1998-1999 Physics Olympiad Preparation Program

— University of Toronto —

Solution Set 1: General

1) Pushy Photons

a) Black is good — it would absorb all the light, and therefore transfer all the momentum of the light. But *silver* would be better — when the light reflects back to where it came from, it reverses its momentum. So this would mean (p– (–p)) = 2p for the momentum transferred to the sail.

b) From the useful bits of info at the bottom of the question sheets, the solar constant is 1.353 kW m^{-2}. For light, E = pc, so by taking derivatives we also have that:

$$\frac{dE}{dt} = \frac{dp}{dt} \cdot c$$

However, *power* P is defined as the rate of change of energy with time, on the left-hand side, and *force* F is defined as the rate of change of momentum with time, on the right-hand side. So we can write:

$$P = \frac{dE}{dt} = \frac{dp}{dt} \cdot c = F \cdot c$$

or

$$F = \frac{P}{c}$$

or actually F = 2P/c if we let the light be reflected.

A soccer field isn't always the same size, but has to be in a certain allowed range of sizes; roughly 100m × 50m might be reasonable. The power P incident on the field is intensity times area: $P = IA = 1.353 \text{ kW m}^{-2} \bullet 5000 \text{ m}^2 = 6.765 \text{ MW}$ (quite a bit of power), so the force on the whole large area would be

$$F = 2 \bullet 6.765 \text{ MW} \ / \ (3 \times 10^8 \text{ m s}^{-1}) = 4.51 \times 10^{-2} \text{ N}$$

Also, F = ma, so the acceleration of the 2,000 kg spacecraft would be:

$$a = 2.25 \times 10^{-5} \text{ m s}^{-2}$$

The velocity after constant acceleration for a time t depends on initial velocity (which is zero, here):

$$v = u + at$$

A day of seconds is (24 hr) • (60 min/hr) • (60 sec/min) = 86,400 s. So the velocity after a day would be 1.94 $\mathrm{m\ s^{-1}}$, which is jogging speed. After a year, it would be 365 times this value, about 710 $\mathrm{m\ s^{-1}}$, which is roughly 10% of the orbital speed of a low-orbit satellite.

c) Again, if the light is reflected, $F = 2P/c = 2 \bullet 10^{12}\ \mathrm{W} / (3 \times 10^{8}\ \mathrm{m\ s^{-1}}) = 7 \times 10^{3}$ N, which is the gravitational force of the smallest automobile. The force is only applied for a trillionth of a second, so the momentum transfer (the impulse) is:

$$p \bullet \Delta t = (7 \times 10^{3}\ \mathrm{N}) \bullet (10^{-12}\ \mathrm{s}) = 7 \times 10^{-9}\ \mathrm{kg\ m\ s^{-1}}$$

which is small, like a big dust-mote moving in a sunbeam, or a raindrop in a very misty rain.

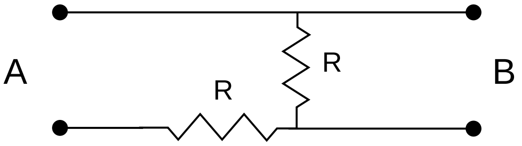
The *pressure* is a different story, though, because this force is applied over a tiny spot:

$$P = F \ / \ A = (7 \times 10^{3}\ \mathrm{N}) \ / \ (\ (10 \times 10^{-6}\ \mathrm{m})^{2}\ ) = 7 \times 10^{13}\ \mathrm{N\ m^{-2}}$$

100 kPa = $100 \times 10^{3}\ \mathrm{N\ m^{-2}}$ is roughly one atmosphere of pressure. So this pressure, briefly exerted by the focussed laser pulse, is something like 700 million atmospheres of pressure. *[Robin]*

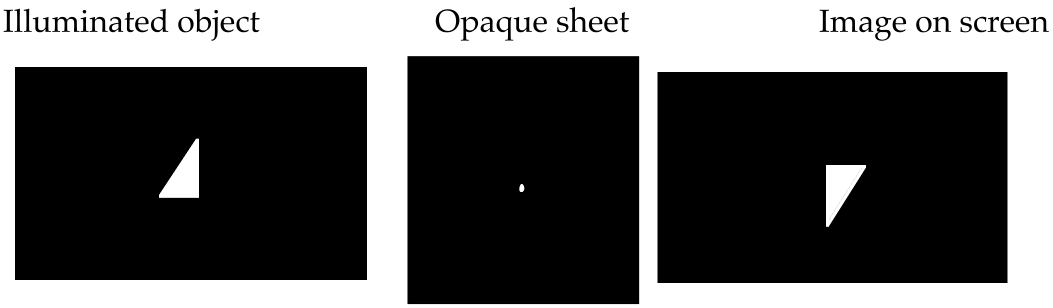
2) Out of sight, hope you don't mind

This simple circuit works, using any two identical resistors. <u>Worth noting:</u> since the terminal-pairs 'A' and 'B' don't behave the same, the circuit must not be symmetric for the pairs of terminals. *[Gnädig/Honyek]*



