1995-1996 Physics Olympiad Preparation Program

– University of Toronto –

Solution Set 1: General Physics

1. (a) We require that there is no net torque about the axis through the point where the two beams connect to the vertical support (recall that torque is $\vec{\tau} = \vec{r} \times \vec{F}$ where $\vec{r}$ is the vector separating the axis about which you are calculating the torque and the point where the force is applied, and $\vec{F}$ is the force). The torque due to gravity on beam 1 is centred through the midpoint of beam 1. Ditto for beam 2. The torque due to the counterweight is the cross product of the gravitational force and the vector between the axis and the end of beam 2. The force diagram for the crane is shown in Fig. 1a). Thus the net torque is

$$\sum \tau = -m_1 g \frac{l_1}{2} + m_2 g \frac{l_2}{2} + m_c g l_2 = 0 \quad (1)$$

$$\Rightarrow m_c = \frac{1}{2}\left(m_1 \frac{l_1}{l_2} - m_2\right) \quad (2)$$

$$= 12,000 \text{ kg},$$

where g is the acceleration of gravity ($g = 9.8$ m/s^2). Note that we can verify that the equation for m_c makes sense. For example, if $m_1 = m_2$ and $l_1 = l_2$, then we would expect $m_c = 0$, which is the case. Can you think of other sanity checks? What about units? Note that we do not have to take into account the torque due to the support cable tension on each beam. Can you understand why not?

(b) We require that the net torque on each arm to be equal. In this case, the tensions on the cable supporting the two beams are forces, T_1 and T_2 that act on the ends of the beams and on the vertical support. Let θ_1 be the opening angle between beam 1 and the cable (as shown in Fig. 1b)). Then the torque acting on beam 1 is

$$\sum \tau = 0 = l_1 T_1 \sin\theta_1 - \frac{l_1}{2} m_1 g \quad (3)$$

$$\Rightarrow T_1 = \frac{m_1 g}{2\sin\theta_1} \quad (4)$$

$$= \frac{m_1 g \sqrt{l_1^2 + h^2}}{2h} \quad (5)$$

$$= 139,000 \text{ N},$$

where h is the height of the vertical support above the beams. Similarily, for beam 2, the requirement that the net torque on the beam is zero gives

$$\sum \tau = 0 = -T_2 \sin\theta_2 + m_2 g \frac{l_2}{2} + m_c g \quad (6)$$

$$\Rightarrow T_2 = \left(\frac{m_2 g}{2} + m_c g\right) \frac{\sqrt{l_2^2 + h^2}}{h} \quad (7)$$

$$= 187,000 \text{ N}.$$

Its interesting that the tension is highest on the cable supporting the counterweight. Them cables are pretty strong. They have to be able to support almost ten tons, and that's without any load being carried.

2. (a) To lift Batman and Ms. Vale up $h = 15$ m, one has to at least expend the work equivalent to raising them that distance in a gravitational field (it could be more–think about the work required

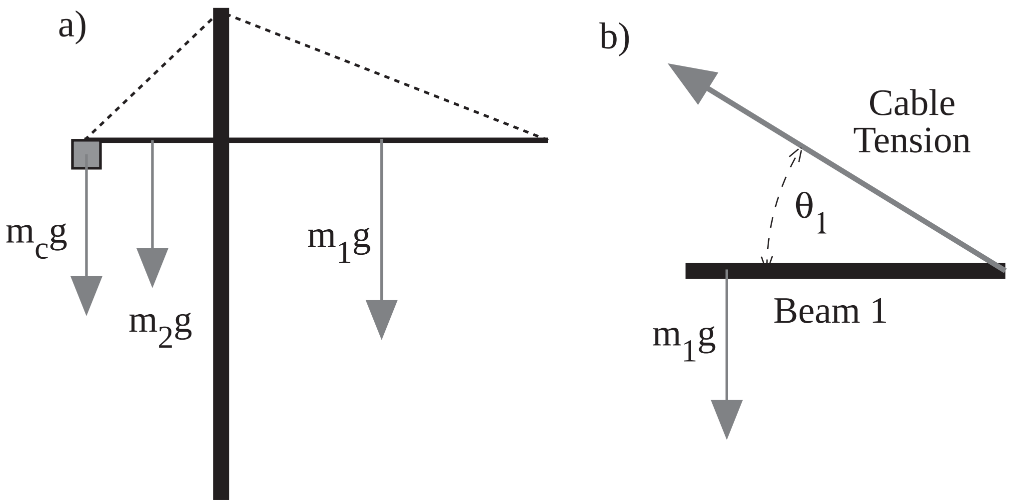

Figure 1: The forces acting on the two beams around the point at shich they are connected to the vertical support are shown in a). Note that we do not consider the the force of tension on the two beams from the support cables. (Why not?) The forces acting on the end of beam 1 due to the cable and the weight of gravity are shown in b).

to accelerate them into motion). This is equal to

$$W = h\,(m_B + m_V)\,g = 17,931 \text{ Joules}, \tag{8}$$

where g is our faithful acceleration due to gravity at Earth's surface.

