

2005-2006 Physics Olympiad Preparation Program

– University of Toronto – Solutions. Set 1: General

Problem 1

A billiard-ball with mass m collides elastically and head-on with another ball that has mass M and is initially at rest. Examine the relationship between the energy lost in the collision by the ball, which was initially moving, and the ratio of masses of the balls.

How can we apply the result to the problem of slowing down neutrons in a nuclear reactor?

Solution

First, we must denote the given and unknown quantities. We will always try to do it in the conventionally accepted way. For our problem:

v_0 – the velocity of the initially moved ball before collision (the “first” ball);

v – the velocity of this ball after the collision;

u – the velocity of the initially motionless ball after the collision (the “second” ball);

$k = M/m$ – the ratio of masses of two balls (variable);

ΔE – the kinetic energy loss of the first ball in the collision (unknown function).

All collision problems are solved with the help of two laws of conservation: conservation of the linear momentum and conservation of energy. Sometimes, when the collision involves the rotational motion of balls, we also need the law of conservation of the angular momentum. In our problem we have a head-on collision that does not cause rotation. We also should consider the billiard-balls moving without friction, and we will not take into account that the balls are actually rolling along the table. The reason for this approximation is that there is no enough information in the text of the problem for more accurate solution. Therefore, the motion of the two billiard-balls will be considered the translational motion of two so-called point masses (the physical objects with negligibly little size). This permits us to state that the problem is a one-dimensional problem, and all velocities coincide with their single component.

Applying the law of conservation of energy, we must be very careful with the consideration of the type of collision. The problem as it is given permits us to solve it only for the perfectly elastic collision, in which the kinetic energy of the system is not changed.

$$\begin{aligned} \frac{mv_0^2}{2} &= \frac{Mu^2}{2} + \frac{mv^2}{2} \\ mv_0 &= Mu + mv \end{aligned} \quad (1.1)$$

The solution for system (1.1) gives us for u and then for the unknown energy the following:

$$\begin{aligned} u &= v_0 \frac{2}{k+1}; \\ \Delta E &= \frac{Mu^2}{2} = 4k \frac{mv_0^2}{2} \frac{1}{(k+1)^2} = 4E_0 \frac{k}{(k+1)^2} \end{aligned} \quad (1.2)$$

The equation (1.2) is the relationship between the energy loss of the first ball ΔE and the ratio of masses k ; E_0 is a parameter. Let us investigate this function using the Calculus course.

The rules for calculation of limits show that $\Delta E \rightarrow 0$ when (a) $k = 0$, and (b) $k \rightarrow \infty$.

Function (1.2) is everywhere positive and continuous, and therefore it must have a maximum between two zero values.

To find the position and the value of the maximum we have to find the maximum for the ratio $k / (k+1)^2$. We can change this calculation by the determination of minimum for the inverse ratio $(k+1)^2/k$.

$$\left[\frac{(k+1)^2}{k} \right]_{\min} = \left[k + \frac{1}{k} + 2 \right]_{\min} = \left[k + \frac{1}{k} \right]_{\min}$$

(A) The minimum can be easily found with the help of derivative of the function:

$$\left[k + \frac{1}{k} \right]' = 1 - \frac{1}{k^2} = 0; \quad k = 1 \quad (\text{we choose only}$$

one solution, as the ratio of masses cannot be negative!). See fig.1.1.

(B) The minimum can be found without derivative using the relationship between the arithmetic mean, or simply mean, and the geometric mean of two quantities, which tells us that the geometric mean cannot exceed the mean.

$$\frac{k + 1/k}{2} \text{ is the mean;}$$

$$\sqrt{k \cdot \frac{1}{k}} = 1 \text{ is the geometric mean.}$$

The condition for these two values can be written as:

$$k + \frac{1}{k} \geq 2 \quad \text{and} \quad \left[k + \frac{1}{k} \right]_{\min} = 2 \quad (1.3)$$

The positive solution of the equation (1.3) gives $k = 1$. For this k , $\Delta E = E_0$ that means that in general the incident ball loses the maximum of its energy when it hits the ball of the same mass.

This conclusion makes clear why the hydrogen atoms must be used to slow down neutrons. The hydrogen atom and a neutron have almost the same mass, and neutrons lose the maximum energy in the collisions with the hydrogen atoms in comparison with atoms of other elements.

That is why, the hydrogenous materials are used in nuclear reactors to slow down neutrons.

Problem 2.

We want to boil one litre of water in a light thin-walled vessel. For this purpose, we start to heat water by an electric heater, with the label "500 W. Made in Wonderland". However, after the temperature becomes 60°C, it stops increasing. We become bored with waiting and switch off the heater. Our measurements show that during the first 20 seconds water becomes 2 degrees cooler.

How many watts contains the "Wonderland's watt"?

Solution

When the temperature of water reached its maximum, the system came to the state of dynamic equilibrium: the loss power became equal to the heater power.

