1994-1995 Physics Olympiad Preparation Program

– University of Toronto –

Solution Set 4: Waves and Optics

1. (a) To solve this problem we need the equation which describes how a curved surface bends light

$$\frac{n_1}{S_o} + \frac{n_2}{S_i} = \frac{n_2 - n_1}{R}$$

where n_1 and n_2 are the indices of refraction of the two media, R is the radius of curvature of the surface, S_i is the image distance and S_o is the object distance. Solving for the image distance

$$S_i = \frac{n_2 R S_o}{(n_2 - n_1)S_o - n_1 R}.$$

At the second surface we have

$$\frac{n_2}{2R - S_i} + \frac{n_1}{x} = \frac{-(n_2 - n_1)}{-R}$$

where x is the image distance we are trying to find and all of the other variables have the same meaning as above. Here, the second object distance $S_o = 2R - S_i$ because a virtual image is formed by the first surface. Solving for x gives

$$x = \frac{n_1 R(2R - S_i)}{(n_2 - n_1)(2R - S_i) - n_2 R}.$$

Putting $n_1 = 1.0$, $n_2 = 1.5$, $S_o = 1.2m$ and $R = 10cm$ gives $S_i = 36cm$ and $x = 6.9cm$. Therefore the image is $6.9cm$ from the crystal ball.

(b) The image is real, inverted and minified.

(c) For the gypsy lady to produce no image means that $x \to \infty$. This corresponds to

$$(n_2 - n_1)(2R - S_i) = n_2 R$$

or $S_i = -10cm$ and $S_o = 5cm$. In other words, that gypsy's mug is awfully close to the crystal ball.

2. (a) This problem is solved by looking at the interference that the two waves undergo at a particular point on the screen. The first thing to do is work out the path length that each wave undergoes on its way to the screen (see figure #1). Consider first r_1

$$r_1 = \sqrt{D^2 + (x - \frac{h}{2})^2}.$$

Using $h << x$ gives

$$r_1 = D\sqrt{1 + \frac{x^2 - hx}{D^2}}$$

and using $D >> x$ gives

$$r_1 = D(1 + \frac{x^2 - hx}{2D^2}).$$

Similarly

$$r_2 = D(1 + \frac{x^2 + hx}{2D^2}).$$

The path difference between the two waves will be $r_2 - r_1 = hx/D$ and phase difference $\Delta = 2\pi xh/\lambda D$. The electric field at the screen will be the superposition of the waves that went through the slits

$$E_{tot} = \frac{E}{2}e^{i\phi} + \frac{E}{2}\exp i(\Delta + \phi)$$

where the factors of two appear because the wave splits in wave in half when it hits the slits. Notice that one wave is phase shifted Δ radians with respect to the other. To find the intensity we multiply E_{tot} by its complex conjugate, which gives

$$I_{screen} = \frac{E^2}{2}(1 + \cos 2\pi\frac{xh}{\lambda D}).$$

(b) If the laser is shifted so that the wave coming from one slit is out phase by π *radians* with respect to the other then the electric field becomes

$$E_{tot} = \frac{E}{2}\exp i(\phi + \pi) + \frac{E}{2}\exp i(\phi + \Delta)$$

Multiplying this by its complex conjugate gives

$$I_{screen} = \frac{E^2}{2}(1 - \cos 2\pi\frac{xh}{\lambda D}).$$

This new formula has its maximum off centre with respect to the two slits, which is the major difference between the interference patterns.

(c) The reason the experimental pattern fades is because the same amount of light doesn't hit all parts of the screen. Most of the light passes straight through the slits, with far less scattered at large angles. This means that the interference pattern is brightest opposite the two slits (ie at $x \simeq 0$).

3. (a) To solve this question we must find out how much energy is needed to evaporate the aluminum, and then how fast the laser can supply this energy. The volume of aluminum to evaporate is $V = (\pi/4)d^2h$ where d is the spot diameter and h is the thickness, which means that the mass of aluminun is $M = V\rho$ with ρ. The heat needed to evaporate this amount of aluminum (neglecting the heat of vapourization) is $H = CM\Delta T$ where C is the heat capacity and ΔT is the temperature difference. Plugging in the numbers gives $H = 126.5\,\mathrm{J}$. The laser beam is attenuated to $P = 50(0.97)^{10} = 36.9\,\mathrm{W}$ by the time it arrives at the missle so it will take 3.4 sec for the laser beam to burn through the missle's skin.

