Canadian Association of Physicists (CAP) High School Exam
is appointed to Thursday, April 5, 2007. To be selected for the National Finals, the successful participation in the CAP Exam is essential.

POPTOR Weekend and Final Exam
to select Ontario students for National Finals will be held at the University of Toronto from May 18 to May 20, 2007

National Olympiad Finals
will be held from June 3 to June 10, 2007

2006-2007 Physics Olympiad Preparation Program

– University of Toronto –

Solutions. Set 5: AC Circuits, Electronics and General

Due April 2, 2007

Problem 1.

In an experiment carried out by S. C. Collins between 1955 and 1958, a current was maintained in a superconducting lead ring for 2.50 yr with no observed loss. If the inductance of the ring was 3.14×10^{-8} H, and the sensitivity of the experiment was 1 part in 10^9, what was the maximum resistance of the ring? (*Suggestion:* Treat this as a decaying current in an *RL* circuit.)

Solution

For an *RL* circuit,

$$I(t) = I_{\max} e^{-\frac{R}{L}t}; \quad \Rightarrow \quad \frac{I(t)}{I_{\max}} = e^{-\frac{R}{L}t} \approx 1 - \frac{R}{L}t = 1 - 10^{-9} \text{ (given)}. \quad (1.1)$$

In eq.1.1 we use the first two terms after decomposition of the exponential function into series. Besides, we assume that the decrease in current might be not more than the given sensitivity (10^{-9}).

The last relationship of eq.1.1 gives that $\frac{R}{L}t = 10^{-9}$ so,

$$R_{\max} = \frac{\left(3.14 \times 10^{-8}\right)\left(10^{-9}\right)}{\left(2.50 \text{ yr}\right)\left(3.16 \times 10^{7} \text{ s/yr}\right)} = \boxed{3.97 \times 10^{-25} \ \Omega}$$

If the ring were of purest copper (not superconducting), of diameter 1 cm, and cross-sectional area of a wire 1 mm^2, its resistance would be at least $10^{-6}\ \Omega$.

Problem 2.

A diode is a device that allows current to be carried in only one direction (the direction indicated by the arrowhead in its circuit symbol). Find in terms of ΔV and R the average power delivered to the diode circuit of Figure 2.1.

Solution

In our solution we will consider a conventionally accepted direction of current from a positive terminal to a negative one. For an alternating current source, the given value $\Delta V = \Delta V_{rms}$ is the root-mean square of the applied voltage. For sinusoidal current $\Delta V_{rms} = \Delta V_{max} / \sqrt{2}$.

One-half of the time, the left side of the generator is positive, the top diode conducts, and the bottom diode switches off. The equivalent resistance is given by

$$\left[\frac{1}{2R}+\frac{1}{2R}\right]^{-1}=R \text{ and the power is } \frac{(\Delta V_{rms})^2}{R}.$$

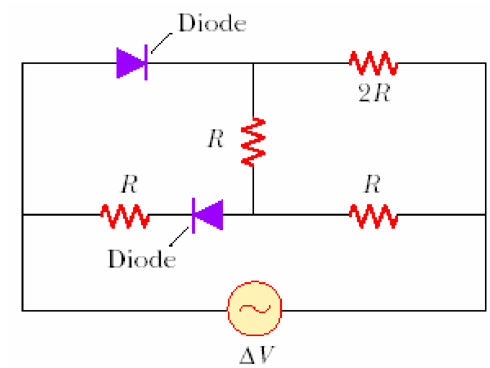

Fig.2.1

The other half of the time the right side of the generator is positive, the upper diode is an open circuit, and the lower diode has almost zero resistance. The equivalent resistance is then given by

$$R_{eq}=R+\left[\frac{1}{3R}+\frac{1}{R}\right]^{-1}=\frac{7R}{4}$$

and power is $$P=\frac{(\Delta V_{rms})^2}{R_{eq}}=\frac{4(\Delta V_{rms})^2}{7R}.$$

The average power during one period of current oscillation may be calculated as follows:

$$P_{av}=\frac{1}{2}\frac{\Delta V^2}{R}+\frac{1}{2}\frac{4(\Delta V)^2}{7R}=\frac{11}{14}\frac{(\Delta V)^2}{R}$$

Problem 3.

A plane electromagnetic wave has an intensity of 750 W/m^2. A flat, rectangular surface of dimensions 50 cm × 100 cm is placed perpendicular to the direction of the wave. The surface absorbs half of the energy and reflects half. Calculate
(a) the total energy absorbed by the surface in 1.00 min and
(b) the momentum absorbed in this time.