The loss power depends on the difference between the temperature of water and the temperature

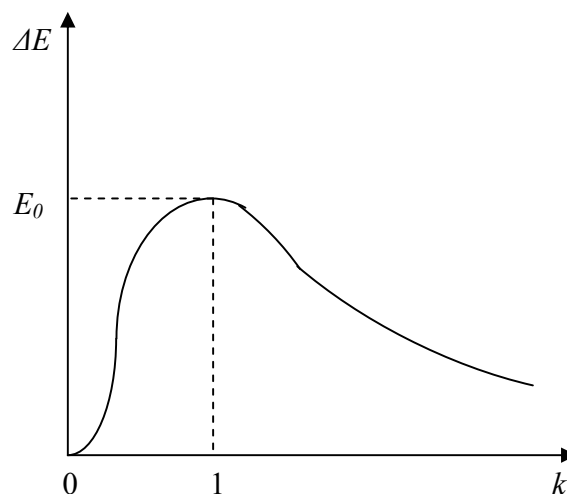


Fig.1.1

of the environment. To simplify the problem, we can consider the temperature change of $\Delta T = 2^\circ$ small enough to not influence the loss power at the beginning of the cooling process during $\tau = 20$ s. This permits to calculate the real power of the heater P as follows:

$$P = \frac{cm\Delta T}{\tau} = \frac{4200(J/kg \cdot K) \cdot 1kg \cdot 2K}{20s} = 420W$$

where c is the specific heat capacity of water; m is the mass of 1 litre of water. The division of Celsius scale and Kelvin scale of temperature is the same. Therefore, $\Delta T = 2^\circ C = 2$ K.

Instead of the indicated “500W” the heater produces only 420 W. We can conclude that one “Wonderland watt” contains $420/500 = 0.84$ W.

Problem 3.

Fig.3.1 shows a bridge circuit used to measure the unknown resistance R_x . Resistances R_1 and R_2 are given; the resistance of a battery is negligible; R is the variable resistance. G is a galvanometer – an electric instrument to measure either currents or voltages. It has a scale with zero in the middle. Galvanometer in a circuit is usually an indicator of zero current or voltage, or of the direction of change of current or voltage.

- Explain the way to measure the unknown resistance with the bridge circuit.
- Propose the method to measure the internal resistance r of the galvanometer using only the parts of the same circuit without additional meter.

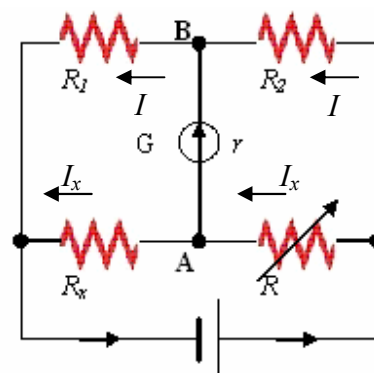


Fig.3.1

Solution

- First, we must decide how to use a galvanometer: either to measure the current in the “bridge”, or to “balance” the voltage in the left and right parts of the circuit. The method of balance is more precise, so we will vary the resistance R to equate the potentials in points A and B of the circuit (fig.3.1). In this case, the galvanometer indicates 0; the current I in the resistors with resistance R_1 and R_2 is the same; and the current I_x in the resistors with R and R_x is the same, but in general is different from the current in the first two resistors. All the above can be combined in the equations:

$$I_x R = I R_2; \quad I_x R_x = I R_1; \quad \Rightarrow \quad \frac{R_1}{R_2} = \frac{R_x}{R}; \quad R_x = R \frac{R_1}{R_2} \quad (3.1)$$

- To measure the unknown resistance r we can replace the resistance R_x by the galvanometer. We cannot use the additional meter, however, it is not forbidden to use a switch S (fig. 3.2). Changing the variable resistance, we should adjust it to the value when the reading of the galvanometer does not change for the positions “on” and “off” of the switch. This will mean that the potential difference in the section AB is zero. We write this condition as follows:

$$I_1 R = I_2 R_2; \quad I_1 r = I_2 R_1; \quad \Rightarrow \quad r = R \frac{R_1}{R_2}$$

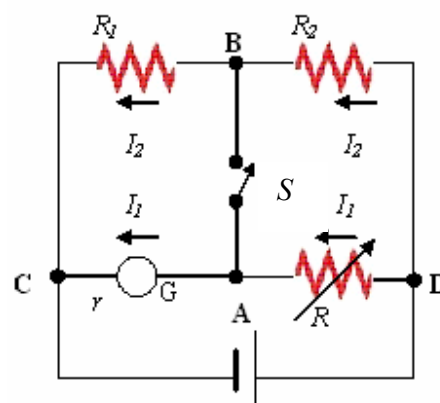


Fig.3.2

Problem 4.

Fine sand is sprinkled on the horizontal membrane, which vibrates in the vertical direction with frequency $f = 500$ Hz. The sand particulates bounce at the height of 3 mm over the level of the position of equilibrium of the membrane.

What is the amplitude of the membrane's vibration?

