1997-1998 Physics Olympiad Preparation Program

– University of Toronto –

Problem Set 6 Solutions: AC Circuits and Electronics

1) Misha's Mom's Medallions

Faraday law of electrolysis states that the mass of substance liberated in electrolysis is proportional to the charge passed:

$m = kq = kIt = (m_0/F)It,$

Where k is the electrochemical equivalent of the substance; m_0 is a molar mass; $F = N_A q_e$ is the Faraday constant which is equal to the product of the Avogadro's number and the electron charge. The mass of medallion $m = Vd = Ahd = m_0 It/F$.

Thus the period of time for silver-plating the medallion $t = FdhA/m_0 I =$ ((9.65)(1.05)(5)(5.7)(10))/((1.1)(1.8)) = 1458 sec = 24.3 min. Therefore, Misha has to pay 4 dollars for his mom's birthday present.

2) Tesla Yes! Edison No!

The electrical power delivered to the factory $P = VI = 55\ kW$. The transmission line conductors have a resistance of 3Ω ⇒ the voltage drop along the line $U = 1{,}500\ V$. Therefore, the output voltage must be $1{,}500\ V + 110\ V = 1{,}610\ V$ to deliver 110 V to the factory. The power loss is $P_1 = 2\ UI = 1{,}500\ kW$, which is 27 times greater than the delivered power.

If we use transformers, we can claculate the turns ratio $n_2/n_1 = V_2/V_1 = 500$ and the current in transmission line $I_2 = I_1 \,.\, (n_1/n_2) = 1A$. The voltage drop along the line $U = 3\ V$, which is only $5 \bullet 10^{-3}$% of $V_2 = 55\ kV$. The power loss in the line now is equal to $P_1 = 6\ W$, which is only $1 \bullet 10^{-2}$% of the value of the delivered power.

3) Analog differentiation and integration

a)we have

$$\frac{q}{C} + R\frac{dq}{dt} = V_{in} \qquad [1]$$

$$R\frac{dq}{dt} = V_{out} \qquad [2]$$

Now, for low frequencies, the capacitor is charging/discharging all the time, and most of the voltage drop occurs across it [$R_{\text{capacitor}} >> R$, if you like]. Hence, we can drop the Rdq/dt in eq. [1]. Differentiating eq. [1] and plugging into [2] gives:

$$\frac{q}{C} \approx V_{in}$$

$$\frac{d}{dt}\left(\frac{q}{C}\right) = \frac{dV_{in}}{dt}$$

$$\frac{dq}{dt} = C\frac{dV_{in}}{dt} = \frac{V_{out}}{R}$$

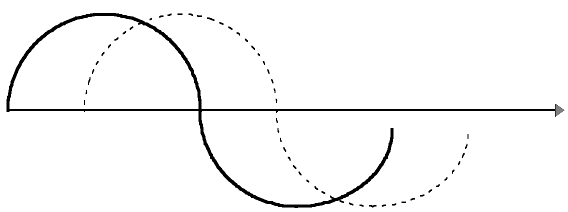
Or,

$$V_{out} = RC\frac{dV_{in}}{dt}$$

[condition: low frequencies; input frequency << $1/RC$]

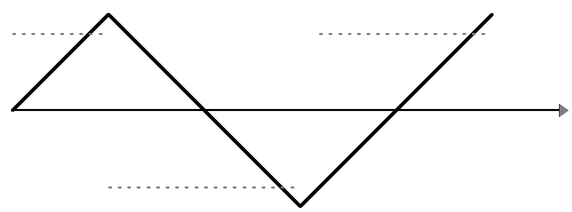
<u>Plots</u>

i)



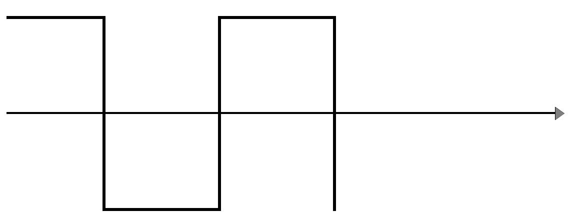
dotted line: output (cosine)
normal line: input (sine)

iii)



dotted line: output
normal line: input

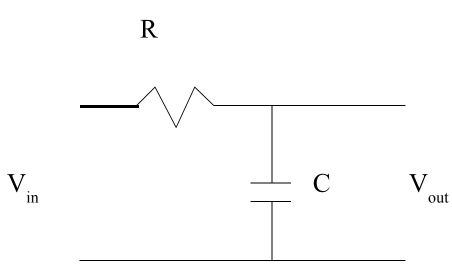
ii)



0-line: output
normal line: input

NOTE: plots are not to scale!

b)Circuit:



we have

$$\frac{q}{C} + R\frac{dq}{dt} = V_{in} \qquad [1]$$

$$\frac{q}{C} = V_{out} \qquad [2]$$

Now, for high frequencies, there is not enough time for the capacitor to be

charged, and most of the voltage drop occurs across the resistor [$R_{\text{capacitor}} << R$, if you like]. Hence, we can drop the q/C in eq. [1].
Differentiating eq. [2] and plugging into [1] gives:

$$R\frac{dq}{dt} \approx V_{in}$$

$$\frac{d}{dt}\left(\frac{q}{C}\right) = \frac{dV_{out}}{dt}$$

$$\frac{dq}{dt} = C\frac{dV_{out}}{dt} = \frac{V_{in}}{R}$$

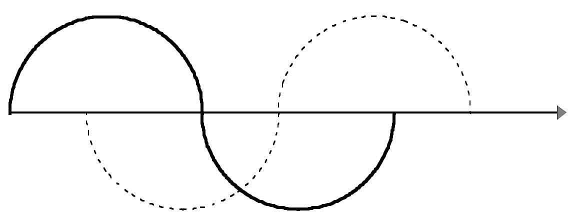
Or,

$$V_{out} = \frac{1}{RC}\int V_{in}dt$$

[condition is: high frequencies; angular frequency of input signal >> $1/RC$]

