need a rectangular prism with its height at least several times greater than its length and width. Put this prism vertically on the table or any other horizontal plane.

How to determine the coefficient of friction between the prism and the plane using only a ruler as a measurement equipment? You can use any facilities for the goals other than measurements.

Solution

Take a piece of thread, make a loop and through it over the prism, as it is shown in the fig.1.
If you start with a loop near the horizontal surface, the prism will slip along the surface. Gingerly raising the thread upper and upper along the prism we will come to the position h_0 of the thread that provides the turning of the prism over its edge without slipping (fig.2). For this position, there will be two horizontal forces of the opposite direction: the force applied to the thread, and the force of static friction. According to the second Newton's law, they must have equal magnitudes because the center of mass of the prism does not participate in the translation motion in the moment when slipping stops.
For this moment, we can write two equations that are always applied to the position of static equilibrium: equation for the forces, and equation for the torques.
For the forces, we have:

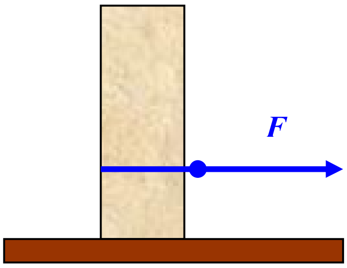

Fig.1

$$F = \mu_S mg \qquad (1)$$

For the torque around the right lower edge, we can write:

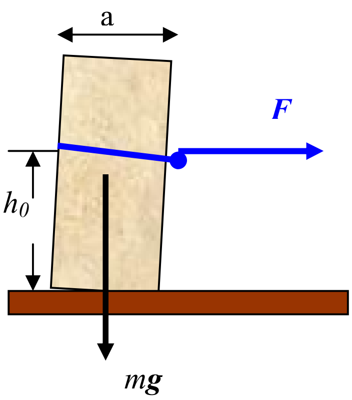

Fig.2

$$Fh_0 = mg\frac{a}{2} \qquad (2)$$

where a is one of the base's sides.
Combination of the eqs. (1) and (2) gives us the solution for the coefficient of static friction μ_S :

$$\mu_S = \frac{a}{2h_0}$$

After measuring a and h_0 with the help of a ruler we can calculate the coefficient of friction.
In every specific case the measurements will result in different value of the coefficient of friction. Nevertheless, the approximate estimation of the numerical result for the two materials (a prism and a surface) should be done using the Internet data for different surfaces.