2008-2009 Physics Olympiad Preparation Program

University of Toronto

Problem set 6: Electronics, Circuits

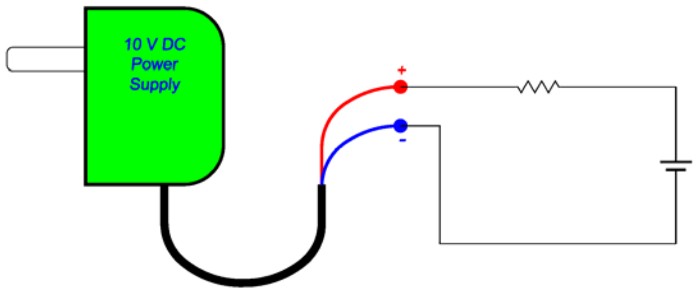
Solutions

Note:

The POPTOR Weekend takes place from **Friday evening 2009 May 15 to Sunday May 17.**

1. You need to charge a 1.5 V NiCad battery and have a 10 V DC power supply left over from a now unused cordless phone. However, the battery has a warning on it that the charging current should not exceed 50 mA. You decide to use the power supply, but place a resistor in series with the battery to limit the current.

a) Sketch the circuit that you will use.



The resistor can be placed in either the positive or negative side of the circuit.

b) When you measure the voltage on the discharged NiCad battery, it is 1.2 V. How big a resistor do you need to limit the charging current to 50 mA?

The voltage drop across the resistor ΔV has to be the difference between the power supply voltage and the discharged battery voltage. At the current limit of 50 m

$$\Delta V = 0.05\,R = 10 - 1.2 = 8.8\ \text{V}$$

$$R \geq 176\ \Omega$$

In practice, it will be difficult to find a resistor of exactly 176 Ω – hence the equal to or greater than relationship.

c) When you begin the charging how much power is lost in the resistor? How much power goes into the battery?

$$\text{Power lost in the resistor} = I^2 R = .05^2 \bullet 176 = 440\ \text{mW}$$

$$\text{Power deposited in the battery} = V I = 1.2 \bullet .05 = 60\ \text{mW}$$

Not exactly an efficient *(or green)* way to charge the battery.

d) If you forget to take the charger off after the battery is fully charged, how much power is deposited in the battery. Where does it go?

When the battery is fully charge the voltage across it will be 1.5 V. The charging current will then be reduce as a result of the reduction in $\Delta V = 10 - 1.5 = 8.5$ V

$$I = \Delta V / R = 8.5 / 176 = 48.3\ \text{mA}$$

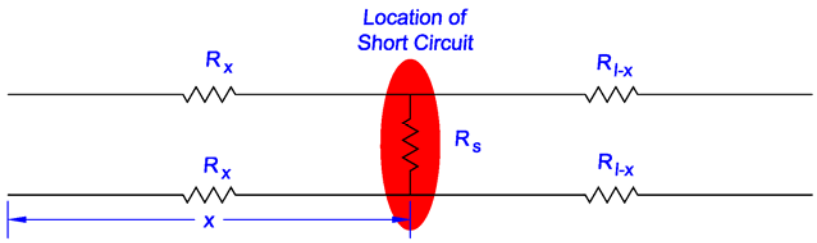
$$\text{Power deposited in the battery} = V I = 1.5 \bullet .0483 = 72.4\ \text{mW}$$

While this is going on, you are also wasting power in the resistor as the rate of

$$I^2 R = .0483^2 \bullet 176 = 411\ \text{mW}$$

2. A signal cable, consisting of two insulated copper wires, each 1.0 mm in diameter, twisted together, links two buildings on campus. A short circuit has developed somewhere along the cable and as it is 2 km long, you decide to try to find the location of the problem and repair the cable – the cost of laying a new cable is somewhat more than the price of the cable. The nature of the "short circuit" is an unknown resistance, R_s, connecting the two wires. To narrow your search you make some measurements, after disconnecting both ends of the cable from the signaling equipment. With the pair of wires at the far end not connected, when you attach a 100 V DC power supply at the near end, you measure a current of 370 mA. When you connect both wires at the far end of the cable, you measure 800 mA. Where is the problem located?