3) Leonardo's *camera obscura*

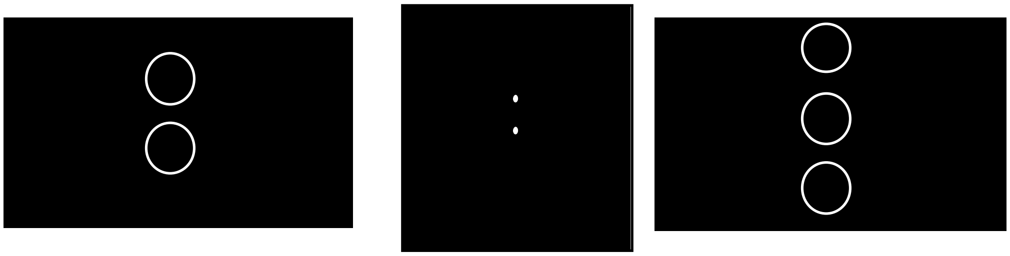
a)


Illuminated object

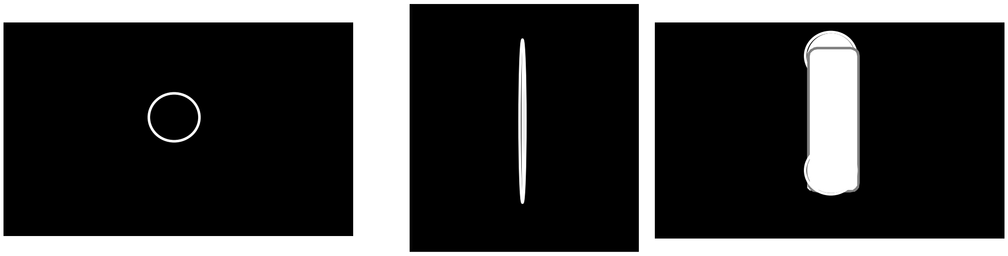
Opaque sheet

Image on screen

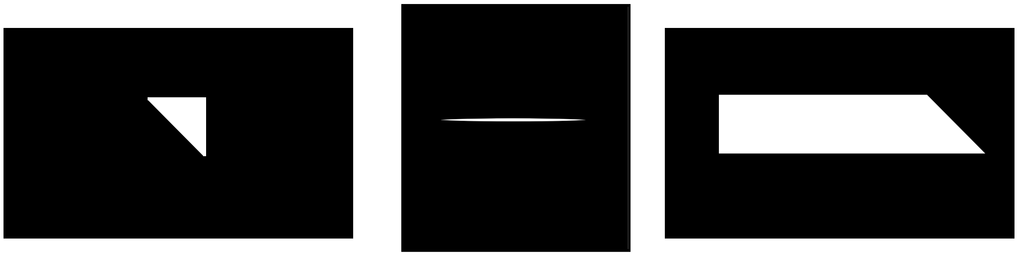
b) Illuminated object Opaque sheet Image on screen



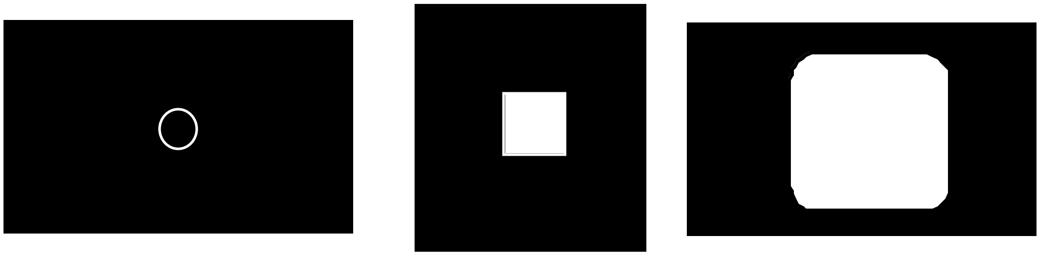
c) Illuminated object Opaque sheet Image on screen



d) Illuminated object Opaque sheet Image on screen



e) Illuminated object Opaque sheet Image on screen



[James]

4) Heat works

We begin with two identical spheres at some uniform temperature. One rests on an insulating table and the other is suspended by an insulating thread. To each sphere the same amount of heat energy, Q Joules say, is transferred.