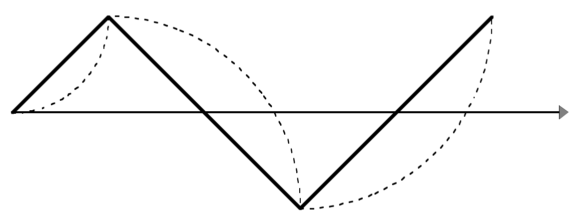
<u>Plots</u>

i)



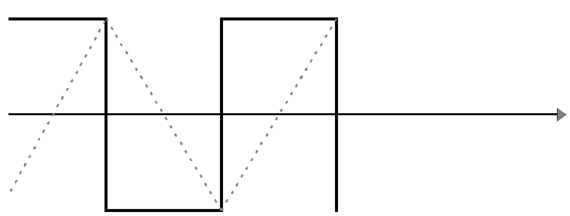
dotted line: output (-cosine)
normal line: input (sine)

iii)



dotted line: output (parabolas)
normal line: input

ii)

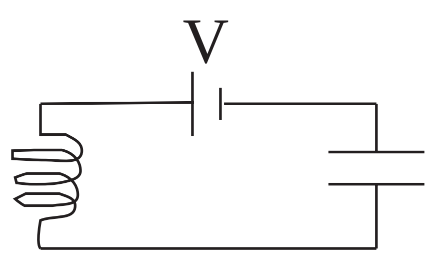


dotted lines: output
normal line: input

NOTE: Plots are not to scale!

4) What goes around, comes around

a) If the voltage across the battery is V, then the voltage across the capacitor is –V in steady state.



b) From the rather direct hints in the question, the voltage across the capacitor looks like: $dV_c/dt \propto I(t)$, where $\propto$ indicates proportional, I(t) is the total current running through the circuit as a function of time, and V_c is the voltage across the capacitor

Again, from the oh-so-subtle hints, the voltage across the inductor is:

$$V_i \propto dI(t) / dt$$

From Kirchhoff's voltage law, the sum of voltages in a circuit must be zero. For this to be true for all time, the change in voltage across one device must equal the opposite change in voltage across the other device. This allows us to write:

$$dV_c/dt = -dV_i/dt$$

$$I(t) \propto -d^2 I(t) / dt^2$$

This is the same as

$$x(t) \propto -d^2 x(t) / dt^2$$

which is the equation for a simple harmonic oscillator, as seen in a previous problem set featuring everyone's favorite oscillator, Bob (no relation to the author).

The solution to this is $I(t) \propto \cos(kt)$ where is k is some constant dependent on the inductance and capacitance in the circuit. If you are not sure if this is a proper solution, substitute it into the equation and verify that the LHS=RHS.

(I am being a little lax here about the boundary conditions, i.e. what is the current at I(0). By writing the solution as cos(kt), I am assuming that the initial current is non-zero, so that we are starting the clock just after the battery has been shorted and removed, not before.)

c) Energy transfers from the electric field in the capacitor to the magnetic field around the inductor and back again.

d) As soon as Frido tries to do something with his circuit, he will no longer have a perfect LC circuit. He must put a resistor in series with it:

As you know, electrical energy is turned into heat in a resistor according to RI^2 where R is the resistance and I is the current flowing through the circuit. Without even solving the circuit equation, you can show that energy is leaving the circuit. Alas, poor Frido has been boondoggled.

5) Operation: 'Amplifier'

a) i) $0V$

ii) $-15V$

iii) $15V$

b) With any op-amp circuit there are two possible results: either the inputs are different and the op-amp is in saturation, or else the inputs are driven so that they are at the same voltage. Feedback allows the the second result in each of the given cases.

i) The current flowing through the feed back loop must be the same current flowing out the *V*- line. We try to find a self-consistent situation were $V- = 0$. For this to happen, the current must be $1V/100$ Ohms. Thus $V_{\text{out}} = -1V$. This circuit inverts the input voltage

ii) Same reasoning as in part (i). $V_{\text{out}} = -5V$. This circuit inverts and amplifies with a gain of five, the input voltage.

iii) $V_{\text{out}} = -1.9V$. This circuits adds and inverts the input voltages.

iv) $V_{\text{out}} = 2V$. This circuit doubles the input voltage (no inversion)

6) Flip-Flopping on a counter proposal

a) If *R* and *S* are both set to 1, there are 2 equally stable output configurations: Q=0 and Q_{bar}=1, or Q=1 and Q_{bar}=0. If we switch *R* to 0 and back to 1, *Q* and Q_{bar} will switch from the stable configuration they started in to the other stable configuration. Thus a pulse in *R* causes the outputs to "flip flop" between the two stable configurations.

b)

pulse	*A*	*B*	*C*
0	0	0	0
1	1	0	0
2	0	1	0
3	1	1	0
4	0	0	1

We can see that if we let the output of *A* be the 2^0 binary digit, *B* be the 2^1 digit, and *C* the 2^2, this counter gives the *binary value* for the number of pulses that have been fed into the input (e.g., 4 = 100 base2)