(b) Let's assume they accelerate to $v_h = 1$ m/s and then rise steadily the rest of the way. The power expended (ignorning the power required for acceleration) is just the force × velocity, which is

$$P = (m_B + m_V)\,g \tag{9}$$

$$= 1,196 \text{ Watts } = 1.6 \text{ hp}. \tag{10}$$

Recall that 1 horsepower is equal to 746 Watts. That's a pretty hefty little motor he has there.

(c) Let's consider why the winch stopped in the first place. There are only a few likely scenarios (given that it wasn't really broken):

i. motor overheated or otherwise failed,
ii. the energy source couldn't continue to supply the needed power, or
iii. the winch is based on some funky technology that I know nothing about that gives out at exactly the right moment in the script.

The fact that it continued to operate and yank Vicky up when Batman let go and the tension dropped indicates that i) is unlikely; the drop in tension itself would not have been correlated with the moment when the motor became operative. Scenario ii) is plausible, as the power level required to haul Vicky up would be less than that required to haul both of them up. Of course, I would go for iii). Hollywood rarely cares about conforming to reality.

3. (a) We can safely assume that Sven and the pulpwood are going to decelerate uniformly, with deceleration a. This is an approximation, as brakes usually fade as they heat up, but we have to start from somewhere, and it isn't such a bad assumption. We can then employ our usual equations for an object of mass m decelerating uniformly over a distance d. If $v_0 = 120$ km/h is Sven's initial speed, then the time required to decelerate is $t = v_o/a$. Thus,

$$d = \frac{1}{2}a\left(\frac{v_o}{a}\right)^2 = \frac{v_o^2}{2a} \tag{11}$$

$$\Rightarrow a = \frac{v_0^2}{2d} = 5.8 \text{ m/s}^2, \tag{12}$$

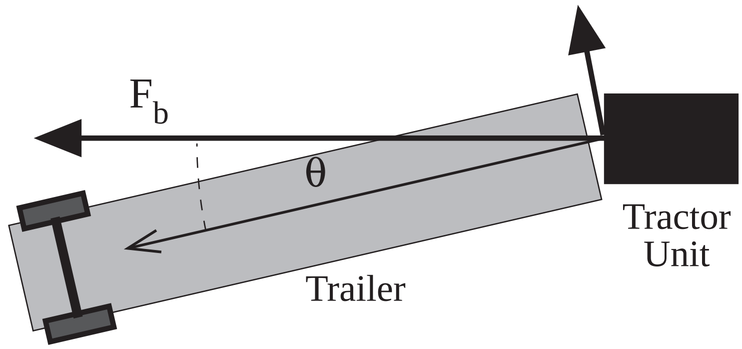

Figure 2: The braking force acting on the trailer.

which is over half a g. That's real heavy braking. I'm guessing Sven ended up with the meat.

(b) The force diagram on the trailer is shown in Fig. 2. The braking force F_b is shown, along with the component of that force perpendicular to the axis of the trailer, $F_p = F_b \sin\theta$, where θ is the angle between the axis of the trailer and the braking force. There is one other force in the problem; the reaction force of the pavement acting on the trailer through its tires to keep the angle θ constant (note that it is not shown in Fig. 2). This reaction force would normally be equal to F_p, so that there is no increase in angular momentum of the trailer (note that the trailer is swinging in uniform motion as it turns the corner). The maximum frictional force is

$$F_f = \mu_s m_w g, \tag{13}$$

where m_w is the weight of the trailer resting on the back axle, which we approximate as half the total trailer weight m_t. Since the braking force is $F_b = m_t a$, the maximum angle between the trailer and tractor unit is given by the requirement

$$\begin{aligned} F_f &= F_p & (14)\\ \Rightarrow \frac{\mu_s m_t g}{2} &= m_t a \sin\theta & (15)\\ \Rightarrow \theta &= \sin^{-1}\left(\frac{\mu_s g}{2a}\right) = 9.7^\circ. & (16) \end{aligned}$$

Not a big angle in this case. Note that it matters not a whit what the trailer weighs.