The equivalent circuit diagram is



where x is the fractional distance along the cable to the short circuit (actual distance is 2000 x).

The resistance of each wire in the cable, $R_w = R_x + R_{1-x}$, can be calculated from the cross sectional area of the wire, its length and the resistivity of copper.

$$R_w = \rho\, l / A = 1.7 \times 10^{-9} \bullet 2000 / \pi\, (.5 \times 10^{-3})^2 = 43.3\ \Omega$$

For the open circuit case (wires not connected at the far end):

$$V / I_o = (2R_x + R_s) = (2 \times R_w + R_s)$$

$$R_s = (V / I_o) - 2 R_w\, x \qquad (1)$$

For the closed circuit case (wires short circuited at the far end), there are two resistances in parallel (R_s and $2R_{1-x}$), added to the $2R_x$.

$$R = 2R_x + \left[\frac{1}{R_s} + \frac{1}{2R_{1-x}}\right]^{-1}$$

$$= 2R_x + \frac{2 R_{1-x} R_s}{2R_{1-x} + R_s}$$

$$= 2 R_w x + \frac{2 R_w R_s (1-x)}{2R_w(1-x) + R_s}$$

$$V / I_c = 2 R_w x + \frac{2 R_w R_s (1-x)}{2R_w(1-x) + R_s} \qquad (2)$$

At this point, some substitutions will make life a little easier. Let $a = V / I_o$, $b = 2R_w$ and $c = V / I_c$. Thus from equation (1) above, $R_s = a - bx$. Substituting these into equation (2) we get

$$c = b\,x + \frac{b\,(a - b\,x)\,(1-x)}{b\,(1-x) + a - b\,x}$$

This gives the following quadratic equation in x:

$$b^2x^2 - 2\,b\,c\,x + a\,c + b\,c - a\,b = 0$$

If you try to solve this equation for x, with the values given, you get imaginary solutions – a result of a copying error that was discovered too late to change – sorry about that. The diameter of the wire should have been 0.8 mm, which makes $R_w = 67.6\ \Omega$ and gives x = 0.64.

3. For alternating current, the resistance of a wire is complicated by the frequency dependence of the depth to which the alternating electric field can penetrate radially into the conductor. This characteristic depth is called the skin depth, δ and is given by

$$\delta = \sqrt{\frac{2\rho}{2\pi f \mu}}$$

where ρ is the resistivity and μ is the magnetic permeability of the material and f is the frequency. For a wire which has a large diameter compared with δ, the resistance can be approximate by assuming that the wire is a hollow cylinder with a wall thickness δ.

a) For an aluminum conductor 5 cm in diameter, calculate the resistance per unit length. For aluminum, $\rho = 2.65 \times 10^{-8}\ \Omega$ m and $\mu = 4\pi \times 10^{-7}$ N A^{-2}.

The resistance R is obtained from the resistivity ρ, the cross sectional area of the conductor A and the length L by $R = \rho L / A$. In this case, the cross sectional area is that of the cylinder of wall thickness δ.

$$\delta = \sqrt{\frac{2\rho}{2\pi f \mu}} = \sqrt{\frac{2 \bullet 2.65 \times 10^{-8}}{2\pi\, 60 \bullet 4\pi \times 10^{-7}}} = 10.6 \text{ mm}$$

For a cylinder with diameter 5 cm and wall thickness 10.6 mm, the cross sectional area is:

$$A = \pi [r_{cond}^2 - (r_{cond} - \delta)^2] = \pi [0.025^2 - (0.025 - 0.0106)^2] = 1.31 \times 10^{-3}\ m^2$$

The resistance per unit length R is

$$R = \rho / A = 2.65 \times 10^{-8} / 1.31 \times 10^{-3} = 2.023 \times 10^{-5}\ \Omega/m$$

b) The hydro-electric power station at Churchill Falls, NL produces 5.4 GW of electricity, most of which is sold to Hydro-Québec, transported over 735 kV transmission lines. If the distance from Churchill Falls to Québec City is 1200 km, how many conductors with the capacity of the one in part (a) are required so that the resistive losses are limited to 1% of the total power produced. Assume that the 735 kV is a peak-to-peak measurement; use the equivalent root-mean-square voltage and current for the calculation ($V_{rms} = V_{p\text{-}p} / \sqrt{2}$)

