1994-1995 Physics Olympiad Preparation Program

– University of Toronto –

Solution Set 6: Electronics

1. (a) This problem is solved by finding the impedance of a capacitor and inductor in parallel with each other. Adding them in parallel gives

$$\frac{1}{Z_{tot}} = \frac{1}{Z_c} + \frac{1}{Z_l} = i\omega C + \frac{1}{i\omega \mathrm{L}} = \frac{1-\omega^2 LC}{i\omega L}$$

which gives the absolute value as

$$Z_{tot} = \frac{\omega L}{\mid 1-\omega^2 LC \mid}.$$

This impedance becomes infinite when $\omega = \sqrt{\frac{1}{LC}}$.

(b) See figure #1.

(c) If the inductor has some resistance then the final form is found by solving

$$\frac{1}{Z_{tot}} = \frac{1}{Z_l + R} + \frac{1}{Z_c}$$

for Z_{tot} which gives

$$Z_{tot} = \frac{R + i\omega L}{1-\omega^2 LC + i\omega RC} = \frac{(R+i\omega L)(1-\omega^2 LC - i\omega RC)}{(1-\omega^2 LC)^2 + (\omega RC)^2}$$

2. To find out what types of filters these are we must remember several things:(i) as frequency increases Z_c decreases and Z_l increases, and (ii) the value of resistance is independent of frequency. Also, the output of the filter circuit depends on the amount of voltage across the last element. It is usually very helpful to look at the circuits in the limits of $\omega = 0$ and $\omega = \infty$ to get an idea what is going on.

(a) The resistance of the capacitors gets very small at high frequencies, which in effect just shorts the signal between the output and the input. Conversly, at low frequencies the resistance of the capacitor is huge, which allows very little input signal to pass to the output. Therefore this is a high pass filter.

(b) This is no filter at all because there are no frequency dependent elements in it.

(c) This is a low pass filter. At high frequencies the inductor has a huge resistance, thus allowing no output signal.

(d) This will be a band rejection filter because of the tank circuit. Its resistance will become huge at some intermediate frequency allowing no signal to be passed.

(e) This is a low pass filter.

3. The trick with this problem is to use the symmetry involved to simplifiy it. Because all of the resistors are of equal value, they will all have the same current flowing through them, which means they will also have the same voltage drop. Because of this none of the resistors in the plane will have any current flowing through them. Because of that the circuit can be redrawn (see figure #2) with the resistors in the plane replaced by a wire. Now it is easy to see that the total resistance will just be $R_{tot} = R/2$!!!

4. (a) We will first need the equation that governs current density in a conductor

$$J = \sigma_c E$$

where J is the current density and E is the electric field. Now we can simplify these equations using $J = I/\pi b^2$ and $E = V/L$ which gives $I = \pi\sigma b^2 V/L$. Combining this with Ohm's law gives the final result

$$R = \frac{L}{\pi\sigma b^2}.$$

(b) Using the hint given, I will approximate the problem as four resistors (see figure #3). The resistance R_w is the resistance of the Warbird, R_o the resistance of the cylinder of length a and radius b minus the resistance of the Warbird and R_p is the resistance of the unaffected ends. Working these out using the formula from part (a) gives

$$R_p = \frac{L-a}{2\pi\sigma_c b^2}$$

$$R_w = \frac{a}{\pi\sigma_w a^2}$$

and

$$R_o = \frac{a}{\pi\sigma_c(b^2 - a^2)}.$$

The next part is to just work out the resistance of the four resistors in the circuit, which becomes

$$R_{tot} = \frac{L}{\pi\sigma_c b^2} - \frac{a^3(\sigma_w - \sigma_c)}{\pi(\sigma_c b^2)^2}$$

where I have assumed $b >> a$. This means that the change in resistance to first order is

$$\delta R = \frac{a^3(\sigma_c - \sigma_w)}{\pi(\sigma_c b^2)^2}.$$

5. (a) The trick to solving this problem is to carefully go through the two cases where the currents are going in opposite directions. Since the diodes only allow current to pass one way through them, there will be two separate paths for the current to follow. If the current is going upwards then it will flow through diodes D_2 and D_3. In the other half of the cycle the current flows in the opposite direction and passes through diodes D_4 and D_1. When adding these up it produces a signal at the resistor like that shown in the figure # 4. If a square wave is used as input then the resistor will have a DC voltage across it at all times.

(b) Adding a capacitor to this circuit smooths the signal at the resistor producing a DC signal with a slight ripple on it. Since the capacitor is so large, it takes a long time to charge and discharge. The charging and discharging will have a time constant of RC, which should be large compared to the oscillating time of the AC signal. If the RC time constant is large enough relative to the oscillating time of the signal, then the ripple on the DC signal is small.

6. (a) This circuit is called a half adder circuit and adds two one digit binary numbers with a carry bit. Its output is in the form of a two bit binary number. The three gates on the top are just an exclusive or gate (XOR). The truth table is

A	B	Carry	Sum
0	0	0	0
1	0	0	1
0	1	0	1
1	1	1	0

(b) The full adder can be made up of two half adders and a OR gate. See figure # 5 for the circuit diagram. The truth table for the full adder is

A	B	C	Carry	Sum
0	0	0	0	0
1	0	0	0	1
0	1	0	0	1
0	0	1	0	1
1	1	0	1	0
1	0	1	1	0
0	1	1	1	0
1	1	1	1	1