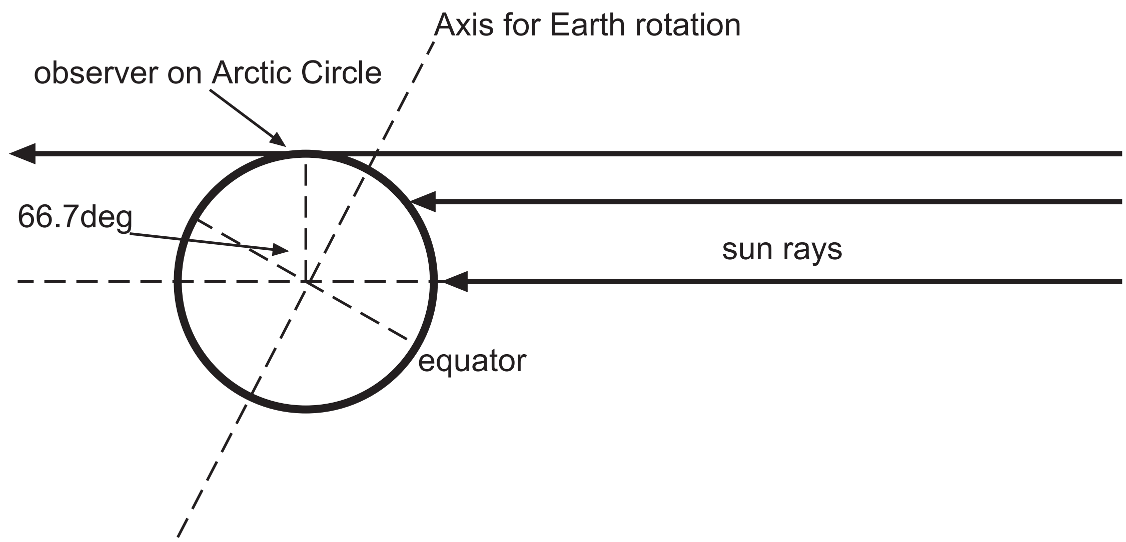
1998-1999 Physics Olympiad Preparation Program

— University of Toronto —

Solution Set 4: Optics and Waves

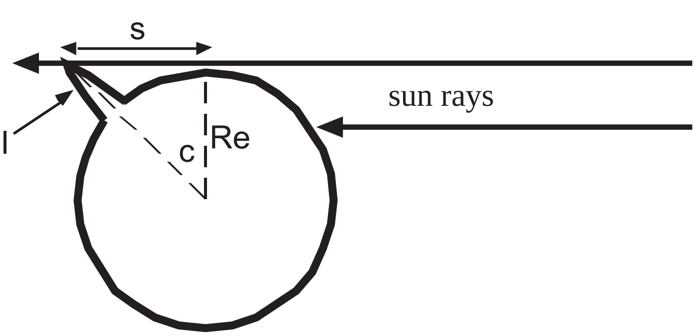
1) The land of the midnight sun (is where, exactly?)

a) At midnight on the summer solstice, the sun rays are tangent to the perfect sphere at the Arctic Circle. The 'side view' looks like:



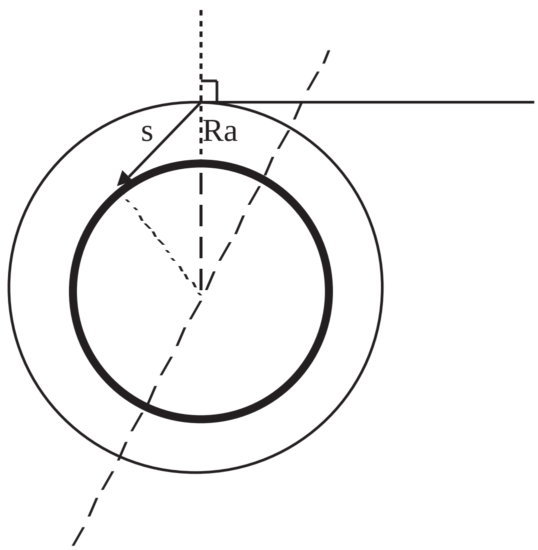
Once the picture is drawn it is obvious that the tilt of the sphere's axis to the plane it sweeps out around the sun is 66.7°.

b) i) Exaggerated for clarity:

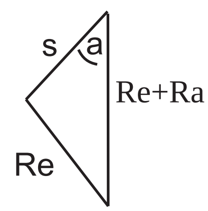


Thus c=arccos(Re/(Re+300m)) and the subtended arc opposite from angle c is 60 km. Thus you are 60 km south of the Arctic Circle.