Solution

Let us denote the intensity as I, the surface area as A, and the time of interaction as t.
(a) The total energy E, absorbed by the surface is

$$E=\frac{1}{2}IAt=\frac{1}{2}(750\text{ W/m}^2)(0.500\text{x}1.00\text{m}^2)(60.0\text{s})=11.3\text{ kJ}.$$

(b) The total energy incident on the surface in this time is $2E$ = 22.5 kJ, with E = 11.3 kJ being absorbed and E = 11.3 kJ being reflected. For electromagnetic wave or for a quantum of electromagnetic radiation – a photon - the relationship between energy E and momentum p is given by: $E = pc$, where c is speed of light. The reflected wave changes the momentum of the surface by twice as large value as the momentum of the incident wave (remember the ball elastically bouncing from the wall). The absorbed wave changes the momentum of the surface by the value of the initial momentum of the wave.
The total momentum p_{total}, transferred to the surface, is given by:
p_{toal} = (momentum from reflection) + (momentum from absorption), or

$$p_{total}=\frac{2E}{c}+\frac{E}{c}=3\frac{E}{c}=\frac{3\times 11.3\times 10^3}{3\times 10^8}=1.13\times 10^{-4}\text{kg}\cdot\text{m/s}$$

Problem 4.

Two circuits like the one shown in Figure 4.1 are identical except for the value of L. In the circuit A the inductance of the inductor is L_A, and in the circuit B it is L_B. The change of current in any RL-circuit is characterized by the time constant $\tau = L/R$. The time constant equals the time interval for the decreasing current to drop by the factor of e (the base of the natural logarithm) and equals the time interval for the increasing current to reach $(1 - e^{-1}) \approx 0.632$ of its maximum value. A switch S is designed so that it is never open, which would cause the current to stop. The switch is thrown to position a at $t = 0$. At $t = 10$ s, the switch is thrown to position b. The resulting currents for the two circuits are as graphed in the Figure 4.2. Assuming that the time constant τ of each circuit is much less than 10 s, estimate roughly the relationship between L_A and L_B?

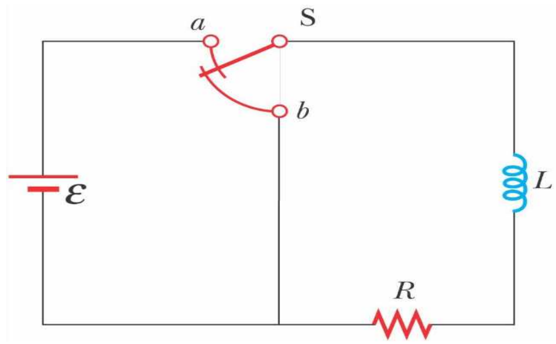

Fig.4.1

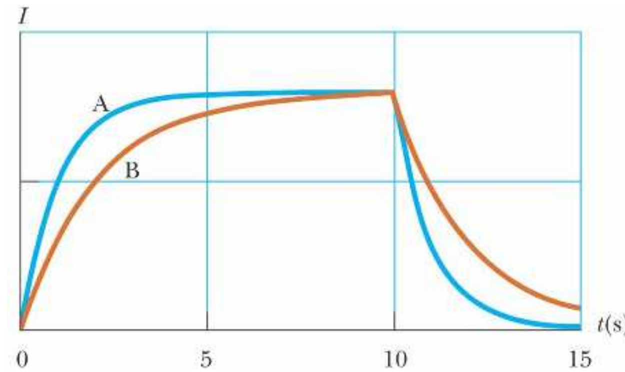

Fig.4.2

Solution

Figure 4.2 shows that circuit B has the greater time constant because in this circuit it takes longer for the current to reach its maximum value and then longer for this current to decrease to zero. The given equation for a time constant $\tau = L / R$ indicates that, for equal resistances R_A and R_B, the condition $\tau_B > \tau_A$ means that $L_B > L_A$. In general, the inductance is a measure of "inertia" of the circuit. The greater the inertia (inductance), the slower the current may be changed.

Problem 5.

A first-year university student performs an experiment with photoelectric effect. He uses an experiment device, sketched in Figure 5.1. The cathode C is illuminated by the beam of light (the beam of photons) and emits electrons with the maximum kinetic energy at the surface of the cathode K_{max}. The relationship among the frequency of photons f, the value of K_{max} and the work function ϕ of the metal of the cathode (the work done on the electron to bring it out of the surface with zero kinetic energy) is give by the Einstein's equation for the photoelectric effect:

$$hf = K_{max} + \phi, \quad (5.1)$$

where h is Planck's constant.

The potential difference between the cathode and the anode A may very.