4. Since a mil is $\frac{1}{1000}$ inch, the cross-sectional area of the conductor is:

$$A = \pi\left[\left(\frac{0.0808\text{ in}}{2}\right)\left(\frac{2.54\times10^{-2}\text{ m}}{1\text{ in}}\right)\right]^2 = 3.31\times10^{-6}\text{ m}^2.$$

The current density therefore is:

$$J = \frac{I}{A} = \frac{20\text{ A}}{3.31\times10^{-6}} = 6.04\times10^{6}\text{ A}\cdot\text{m}^{-2}.$$

The electric field intensity is:

$$E = \frac{J}{\sigma} = \frac{6.04\times10^{6}\text{ A}\cdot\text{m}^{-2}}{5.8\times10^{7}\ \Omega^{-1}\cdot\text{m}^{-1}} = 1.04\times10^{-1}\text{ V}\cdot\text{m}^{-1}.$$

The voltage drop is given by:

$$V = E\ell = (1.04\times10^{-1}\text{ V}\cdot\text{m}^{-1})(50\text{ ft})(12\text{ in}\cdot\text{ft}^{-1})(0.0254\text{ m}\cdot\text{in}^{-1}) = 1.59\text{ V}.$$

The resistance is given by:

$$R = \frac{V}{I} = \frac{1.59\ \text{V}}{20\ \text{A}} = 7.95 \times 10^{-2}\ \Omega.$$

Since $\sigma = \rho\mu$, where ρ is the charge density:

$$\rho = \frac{\sigma}{\mu} = \frac{5.8 \times 10^{7}\ \Omega^{-1} \cdot \text{m}^{-1}}{0.0032\ \text{m}^{2} \cdot \text{V}^{-1} \cdot \text{s}^{-1}} = 1.81 \times 10^{10}\ \text{C} \cdot \text{m}^{-3}.$$

Since $J = \rho U$, where U is the drift velocity:

$$U = \frac{J}{\rho} = \frac{6.04 \times 10^{6}\ \text{A} \cdot \text{m}^{-2}}{1.81 \times 10^{10}\ \text{C} \cdot \text{m}^{-3}} = 3.34 \times 10^{-4}\ \text{m} \cdot \text{s}^{-1}.$$

With the above drift velocity, an electron will require approximately 30 seconds to move a distance of 1 cm in the #12 copper conductor.

5. The image distance, v, can be calculated from the lens makers' equation:

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f},$$

which yields a distance of:

$$v = \frac{uf}{u - f},$$

where f is the focal length and u is the object distance.

The Rayleigh criterion, $\theta = \frac{1.22\lambda}{D}$, where θ is the limiting angle of resolution, λ is the wavelength, and D is the diameter of the aperture, can be used to calculate the blur diameter:

$$d = \theta v = \frac{1.22\lambda v}{D}.$$

The defocus blur is calculated by similar triangles as:

$$d = \frac{(v - f)D}{f}.$$

Equating the diffraction blur and the defocussing blur gives:

$$D = \sqrt{\frac{1.22\lambda v f}{(v - f)}} = \sqrt{1.22\lambda u}.$$

For $\lambda = 550$ nm and $u = 15$ m, we end up with $D = 3.2$ mm.

6. (a) The digital logic function is an OR expression encompassing the combinations that lead to a correct process, namely:

$$X = A \cdot C \cdot S + \bar{A} \cdot \bar{C} \cdot \bar{S} + C.$$

The circuit is sketched in Fig. 3a).

(b) Operating on the second term of the above expression with one of DeMorgan's laws of Boolean algebra ($\overline{A \cdot B} = \bar{A} + \bar{B}$) gives:

$$X = A \cdot C \cdot S + \overline{A + C + S} + C.$$

Applying the redundancy law of Boolean algebra ($A + A \cdot B = A$) yields:

$$X = (C + A \cdot C \cdot S) + \overline{A + C + S} = C + \overline{A + C + S}.$$

The simplified circuit is sketched in Fig. 3b).

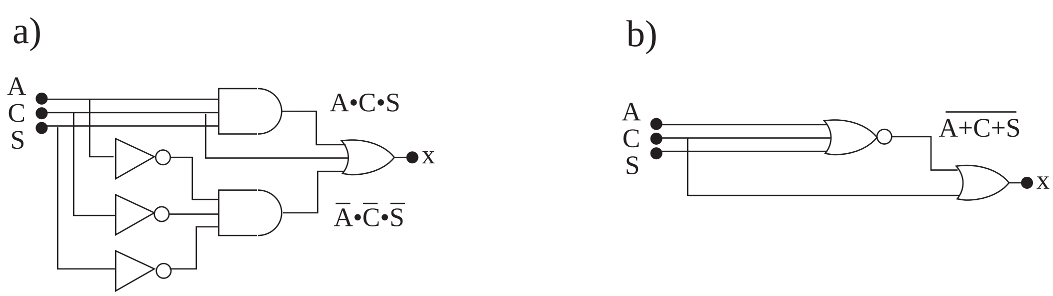

Figure 3: The circuit to handle the logic of the starship *Compromise* transporter control logic is shown in a). A simplified circuit is shown in b).