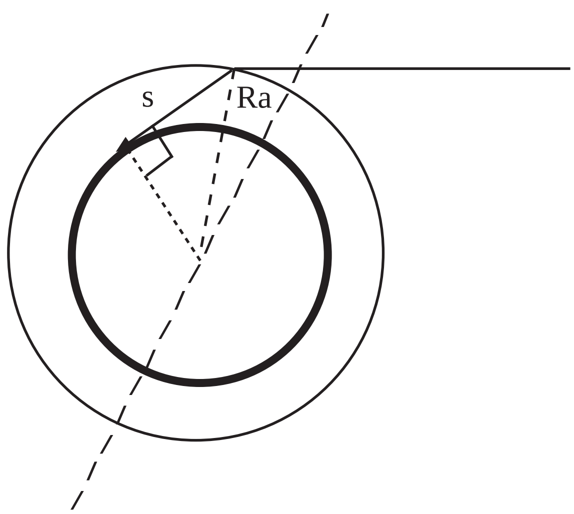
ii) Refraction causes the rays to bend at the air/vacuum interface. Modelling this as a single interface isn't exactly right (the interface is continuous and still causes refraction due to the index gradient). Snell's law: sin(90°)=n sin(a) as defined in the cartoons below:

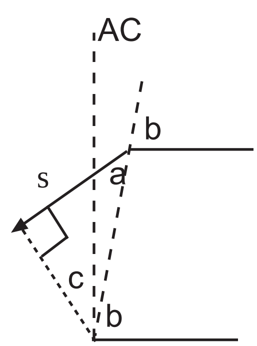


So:



Thus a=88.6°. Solving for s gives an imaginary number (i.e., with $\sqrt{-1}$). What went wrong? We assumed that the refracted ray that intersected the interface exactly over the Arctic Circle would eventually intersect the ground, but it does not. If the interface were lower, the index difference larger, or the Earth bigger, it could intersect the Earth's surface. Thus the most southern ray to intersect the ground intersects the atmosphere at a point towards the sun:





As shown on the diagram, this refracted ray is tangent to the earth surface. We find s=190 km. We still have to find out how far south we are. For this triangle, the refracted angle of the ray in the atmosphere (angle: a) is 88.19° to the normal. The incident angle (angle: b) is 88.85° to the normal. Thus we find that angle c is (90–88.19–(90–88.85)) = 0.66°. Using the value of Re, the corresponding arc length is 69 km. This is the distance you are south of the Arctic Circle. *[James]*

2) Making light of photons

a) The output at (1) is the sum of the two beams. Remember that though the intensity after one reflection followed by transmission through the beam splitter is 0.25, the electric field changes by 0.5 since $I \sim E^2$.

The math in this problem is easier if you replace 2*cos(a) by exp(ia)+c.c. where c.c. indicates complex conjugate (*i.e.*, in this case c.c.=exp(–ia)). Convince yourself that this relationship is true, using the fact (you may not know) that exp(ia) = cos(a) + i sin(a), where i = $\sqrt{-1}$. The problem still can be done by writing cos() and sin() everywhere, but this *phasor notation* is easier and extremely useful!

Thus the sum of the two beams at position (1) is:

$$E_{total} = \frac{E_0}{4}\left[\exp(i\pi)\left\{\exp\left(i2\pi v\left(\frac{2\cdot x1 + x0}{c} - t\right)\right) + \exp\left(i2\pi\left(\frac{2\cdot x2 + x0}{c} - t\right)\right)\right\}\right] + c.c.$$

where x0 is the distance from the beam splitter to position (1).

Rewriting:

$$E_{total} = \frac{E_0}{4}\exp\left(i\left\{2\pi v\left(\frac{2\cdot x2 + x0}{c} - t\right) + \pi + \frac{2\pi\cdot v\cdot x0}{c}\right\}\right)\left\{\exp(i2\pi\cdot v2\cdot Dx) + 1\right\} + c.c.$$

where Dx is (x1–x2).