In the absence of external forces we would expect each sphere to have an equal temperature change. However, since gravity is present and the volumes of the sphere will increase therefore their *centres of mass will move*.

Since the ball on the table will have its centre of mass *rise*, some heat energy will be used to do work against gravity. For the hanging ball, however, the situation is reversed. Therefore the resting ball will have a lower final temperature than that of the hanging ball.

Suppose each sphere is 100 g, initially at some temperature T_I K (the density of aluminum is 2.7×10^3 kg m^{-3}) and 250 KJ of heat is added to each. Let Δr_A and ΔT_A denote the change in radius of the sitting sphere, sphere A and Δr_B and ΔT_B denote that for the hanging sphere, sphere B.

The initial radius of each sphere can be calculated with

$$\frac{4}{3}\pi r_i^3 = \frac{[\text{mass of aluminum}]}{[\text{density of aluminum}]} = \frac{\frac{1}{10}\text{kg}}{2.7 \times 10^3 \text{ kg m}^{-3}}$$

then $$r_i = \left[\frac{3 \times 10^{-3}}{27\,(4\pi)}\right]^{\frac{1}{3}} \approx 0.0207 \text{ m}$$

Assuming a linear expansion coefficient to apply over the entire uniform heating we have (to first approx.),

$$\Delta r_A = [2.31 \times 10^{-5}\, \Delta T_A] r_i = 4.777$$

and

$$\Delta r_B = [2.31 \times 10^{-5}\, \Delta T_B] r_i = 4.777$$

Hence

$$\Delta T_A = \frac{1}{2.31 \times 10^{-5}} \quad \frac{\Delta r_A}{r_i}$$

and

$$\Delta T_B = \frac{1}{2.31 \times 10^{-5}} \quad \frac{\Delta r_B}{r_i}$$

The heat capacity, C, of 100 g of aluminum is simply

$$C = 24.4 \frac{\text{J}}{\text{K} \cdot \text{mol}} \cdot \left(\frac{27}{27 \text{ g/mol}}\right) \cdot (100 \text{ g})$$
$$= 90.3704 \frac{\text{J}}{\text{K}}$$

Where we have used the molar mass of aluminum to be 27 g/mol.

Clearly the centre of mass of sphere A will rise by a distance equal to Δr_A and the centre of mass of sphere B will fall a distance equal to Δr_B.

Since the heat energy transferred must balance the change in potential energy plus the energy due to the temperature change we have

$$
\begin{aligned}
250\,000 &= \mathrm{mg}\ \Delta \mathrm{r}_\mathrm{B} + \mathrm{C}\Delta \mathrm{T}_\mathrm{A} \\
&= \frac{9.81}{10}(4.777 \times 10^{-7})\Delta \mathrm{T}_\mathrm{A} + 90.3704\ \Delta \mathrm{T}_\mathrm{B}
\end{aligned}
$$

Thus

$$250\,000 = [4.777 \times 10^{-7} + 90.3704]\ \Delta \mathrm{T}_\mathrm{A}$$

Similarly

$$250\,000 = [4.777 \times 10^{-7} + 90.3704]\ \Delta \mathrm{T}_\mathrm{B}$$

Therefore, the final difference in temperatures is

$$
\begin{aligned}
\Delta T &= \Delta T_B - \Delta T_A \\
&= 250\,000\left[\frac{1}{90.3704 - 4.777 \times 10^{-7}} - \frac{1}{90.3704 + 4.777 \times 10^{-7}}\right] \\
&\approx 250\,000\left[1.170 \times 10^{-10}\right] \\
&\approx 2.925 \times 10^{-5}
\end{aligned}
$$

Therefore the difference in temperature is approx 2.87 × 10^{-5} K. Pretty small! *[Gnädig/Honyek & Peter]*

5) Death of an Atom

The energy given off in a time Δt is roughly

$$\Delta E = \frac{kq^2 a^2 \Delta t}{c^3}$$

In 1 orbit (time T),

$$\Delta E = \frac{kq^2 a^2 T}{c^3}.$$

We will assume the radius of an orbit stays constant for any one cycle (the electron is actually spiralling in, so this is not really true). The radius is R ≈ 0.5 Angstroms. The electron is kept in orbit by the attraction between it and the nucleus. We have $m\frac{V^2}{R} = \frac{kqq}{R^2} = ma$, where m is the mass of the electron and V is the orbital speed. The charge of the nucleus is +q for Hydrogen. Plugging all this in yields the nice expression:

$$E_{cycle} = \frac{2\pi k q^2}{R}\left(\frac{V}{c}\right)^3.$$

Here, this equals about $4.4*10^{-24}$ J / cycle [this should be negative, but we will remember that this is the energy lost]