For the 735 kV transmission line, the Vrms is

$$V_{rms} = V_{p\text{-}p} / \sqrt{2} = 519.7 \text{ kV}$$

At this voltage, the current to be transmitted is

$$I_{rms} = P_{CF} / V_{rms} = 5.4 \times 10^9 / 519.7 \times 10^3 = 10.39 \text{ kA}$$

The power loss in a conductor due to its resistance is I^2R. So Rt, the total resistance of all the conductors used, for 1% power loss is given by:

$$0.01\, P_{CF} = I^2 R_t; \qquad R_t = 0.01 \bullet 5.4 \times 10^9 / (10.39 \times 10^3)^2 = 0.5002\ \Omega$$

From (a), the resistance of each conductor $R_c = 2.023 \times 10^{-5} \bullet 1.2 \times 10^6 = 24.28\ \Omega$, so the number of conductors required ($\# = R_c / R_t$) will be at least 49. In practice, bulk electrical power is transmitted in 3 phases, with the phases of the sine wave separated by 120° in time, so transmission line conductors occur in multiples of 3. In addition, at these voltages, to minimize losses due to corona discharge, each phase is transmitted as a closely spaced bundle of 4 conductors.

c) The generators at Churchill Falls produce the electricity at 15 kV (assume $V_{p\text{-}p}$). To show the advantage of the stepped up transmission voltage, calculate the number of conductors as in (a) required to transmit the power at the generated voltage. Are there any disadvantages to the higher transmission voltages?

$$V_{rms}(\text{generator}) = 15 / \sqrt{2} = 10.61 \text{ kV}$$

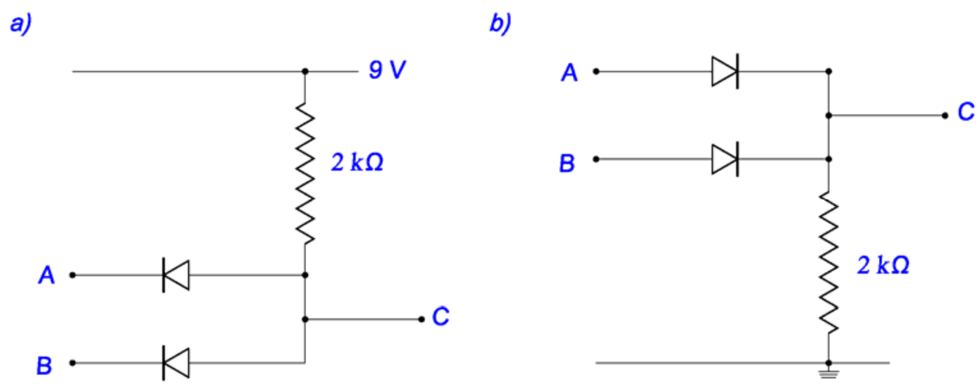
$$I_{rms}(\text{generator}) = P_{CF} / V_{rms}(\text{generator}) = 5.4 \times 10^9 / 10.61 \times 10^3 = 5.091 \times 10^5 \text{ A}$$

$$R_t = 0.01 \bullet 5.4 \times 10^9 / (5.091 \times 10^5)^2 = 2.083 \times 10^{-4}\ \Omega$$

$$\# \text{ of conductors} = R_c / R_t = 24.28 / 4.167 \times 10^{-4} = 116{,}533 \text{ !!!!}$$

Some disadvantages of the higher transmission voltage are the losses due to corona discharge and radiation. The latter contributes to resonances in the ionosphere and to the electromagnetic noise that we generate, both of which are observable from space.