This can be rewritten in terms of cosines but for the purposes of the rest of this question, this form is easier to deal with.

ii) From the above expression, you can see that there is no electric field at position (1) when $\exp(i2\pi v2Dx) + 1 = 0$ ie. when $Dx = n\frac{c}{(4v)}$ where n=..,–3, –1,1,3,.. Thus, if the difference in distance is 1/4 of the wavelength (recall wavelength=c/ν), no electric field is found at (1) and we have total destructive interference. Where does the energy go? Consider the electric field that is reflected back in the direction of the original beam. Due to fact that one beam undergoes three reflections and the other only one (instead of two and two), you can show using the same method as above that the MAXIMUM reflected occurs when $Dx = n\frac{c}{(4v)}$ where n=..,–3, –1,1,3,.. . Thus if the beam does not exist at position (1), it is reflected back the way it came. Energy is not lost.

iii) There is some subjectivity to what it means to 'clearly distinguish' but I would argue that you can clearly distinguish between a maximum and a minimum output. Maxima at (1) occur when $Dx = n\frac{c}{(4v)}$ where n=..,–2,0,2,.. So we can write

$(Dx_{max} - Dx_{min}) = \frac{0 - c}{(4v)}$, i.e., 1/4 of the wavelength.

The wavelength is 632 nm, so the smallest distance change you could clearly measure is 158 nm. That is pretty small. To contrast, atomic spacing in a solid is on the order of 5×10^{-10} m, so you could measure a distance corresponding to ~300 atoms.

iv) First assume the ether exists and is moving relative to your laboratory. This seems to make sense unless the ether is held to the surface of the Earth due to gravity. That would require the ether to have sizable mass. The ether must exist in space though since light travels in space, and then you would get some interesting light bending effects due to the concentration of ether around massive objects. Also you would still have ether existing in vacuum, thus you would have something with mass existing in a vacuum and now my head is starting to hurt. Even if the ether is not moving , you could put your apparatus in a vehicle, such that you would be moving relative to the ether.

The trick is to rotate your interferometer and see if you can see a change in the light output at (1). If one arm is parallel to the ether motion and the other is perpendicular to it, you find that destructive interference occurs if

$$\left(\exp\left(i2\pi v\left(\frac{x1}{(c+V)}+\frac{x1}{(c-V)}-\frac{2x2}{c}\right)\right)+1\right)=0$$

where V is the speed of the ether relative to your interferometer. Rotate the interferometer by 90º and this term changes to:

$$\left(\exp\left(i2\pi v\left(\frac{2x1}{c}-\frac{x2}{c+V}-\frac{x2}{(c-V)}\right)\right)+1\right)$$

Define the minimum V such that by rotating your interferometer, you go from a case of destructive interference to a constructive interference. (You might argue that this is too much to require; the output intensity only needs to change by a small amount. This is really determined by the noise in your system, which is a subject we are not going to discuss here, so we take the strongest case required.). Thus:

$$\left(\frac{x1}{(c+V)}+\frac{x1}{(c-V)}-\frac{2x2}{c}\right)=\frac{1}{(2v)} \qquad \text{(Destructive)}$$

$$\left(\frac{2x1}{c}-\frac{x2}{c+V}-\frac{x2}{(c-V)}\right)=0 \qquad \text{(Constructive)}$$

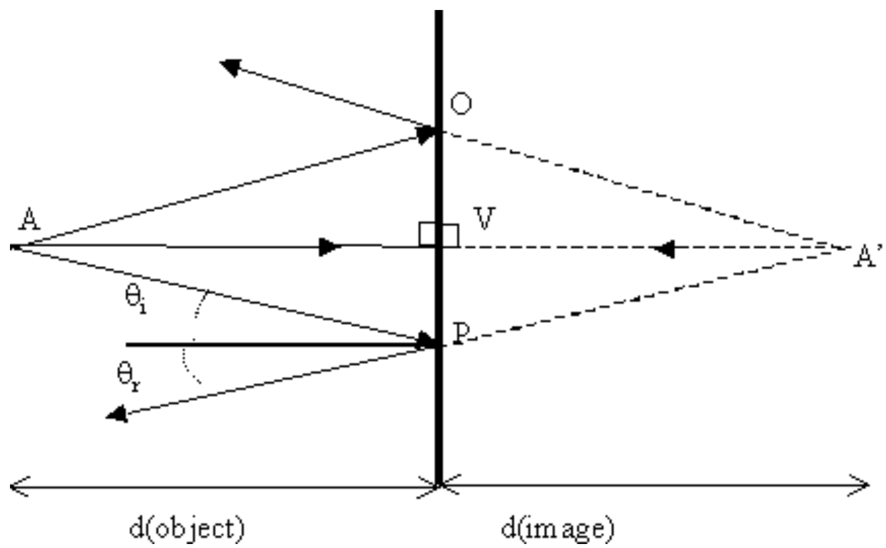
Simplifying this gives:

$$V^2=\frac{\frac{c^2}{(4v)}}{\left(x1+x2+\frac{c}{4v}\right)}$$

To make your scheme as sensitive as possible, you want to make x1+x2 as large as possible. Say 19th century technology limits you to x1+x2=1m. For a source of wavelength 632 nm, you would require $V \sim 1\times10^5$ m/s. Increasing x1+x2 to 100m reduces minimum V required to be observed to $\sim 1\times10^4$ m/s. *[James]*

