1995-1996 Physics Olympiad Preparation Program

– University of Toronto –

Solution Set 6: Electronics

1. (a) The feedback voltage, i.e. the voltage at the "-" terminal of the op amp, can be written as:

$$v_1 = v_0 - (i + i_{\text{in}})R_2.$$

Since $i = v_1/R_1$ and $i_{\text{in}} = v_d/R_d = v_0/A_{OL}R_d$:

$$v_1 = \left[\frac{1 - R_2/A_{OL}R_d}{1 + R_2/R_1}\right] v_0.$$

But

$$v_d = v_1 - v_2 = \frac{v_0}{A_{OL}},$$

or

$$v_2 = v_1 - \frac{v_0}{A_{OL}}.$$

Combining and rearranging the above expressions gives:

$$A_v = \frac{v_0}{v_2} = \frac{R_1 + R_2}{R_1 - \frac{R_1 R_2}{A_{OL} R_d} - \frac{R_1 + R_2}{A_{OL}}}.$$

(b) If $i_{\text{in}} = 0$, then the currents through R_2 and R_1 are equivalent and:

$$\frac{v_0 - v_1}{R_2} = \frac{v_1}{R_1},$$

and

$$A_v \equiv \frac{v_0}{v_2} \approx \frac{v_0}{v_1} = 1 + \frac{R_2}{R_1}.$$

2. Gill's circuit is a *Bridge circuit* and the problem is solved by calculating the *Thevenin* equivalent voltage and impedance. The impedance of a resistor and inductor in series is given by the complex expression $Z = R + i\omega L$, where R is the resistance, ω is the angular frequency, and L is the impedance. The quantity $X_L = \omega L$ is called the inductive reactance.

The equivalent impedance at terminals AB with the source set equal to zero is:

$$Z' = \left[\left(\frac{1}{12\ \Omega + i(5\ \text{kHz} \cdot 4.8\ \text{mH})}\right) + \frac{1}{21\ \Omega}\right]^{-1} + \left[\left(\frac{1}{30\ \Omega + i(5\ \text{kHz} \cdot 12\ \text{mH})}\right) + \frac{1}{50\ \Omega}\right]^{-1},$$

which simplifies to:

$$Z' = 47.3\ \Omega \text{ at } 26.8°.$$

With the circuit open at AB, the current on the left side of the bridge is:

$$I_1 = \frac{20\ \text{V}}{21\ \Omega + 12\ \Omega + i(5\ \text{kHz} \cdot 4.8\ \text{mH})},$$

and the current on the right side of the bridge is:

$$I_2 = \frac{20\ \text{V}}{50\ \Omega + 30\ \Omega + i(5\ \text{kHz} \cdot 12\ \text{mH})}.$$

If we assume that point A is at a higher potential than point B, then we have:

$$V' = V_{AB} = I_1(12\ \Omega + i(5\ \text{kHz} \cdot 4.8\ \text{mH})) - I_2(30\ \Omega + i(5\ \text{kHz} \cdot 12\ \text{mH})).$$

Substituting and simplifying gives:

$$V' = 329\ \text{mV at } 170.5^\circ.$$

3. Just like in many other nightmares, things here didn't proceed optimally. The function, $f = A \cdot B + \bar{A} \cdot B$, can be simplified to B, so no NOR gates are needed and the output can simply be wired to the input B. The question, however, insists that we design a logic circuit using NOR gates, so here goes.

   Since $f = A \cdot B + \bar{A} \cdot B$, then:

$$\bar{f} = \overline{\overline{\overline{A \cdot B + \bar{A} \cdot B}}}.$$

   Applying deMorgan's Laws ($\overline{A + B} = \bar{A} \cdot \bar{B}$, and $\overline{A \cdot B} = \bar{A} + \bar{B}$) gives:

$$\bar{f} = \overline{\overline{\overline{(A \cdot B)} \cdot \overline{(\bar{A} \cdot B)}}},$$

   and:

$$\bar{f} = \overline{\overline{(\bar{A} + \bar{B}) \cdot (A + \bar{B})}},$$

   and:

$$\bar{f} = \overline{\overline{(\bar{A} + \bar{B})} + \overline{(A + \bar{B})}}.$$

   Therefore:

$$f = \overline{\overline{\overline{(\bar{A} + \bar{B})} + \overline{(A + \bar{B})}}}.$$

   The circuit of NOR gates is given in Figure 1.

4. (a) Since levels of sound pressure, or intensity, are related to signal (voltage) levels in the electronics, and the data word length for a single sample is 16 bits, the number of levels of sound pressure is:

$$2^{16} = 65,536\ \text{levels}.$$

   (b) The Danish mathematician, Sven Nyquist, proved what is now called *Nyquist's Theorem*, which states that a sampling rate of at least **double** the highest recorded frequency is required for the reproduction of sine waves of the highest recorded frequency, and of any frequencies below that point. Since any waveform can be decomposed into a superposition of sinusoidal waves, the sampling rate for frequencies detectable by the human ear needs to be at least 40 kHz ($2 \times$ 20 kHz).

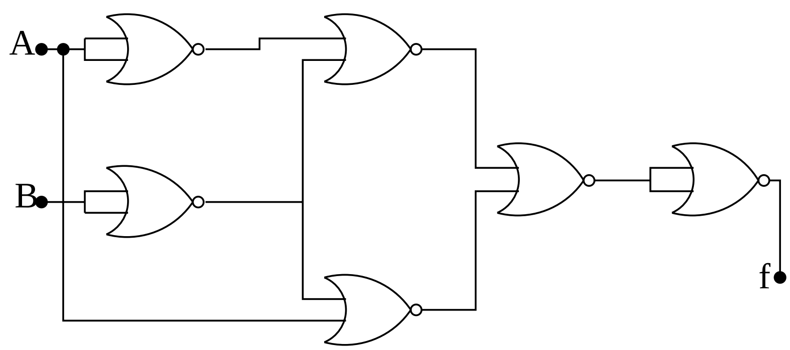

Figure 1: The NOR gate configuration in Problem 3.