4. For each of the circuits in the figure below, calculate and make a table of the voltage appearing at point C when the voltage at points A and B are either 0 or 5V (four conditions). The diodes have a voltage drop of 0.2 volts when conducting. What logic function do these circuits perform?



a)

V_A	V_B	V_C
0	0	0.2
5	0	0.2
0	5	0.2
5	5	5.2

Function: logical AND

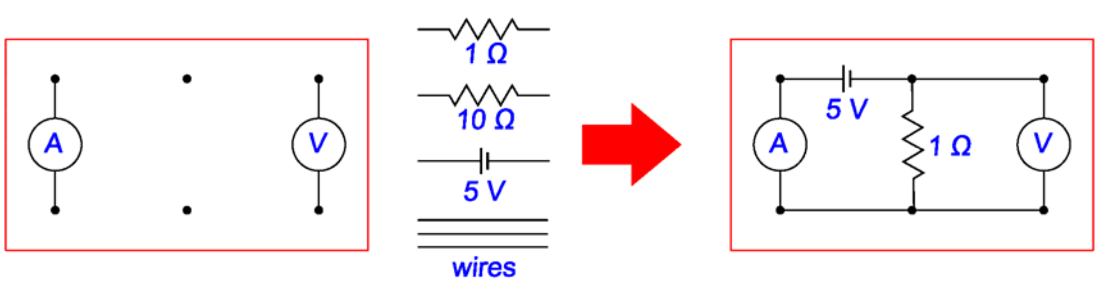
b)

V_A	V_B	V_C
0	0	0
5	0	4.8
0	5	4.5
5	5	4.8

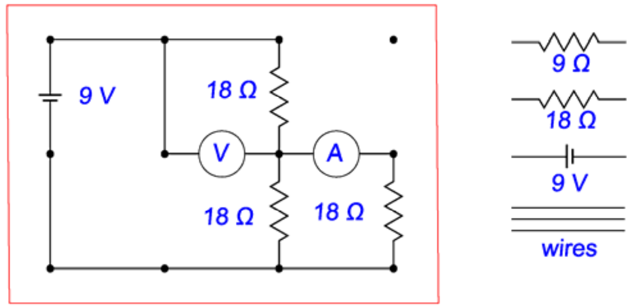
Function: logical OR

5. On the figures below and on the next page, connect the dots using the components provided so that the voltage and current measurements show the values that would be given by the resulting circuit. You may not need to use all the dots or components, but you may need to use a dot for more than one connection. You also may use a particular component more than once in the circuit. The V and A symbolize ideal voltmeters and ammeters (i.e. the voltmeter draws negligible current and the ammeter causes a negligible voltage drop).

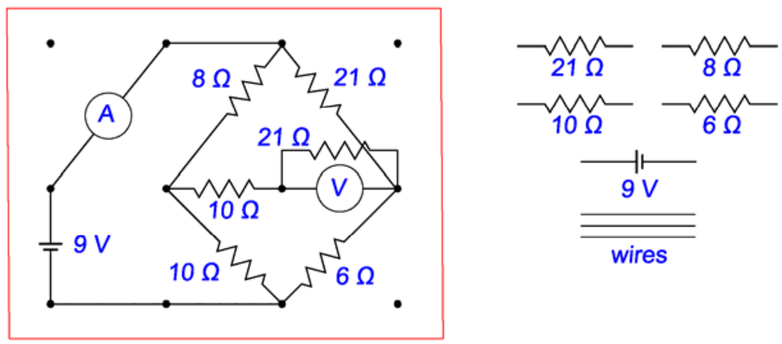
Example: the voltmeter reads 5 V, the ammeter 5 A



a) The voltmeter reads 6 V, the ammeter, 167 mA



b) The voltmeter reads 2 V, the ammeter, 833 mA



This problem too, was the victim of a copy error – the voltage was intended to be 3 V, not 2 V. However, in this case it just made the problem more interesting as there were sufficient resistors to put in a ~ 2:3 voltage divider across the middle of the bridge. But then, if you do a proper analysis of the network (looking at the voltage loops and current nodes), you find that the additional pathway across the middle increases the overall current to 858 mA and decreases the voltage measurement to 1.571 V. To get to within the precision of the voltage and current measurements, you have to increase the overall resistance of the voltage divider. Using ~10 of each resistor in series gets you there.

c) The voltmeter reads the waveform shown below on the right. (For this one you can pretend you have an oscilloscope.)