3) Imagining imaging

i)



From the law of reflection,

$$\theta_i = \theta_r$$

Also $\theta_i + \theta_r$ is the exterior angle of the triangle AA'P, and is therefore equal to the sum of the alternate interior angles ∠VAP and ∠VA'P.

But ∠VAP = θ_i , and therefore ∠VAP = ∠VA'P. This makes the triangles VPA and VA'P congruent, in which case d(object) = d(image).

As for the magnification, this can be determined using the angle between two rays of light leaving a point on the object, and the corresponding angle the rays seem to form when traced back to the virtual image. We have already shown that ∠VAP = ∠VA'P, which is true of all object/image points. Thus the apparent size of an image divided by the size of the original object, or the magnification is +1. The virtual image is life-size and erect.

ii) To demonstrate that one focus is the virtual image of the other we will prove that the angles the incident ray and reflected ray make with the tangent are equal. This is most easily done with the following geometry in mind.

On the diagram given in the question label the following:

the focus F → object point with coordinates (–c, o)

the focus F′ → image point with coordinates (c, o)

the incident ray (leaving F) → line ℓ_2

the reflected ray (`leaving' F′) → line ℓ_1

the point of reflection on the surface of the F′ branch of the hyperbola → point A with coordinates (x_o, y_o)

Also draw in the tangent line at point A → label it line ℓ

Then label

the angle between ℓ_2 and ℓ → angle α

the angle between ℓ and ℓ_1 → angle β

where both α and β are counter clockwise angles.

So we want to prove α = β. Suppose the curve is in `standard position' so that the equation is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\therefore \ \frac{2x}{a^2} - \frac{2yy'}{b^2} = 0 \rightarrow y' = \frac{b^2}{a^2}\frac{x}{y}$$

Substituting (x_o, y_o) yields the slope of the tangent line,

$m = \frac{b^2}{a^2}\frac{x_o}{y_o}$ → gives the equation of the tangent → $\frac{x_o x}{a^2} - \frac{y_o y}{b^2} = 1$

Now we can use the formula for the tangent of the counterclockwise angle from one line ℓ_1 to another ℓ in terms of their respective slopes m_1 and m, namely

$$\tan \alpha = \frac{m - m_1}{1 + mm_1}$$

where the slope of the line ℓ_1 is $m_1 = \dfrac{y_o - 0}{x_o - c}$

plugging in the values for the slope you get something not as bad as it seems,

$$\tan\alpha = \frac{\dfrac{b^2 x_o}{a^2 y_o} - \dfrac{y_o}{x_o - c}}{1 + \left(\dfrac{b^2 x_o}{a^2 y_o}\right)\left(\dfrac{y_o}{x_o - c}\right)} = \frac{b^2 x_o^2 - b^2 c x_o - a^2 y_o^2}{a^2 y_o (x_o - c) + b^2 x_o y_o}$$

$$= \frac{-b^2 x_o c + (b^2 x_o^2 - a^2 y_o^2)}{(a^2 + b^2) x_o y_o - a^2 c y_o}$$

$$= \frac{-b^2 x_o c + b^2 a^2}{c^2 x_o y_o - a^2 c y_o} = \frac{b^2(-x_o c + a^2)}{c y_o (c x_o - a^2)} = \frac{-b^2}{c y_o}$$

where we have used the fact that $a^2 + b^2 = c^2$ and $b^2 x_o^2 - a^2 y_o^2 = a^2 b^2$ for a hyperbola.

The same calculation with –c replacing c gives

$$\tan(-\beta) = \frac{b^2}{c y_o} \qquad \text{so } \tan(\beta) = \frac{-b^2}{c y_o}$$

so $\tan(\alpha) = \tan(\beta)$, and therefore $\alpha = \beta$.

In other words reflected rays appear to come from F′ or the focus F′ is the virtual image of F. The same argument can be made for the reverse situation (F′ the object point and F the virtual image). Thus each focus is the virtual image of the other.

The magnification is *not* well defined. This is clear when you compare the angle between two rays of light leaving a point on the object with the corresponding angle the rays seem to form when traced back to the virtual image.