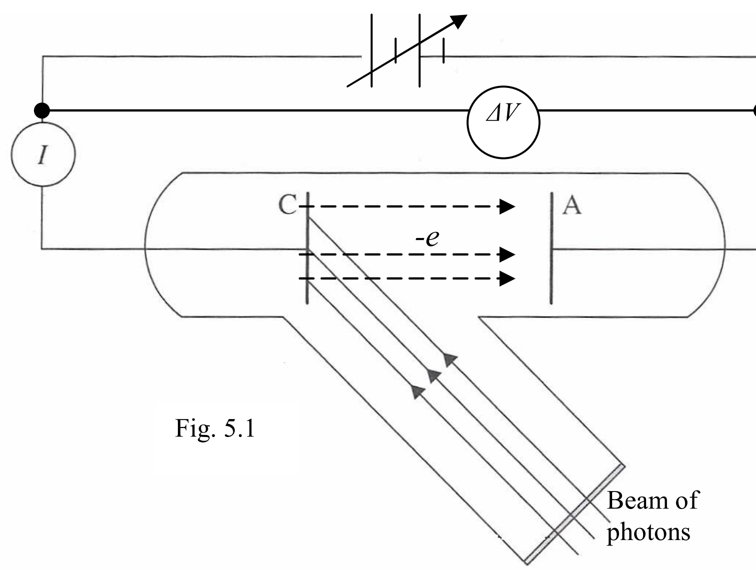

Fig. 5.1

The student connects the negative terminal of the power supply to the anode and tries to "stop" electrons by increasing the applied voltage. The student measured current and voltage in the external circuit as shown. For each wavelength of the incident beam of photons the student finds such potential difference $-\Delta V$ that the current I becomes zero. This threshold value is called the *stopping potential*. The following table shows data obtained by the student in the experiment.

Wavelength of the light, nm	Stopping potential, V
588	**0.67**
505	**0.98**
445	**1.35**
399	**1.63**

Using these data, determine
(a) an experimental value for Planck's constant (in joule-seconds) and
(b) the work function (in electron volts) for the surface.

Solution

Upon the definition of the stopping potential, the work $e\Delta V$ done by the applied electric field is spent to zero the initial kinetic energy of electrons, or $e\Delta V = K_{max}$. If we calculate this work in electron volts, its value will be numerically equal to the stopping potential in volts. Now, we can recalculate the given tabulated data in terms of frequency and maximum kinetic energy of electrons.

λ, nm	f, 10^{14} Hz	K_{max}, eV
588	5.10	0.67
505	5.94	0.98
445	6.74	1.35
399	7.52	1.63

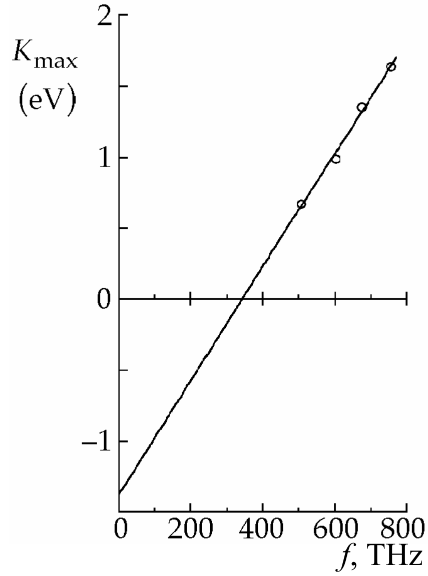

Fig. 5.1

As a result, we can plot K_{max} versus f (Fig. 5.1).
(a) According to eq. 5.1, the slope must give Planck's constant. The slope may be determined approximately by averaging the three slopes, obtained from the table. On the other hand, the slope may be found for the given four plots in fitting procedure applying the least square method to a straight line:

$$\text{slope} = \frac{4\Sigma(x_i y_i) - \Sigma x_i \Sigma y_i}{4\Sigma x_i^2 - (\Sigma x_i)^2} = \frac{5.22\times10^{14}}{1.30\times10^{29}} = 4.015\times10^{-15}\, eV\cdot s = 6.4\times10^{-34}\, J\cdot s$$

Frequencies are the x-values, and kinetic energies are the y-values.
The difference between the obtained value and the value from a handbook ($6.6\text{x}10^{-34}$J s) is about 3%.
(b) Equation 5.1 permits to find the work function ϕ as a line and a vertical axis intersection. The least square method gives the following formula to calculate this value:

$$\phi = \left| \frac{slope \times \Sigma x_i - \Sigma y_i}{4} \right| = 1.2\ \text{eV}$$

The obtained value is too small to be a work function of a pure monometallic surface. However, the value may belong to an oxide or a thin film of one substance on the surface of the other one.