S	R	CLK	Q	$\overline{Q}$
0	0	0	Q	$\overline{Q}$
0	0	1	Q	$\overline{Q}$
0	1	0	Q	$\overline{Q}$
0	1	1	0	1
1	0	0	Q	$\overline{Q}$
1	0	1	1	0
1	1	0	Q	$\overline{Q}$
1	1	1	?	?

Table 1: The truth table for the gated R-S flip-flop in Problem 5.

(c) The dynamic range of CDs is nominally said to be at least 90 dB. In this case, the decibel is a logarithmic measure of the relative highest and lowest power levels possible with the CD format. The decibel, when used to compare two power levels, is defined as $10\log_{10}\frac{P_2}{P_1}$, where P represents power. We know that CDs offer 65,536 *voltage* levels, and thus, since $P = V^2/R$ (V is voltage and R is resistance), the maximum dynamic range is:

$$20\log_{10} 65,536 \doteq 96.3 \text{ dB}.$$

(d) For vinyl long playing records, the stylus spirals along a groove from the outer radius to the inner one while the turntable rotates at a fixed angular velocity. For CDs, the laser pickup instead tracks a spiral from the inside radius to the outer one, and the disc is rotated with a variable angular velocity to provide a constant (linear) speed difference between the pickup and the track for any given radius on the disc. The linear relative speed at the outer radius is:

$$\left(\frac{120 \text{ mm}}{2}\right)(200 \text{ rpm}) = 12 \text{ m/s}.$$

Equating this with the linear relative speed where the magnitude of the angular velocity is 500 rpm gives an inner radius of 24 mm. Now that we have the inner and outer radii, we can calculate the active area of a CD as:

$$\pi\left((60 \text{ mm})^2 - (24 \text{ mm})^2\right) = 9.5 \times 10^{-3} \text{ m}^2.$$

The storage capacity is approximated by using the maximum playtime and the digital sampling rate:

$$(74 \text{ min})(60 \text{ sec/min})(44.1 \times 10^3 \text{ words/sec})(2 \text{ bytes/word}) = 3.92 \times 10^8 \text{ bytes}.$$

The data density is therefore:

$$\frac{3.92 \times 10^8 \text{ bytes}}{9.5 \times 10^{-3} \text{ m}^2} = 4.1 \times 10^{10} \text{ bytes/m}^2.$$

5. (a) The truth table for this sequential logic circuit, which is a gated R-S flip-flop, is given in Table 1.

(b) The principle of this R-S flip-flop can be used to store information in a clocked computer memory register.

(c) Refer to Figure 2 for an equivalent configuration made up of NAND gates.

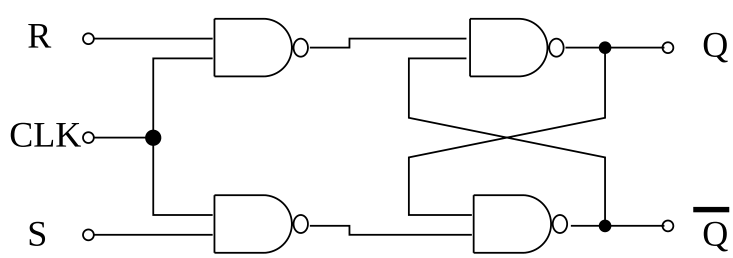

Figure 2: The NAND gate equivalent to Problem 5.

6. Combining the the given attenuation and interterminal resistance information suggests that there's a passive bandpass filter in the black box. Refer to Figure 3 to see the circuit. Since we're only concerned with amplitudes here (we can ignore the phase), we can deal with the equation $V = I|Z|$, where Z is the impedance. The ratio of the output over input voltage amplitudes is therefore given by:

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{R}{\sqrt{(2\pi f L)^2 + r^2 - \left(\frac{1}{2\pi f C}\right)^2 + R^2}}.$$

The resonant frequency from the above equation, f_o, is calculated to be:

$$f_o = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}.$$

We know that the resistance r exists because of the nonzero attenuation (20%) at the resonance frequency f_o. At resonance:

$$\frac{V_{\text{out}}}{V_{\text{in}}} = 80\% = \frac{R}{\sqrt{R^2 + r^2}}.$$

The relationship between r and R is therefore:

$$r = 0.75R.$$

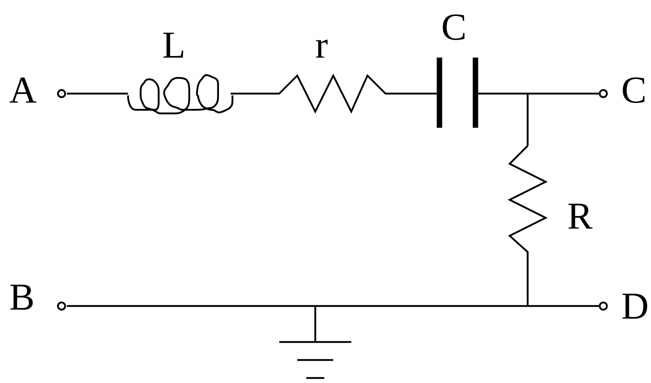

Figure 3: The passive LRC circuit in the black box of Problem 6.