If for example you consider the rays drawn in the diagram (given in the question) and the corresponding angles they make with the x-axis (our second ray reflected back along the x-axis) the magnification, M

$$M = \frac{\tan(F')}{\tan(F)} \to \infty$$

Since the angle $F' \to 90^\circ$

If another two rays are taken an entirely different value for M can be obtained. Thus the magnification depends on which pair of rays you are tracing out and thus cannot be well defined. A proper image isn't really formed. *[Carrie]*

4) Correct time and temperature, at the tone…

i) Let the initial and final lengths, due to a temperature change of $\Delta T K$, be L and L' respectively. The respective periods are then:

$$T = \sqrt{\frac{g}{L}} \qquad \text{and} \qquad T' = \sqrt{\frac{g}{L'}}$$

Since $L' = L(1 + \alpha\Delta T)$, we have

$$T' = \sqrt{\frac{g}{L(1+\alpha\Delta T)}}$$
$$= (1+\alpha\Delta T)^{-\frac{1}{2}}T$$

Thus the required ratio is $\frac{T'}{T} = (1+\alpha\Delta T)^{-\frac{1}{2}}$

ii) Now for brass we have $\alpha = 1.9\times 10^{-5}K^{-1}$ and we're told $\Delta T = 20°C = 20K$

Since $T = 1.000$ the new period is

$$T' = (1+1.9\times 10^{-5}(10))^{-\frac{1}{2}}$$
$$= 0.9998$$

Since the new period is shorter the clock will measure time faster and so gain time. Since the now warmer clock measure is in just 0.9998 s. Therefore the clock gains time by a factor of $\frac{1}{T} = 1.0002$ per unit of real time.

Thus after one day (as measured by the clock initially cold) the warm clock gains 0.0002 days = 0.0002(24) (60) (60) s = 17.28 s.

Therefore the clock when warmer by 20 K gains approximately 17.28 s per day.

iii) In the case of a stretched string, it isn't exactly the change in lengths which change the frequency. The string is stretched to a length L, no matter what. Instead, as the string expands, it becomes less taut — the tension in the string isn't as great, since the string doesn't need to be stretched as much.

For transverse waves in a stretched string we use the equation $c = f\lambda$ where c is the speed of the wave, f the frequency and λ the wavelength. The first harmonic vibrates with the only nodes at the end points of the string. This distance L is then equal to one half the wavelength λ. Thus $f = \frac{c}{2L}$.

The wave speed c is given by $\sqrt{\tau/\mu}$ where τ is the tension in the string and μ the mass density per unit length. Thus $f = \frac{1}{2L}\sqrt{\frac{\tau}{\mu}}$.

We can determine τ from the formula for Young's modulus, namely $\frac{\tau/A}{\Delta l/l} = Y$, where A is the cross sectional area Δl is the change in length due to the applied tension, l is the initial length and Y is 'Young's modulus,' a constant.

So we have $f = \frac{1}{2L}\sqrt{\frac{Y\,A\,\Delta l}{l\,\mu}}$

Initially we have string-length l_{o}, and the string is stretched out by an amount

$$\Delta l = (L - l_o)$$

and $\mu = \rho A$

where ρ is the density of steel at the initial temperature. After the temperature increase ΔT we have

length l' prior to being stretched, and stretch amount $\Delta l'$

$$l' = l_o(1 + \alpha\,\Delta T)$$

$$\Delta l' = L - l'$$

$$= L - l_o(1 + \alpha\,\Delta T)$$

The linear mass density doesn't change, because we always stretch to length *L*. The cross-sectional area A *will* increase as the metal expands, but our Young's modulus, assumed to be constant, is defined in terms of the initial A. (We would have been better off to assume that Y•A was constant).

Therefore

$$f = \frac{1}{2L}\sqrt{\frac{YA\,\Delta l}{\mu\;\,l_o}} = \frac{1}{2L}\sqrt{\frac{YA}{\mu}\frac{\{L - l_o\}}{l_o}} = \frac{1}{2L}\sqrt{\frac{YA}{\mu}\left\{\frac{L}{l_o} - 1\right\}} \quad \textit{and}$$

$$f' = \frac{1}{2L}\sqrt{\frac{YA\,\Delta l\copyright}{\mu\;\,l\copyright}} = \frac{1}{2L}\sqrt{\frac{YA}{\mu}\frac{\{L - l\copyright\}}{l\copyright}} = \frac{1}{2L}\sqrt{\frac{YA}{\mu}\left\{\frac{L}{l\copyright} - 1\right\}} = \frac{1}{2L}\sqrt{\frac{YA}{\mu}\left\{\frac{L}{l_o(1 + \alpha\Delta T)} - 1\right\}}$$

It follows:

$$\frac{f'}{f} = \sqrt{\frac{L - l_o(1 + \alpha\,\Delta T)}{(L - l_o)\;(1 + \alpha\,\Delta T)}}$$

and this is the required ratio.