(b) The angular accuracy needs to be enough to keep the laser aimed at the same spot on the missle for the entire 3.4sec. Assuming the beam can't move by more than its radius gives an accuracy of

$$tan\theta < \frac{5 \times 10^{-4}}{10^4}$$

or the drift in θ must be less than $8.4e - 7$ degrees/sec.

(c) To defend against this sort of attack one could: rotate the missle, thicken the skin, make the skin reflecting, increase heat transfer over the skin, have dummy warheads, send up chaffe, or as some of you suggested, not fire the missles, use a cloaking device, blast laser system and finally, don't piss the guys at ICBM control off.

4. (a) The thing that makes holograms different from normal pictures is that holograms record phase information, in addition to amplitude information on the film. A picture only records how intense the light is, which gives no information about depth. A hologram on the other hand is just a recorded interference pattern of the image. The depth information about an image is preserved on the plate in the form of an interference pattern.

(b) To create a hologram one needs to produce an interference pattern of the desired object. This is done by splitting the laser beam into a reference beam and an object beam (see figure #2). The probe beam is reflected off the object, and then interferes with the reference beam. Because the object has depth to it, the reflected beam will not have the same phase across it when it reaches the holographic plate. It will then interfere with the uniform reference beam, thus producing an interferomic record of the object.

(c) It makes no difference wether the holographic plate is a negative or a positive. Since the information is just an interference pattern, the difference between positives and negatives will only effect the overall phase of the pattern and will not effect the final image.

5. (a) I will first work out the back focal length using the Lensmakers equation. At the first lens we can write

$$\frac{1}{f_1} = \frac{1}{S_{i1}} + \frac{1}{S_{o1}}$$

or

$$S_{i1} = \frac{S_{o1} f_1}{S_{o1} - f_1}.$$

This will be the position of the image after the first lens. The same can be done for the second lens, which gives

$$S_{i2} = \frac{S_{o2} f_2}{S_{o2} - f_2}$$

Since the lenses are located a distance d away from each other we can write $S_{o2} = d - S_{i1}$. Using these three equations to solve for S_{i2} while eliminating S_{o2} and S_{i1} gives

$$\text{back focal length} = S_{i2} = \frac{f_2(d - f_1)}{d - f_1 - f_2}$$

Doing the same analysis but in the other direction gives

$$\text{front focal length} = \frac{f_1(d - f_2)}{d - f_1 - f_2}.$$

(b) If we set $d = f_1 + f_2$ then the system acts like it isn't there. Putting plane waves in one end produces plane waves at the other. Note that the magnification of the system will not likely be unity.

(c) If $d = 0$ in the above equations then

$$\text{back focal length} = \text{front focal length} = \frac{f_1 f_2}{f_1 + f_2}$$

or in other words it doesn't matter which side the image is placed on. We can write

$$\frac{1}{f_{eff}} = \frac{1}{f_1} + \frac{1}{f_2}.$$

This is just a modified version of the thin lens equation.

6. (a) The Aurora are caused by low energy solar electrons that are caught by the Earth's magnetic field. They are then sped up and enter the atmosphere at the magnetic North Pole. These electrons excite nitrogen and oxygen which produce the characteristic blue of the Aurora. The reason it is strongest over Canada is that the magnetic North Pole is centred over ??? Island in the Canadian Arctic.

(b) Stars twinkle because the atmosphere that the light passes through to get to our eyes is turbulent. This is due to uneven heating of the atmosphere by the sun. This turbulence causes random refracting of the light which we see as a twinkle. This twinkle also appears for larger bodies (eg. the moon) but is much less noticable.

(c) The sound from the seashells is actually produced by air currents that pass by the shell and excite the shell's air volume at its natural resonances. Since these air currents are not constant, the resonances will be excited sporadically giving the impression of ocean waves.

(d) When lightning strikes a tree, the current in the lighning stroke needs a path to ground. If the bark of the tree is wet, then the current will pass over the outside of the tree leaving it undamaged. If, on the other hand, the tree isn't wet then the current will pass into the tree and descend through the sap. When this happens the sap is superheated and expands, which ends up turning the tree into toothpicks. The Oak tree has especially rough bark which makes it difficult for the current to find a path to ground along the bark. It is therefore not that Oak trees get struck more by lightning, just that they are more likely to be destroyed when hit.