Solution

Let us denote the amplitude of the membrane's oscillation as A , and the angular frequency as ω . In the vertical direction, we can write the displacement y of the membrane as a function of time in harmonic motion:

$$y = A \sin \omega t \quad (1.1)$$

The velocity v of the membrane and its acceleration a can be written as:

$$v = A \omega \cos \omega t; \quad a = -A \omega^2 \sin \omega t \quad (1.2)$$

These expressions can be obtained with the help of derivatives according to the definition of the instantaneous velocity and instantaneous acceleration:

$$v = y'; \quad a = v'$$

We can also find the expressions for the velocity and acceleration without derivatives. A mass-spring system is a good example of a harmonic oscillator. When $t = 0$ (we start to observe oscillations), the mass is in the position of equilibrium of the spring. At the same point velocity is always the greatest. It means that velocity is proportional to the cosine function of time. To obtain the unit of velocity, m/s, we can combine the given parameters only in one way that is shown in equation (1.2). For the same system according to the Hooke's law, force and therefore acceleration is proportional to the displacement with the negative direction. And again we can find the coefficient with the help of unit analysis.

A grain of sand experiences the force of gravity mg and the force of reaction from the membrane N . All forces are directed along y-axis; therefore, the equation of motion (2nd Newton's law) is following:

$$ma = N - mg \quad (1.3)$$

At the instant t_0 when the grain takes off from the membrane, $N = 0$, and $a = -g$. Obtained condition in combination with eq.(1.2) permits us to calculate this time moment as follows:

$$-g = -A \omega^2 \sin \omega t_0; \quad \sin \omega t_0 = \frac{g}{A \omega^2}$$

At the same moment, the displacement and velocity of the grain are:

$$y_0 = A \sin \omega t_0 = \frac{g}{\omega^2}; \quad v_0 = A \omega \cos \omega t_0$$

After this instant, the grain rises to some height Δy under the only gravitational force. The law of conservation of energy gives:

$$mg\Delta y = \frac{mv_0^2}{2}; \Rightarrow \Delta y = \frac{v_0^2}{2g}$$

In respect to the level with zero displacement the altitude of a grain is: $h = y_0 + \Delta y$, and

$$h = \frac{g}{\omega^2} + \frac{\omega^2 A^2 \cos^2 \omega t_0}{2g} = \frac{g}{\omega^2} + \frac{\omega^2 A^2 (1 - \sin^2 \omega t_0)}{2g} = \frac{g}{2\omega^2} + \frac{A^2 \omega^2}{2g}$$

It gives for the amplitude of the membrane:

$$A = \frac{\sqrt{2gh\omega^2 - g^2}}{\omega^2} = \frac{\sqrt{2gh(2\pi f)^2 - g^2}}{(2\pi f)^2} = 7.72 \cdot 10^{-5} \text{ m} = 0.0772 \text{ mm}$$

(The value for g was taken 9.80 m/s^2).

Problem 5

You can manufacture the equipment for this experimental problem yourself, or try to imagine the result.

To measure acceleration, a bent arc-shaped sealed glass tube filled with water is used. There is a little air bubble inside the tube (Fig. 5.1).

- Sketch the diagram for this experiment.
- Explain the method of measurement in detail.
- Find the relationship between the position of the bubble and acceleration of the tube.

Solution

First, we must understand that the tube is drawn in a vertical plane. If the tube is moving with acceleration \mathbf{a} , the bubble is motionless when the buoyant force \mathbf{F}_A is perpendicular to the tube's wall (fig 5.1). Actually, it means that the buoyant force is directed along the radius of the tube. \mathbf{N} is the normal force of reaction. For the air bubble we can neglect the force of gravity and $m\mathbf{a}$, because the mass of the air is very small.

The buoyant force is applied to the centre of mass of the displaced liquid. Let us imagine an identical tube without the air bubble. The forces applied to the same volume are shown on the fig.5.2. This volume of liquid is in equilibrium when buoyant force of the same value is directed in the same direction, as it was in the tube with a bubble. Therefore, we can use the condition for this equilibrium for our problem:

$$\tan \alpha = \frac{ma}{mg} = \frac{a}{g} \Rightarrow a = g \tan \alpha$$

To perform measurements, we need to put a scale on the upper side of the tube, and always maintain the tube in the vertical plane. As the value of acceleration is proportional not to the angle, but to its tangent, the scale cannot be linear, and the accuracy of measurements will decrease with the increase in acceleration.

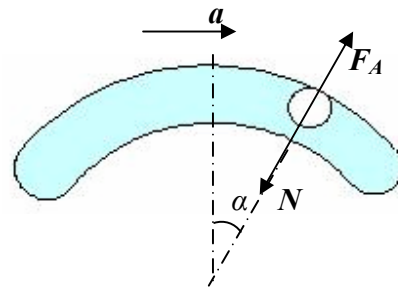


Fig.5.1

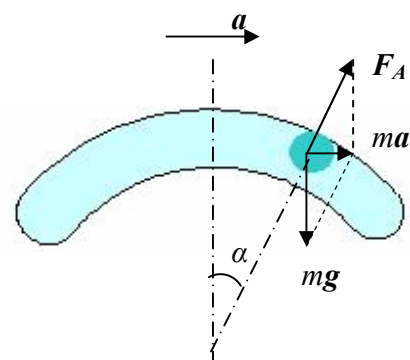


Fig.5.2