Now, using $\alpha = 1.1 \times 10^{-5}\,K^{-1}$, $\Delta T + 20K$, $f = 440\,HZ$, $l_o = 0.795\,m$ *and* $L = 0.8\,m$

we have:

$$\frac{f'}{440} = \sqrt{\frac{0.8 - 0.795(1 + 0.00022)}{(0.8 - 0.795)\cdot(1 + 0.00022)}} = 0.9822$$

$$f' = 432.2\ Hz,$$

Therefore the final frequency of the now-warmer guitar is 432.2 Hz, which is out of tune by about 8 Hz. *[Simal, James & Robin]*

5) Toys, Toys, Toys

The masses are equal, so after the collision the "lattice" will move at **V** m/s [right]; x(t) = Vt is the position as a function of time.

First, note that momentum will be conserved, so that at any point in time we will

have:

$$\begin{aligned} mV &= mL + mR \\ V &= L + R \end{aligned} \qquad [1]$$

where L and R are the velocities of the left and right balls, respectively (they clearly point in the x-direction, so we drop the vector sign).

The position of the centre of mass is given by:

$$x_{c.m.} = \frac{mx_L + mx_R}{2m} = \frac{1}{2}(x_L + x_R) \qquad [2]$$

where x_L and x_R are the positions of the left and right balls (in the laboratory frame), respectively.

Hence,

$$V_{c.m.} = \frac{dx_{c.m.}}{dt} = \frac{1}{2}(L + R) = \frac{V}{2} \quad \text{using [1].}$$

In the centre of mass frame then, which moves at V/2 m/s [right], the velocities of the balls become:

$$L' = V - \frac{V}{2} = \frac{V}{2}; \qquad R' = 0 - \frac{V}{2} = -\frac{V}{2}$$

at t = 0. (the balls are thus approaching each other)

The situation is perfectly symmetric and we can right away conclude that after the two balls collide, which will take a time of L/2/(V/2) = L/V their velocities will be:

$$L'' = -\frac{V}{2}; \qquad R'' = \frac{V}{2}$$

(the balls are going away from each other)

Next, the balls will be pulled by the string, and conserving momentum and energy yields that after the collision we will have:

$$L''' = L' = \frac{V}{2}; \; R''' = R' = -\frac{V}{2}$$

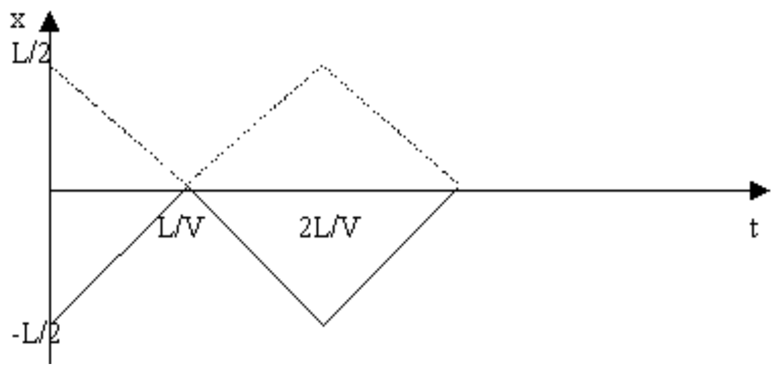
The balls therefore oscillate with a period of 2(L/V). Note that the motion is not simple harmonic, as acceleration $= 0 \neq -kx$.

The motion may be written as

$$L'(t) = \frac{V}{2}, \quad t < \frac{L}{V} \left(\text{mod}\frac{2L}{V}\right) \qquad x_L{}'(t) = \frac{V}{2}t, \quad t < \frac{L}{V} \left(\text{mod}\frac{2L}{V}\right)$$

$$L'(t) = -\frac{V}{2}, \quad t > \frac{L}{V} \left(\text{mod}\frac{2L}{V}\right) \qquad x_L{}'(t) = -\frac{V}{2}t, \quad t > \frac{L}{V} \left(\text{mod}\frac{2L}{V}\right)$$

the primes denote centre of mass frame; "mod" gives the remainder after division, so using it we can figure out what part of the period t is in. If you have never seen "mod" before, don't sweat it. I think it is (more or less) clear the graph looks something like this:



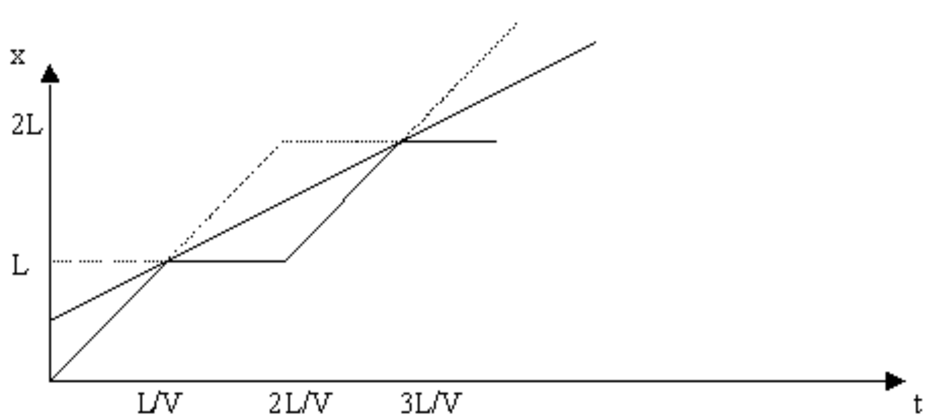
(the dashed line corresponds to the right ball, the solid corresponds to the left one)

Since we have the motion in the centre of mass frame, and we know its velocity, we simply transform to the laboratory frame. This gives:

$$L(t) = V, t\,\text{mod}\frac{2L}{V} < \frac{L}{V} \qquad x_L(t) = Vt, t\,\text{mod}\frac{2L}{V} < \frac{L}{V}$$

$$L(t) = 0, t\,\text{mod}\frac{2L}{V} > \frac{L}{V} \qquad x_L(t) = const, t\,\text{mod}\frac{2L}{V} > \frac{L}{V}$$

Plotting this gives:



The solid line is the motion of the left ball; the dashed that of the right; the thick line is the average of the two, which is also the motion of the centre of mass.

The system looks somewhat like a centipede – first the back moves, with the front being stationary, then the front moves, and so on.

Note that if we had a body of mass 2m colliding with one of mass m we wouldn't get the same results as here – our system loses energy (it's a soft object, like jello).

In an n-particle lattice, the speed of the centre of mass would be V/n.

It is instructive to try this by brute force – keeping the same symbols as in i) (x_L, L

for left object, x_R, R for right)

$$L + R = \frac{dx_L}{dt} + \frac{dx_R}{dt} = V$$

$$mV^2 = m\left(\frac{dx_L}{dt}\right)^2 + m\left(\frac{dx_R}{dt}\right)^2 + k(L - (x_R - x_L))^2$$

This is a system of differential (momentum and energy) equations that just screams out "go away" [NOTE: if you rearrange it – it takes <u>lots</u> of work – you end up with something workable, but not at the high school level]

Let's take the hint and go to the centre of mass frame. From ii), the centre of mass still moves at V/2 m/s [right]. In that frame we once again get the balls approaching at equal speeds (V/2). Since the compression on each half of the spring will be equal, we can treat the system as two springs of length L/2 (k' = 2k).

The motion is simple harmonic with a period:

$$T = 2\pi\sqrt{\frac{m}{2k}}$$

The motion of the left ball is

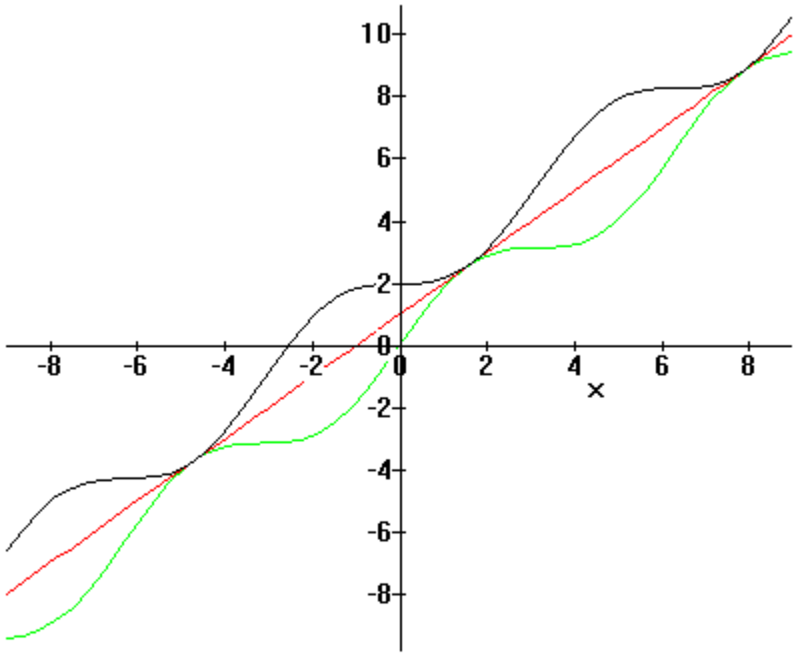
$$x_L{}'(t) = \frac{V}{2}\sqrt{\frac{m}{2k}}\sin\left(\sqrt{\frac{2k}{m}}t\right)$$