The total energy of the orbit is: $-\frac{kqq}{2R}$

compared to that for a gravitationally orbiting object:

$$-\frac{Gmm}{R}.$$

The electron will presumably "die" when all this energy is exhausted. If we assume that the energy loss per cycle is as in i) (not really true), we will have:

$$\frac{kqq}{2R} = n\left(\frac{2\pi k q^2}{R}\left(\frac{V}{c}\right)^3\right),$$

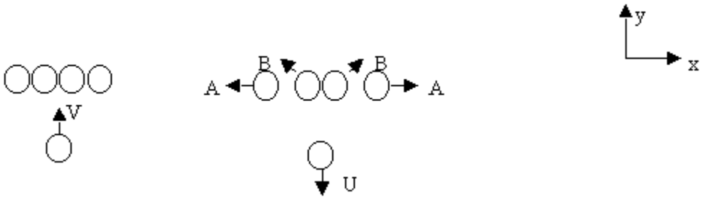
where n is the number of cycles. Plugging in the stuff, n = $3.4*10^5$.

The time, then, for this to happen is simply nT (T is the period) which comes out to about $6*10^{-11}$ s. It is not really meaningful to carry the first digit because of all the approximations we made. Surprisingly, doing a more accurate analysis leads a very similar answer. Either way, this is a very short time, and we have to conclude that the classical answer cannot be right. *[Peter]*

6) Stupid bet tricks

i) First, notice that when the balls collide all forces between them (and hence the momentum transferred) will be normal to the surfaces (that is the only point of contact between them). This is because there are supposed to be no frictional forces.

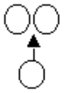
Let the labeling be as follows:



If we tried to do the question directly, we couldn't – we have 4 unknowns (U, B, A, direction of B) and only 2 equations (energy, momentum).

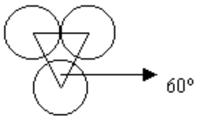
However, since the collisions last a very small amount of time we can split the problem into the two middle balls interacting with the cue ball and THEN the two middle balls interacting with the outside balls.

Doing this we get:



where the final speeds of the pair of balls are C (symmetric).

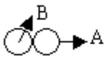
Additionally, we see that the momentum of each ball must be at a 30° angle to the normal:



So, $\frac{C_x}{C_y} = \tan(30) = \frac{1}{\sqrt{3}}$.

Solving, $C = \frac{2\sqrt{3}}{5}V$ (direction is 30° to y-axis), $U = \frac{1}{5}V$ [negative y direction].

Now, the two remaining collisions (which are symmetric, so we only solve one – the right hand side one).



We could write down all equations and solve this by brute force, or we could simply notice that Cy will be unchanged because there are no tangential forces (the balls are slippery – they don't "stick"). Hence, $B_y = C_y$.

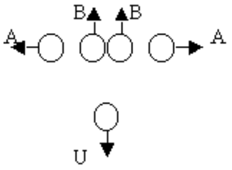
Solving the remaining equations yields

$$A = \frac{C}{2} = \frac{\sqrt{3}}{5}V$$

$$B = \frac{3}{5}V$$

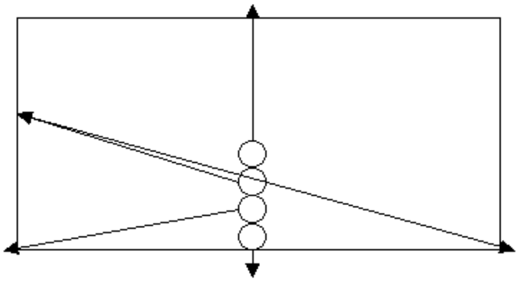
(the direction of B is [positive y], that of A is [positive x-axis]).

So we have:



And it seems the trick shot didn’t work – only the outside balls have been sunk.

ii) When you hit the balls hard they deform slightly and do “stick” to one another. That means that there is a tangential force between the outside most and middle balls (remember in the solution above we assumed no tangential forces to solve this collision). Numbering the balls 1 to 4 from left to right [see picture above] yields the following: 2 has a counterclockwise spin and 3 has a clockwise spin; moreover, the 2 centre balls now also have a horizontal velocity component [why ?]. Upon hitting a bank, which is fairly soft, the spin of ball #3 will make it rebound at more or less the same angle as it came in at — so 3 will go into another corner pocket. Now that you are completely confused look at the picture to make some sense of this:



[Robin is still trying to get a video picture of this pool-shot onto the website,at www.physics.utoronto.ca/~poptor]

[Peter]