[this gives the correct initial conditions – $x_L'(0) = 0$, L(0) = V/2]

Thus, in the outside frame, the motion looks like this:

$$x_L(t) = \frac{V}{2}t + \frac{V}{2}\sqrt{\frac{m}{2k}}\sin\left(\sqrt{\frac{2k}{m}}t\right)$$

Plotting gives (not to scale!):

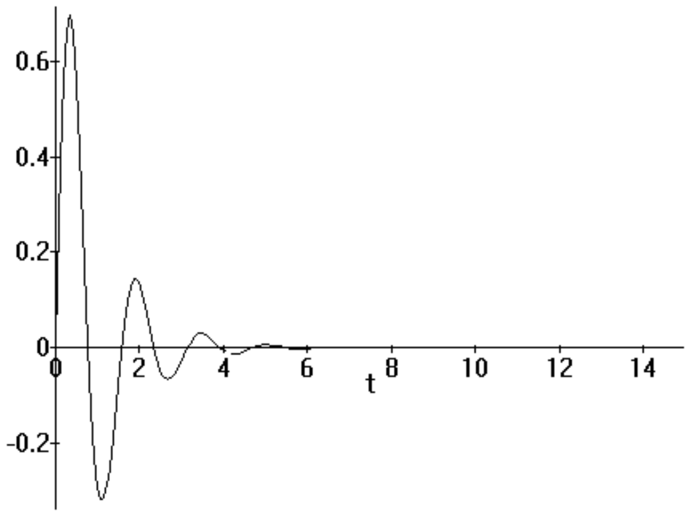


The black line is the right ball, green line is the left ball, straight line is the motion of centre of mass. The motion looks once again like a centipede, but kind of weird (non-uniform).

This is very similar to ii), except that the solution is now

$$x_L{}'(t) = \frac{2\frac{V}{2}e^{-\sqrt{\frac{\gamma t}{m}}}\sin\left(\frac{1}{2}\sqrt{4\frac{k}{m}-\left(\frac{\gamma}{m}\right)^2}\,t\right)}{\sqrt{4\frac{k}{m}-\left(\frac{\gamma}{m}\right)^2}}$$

Plotting gives something like this:

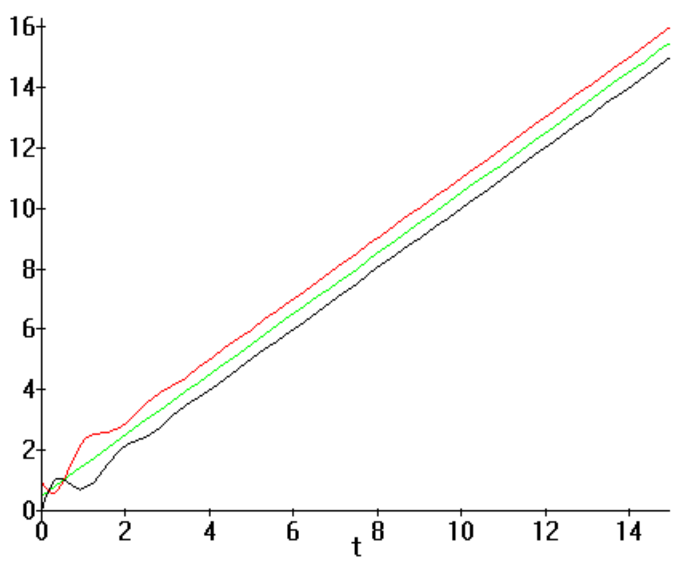


Note that the particle's motion eventually decays and remains at x = 0.

In the outside frame, then:

$$x_L(t) = \frac{V}{2}t + \frac{V e^{-\sqrt{\frac{\gamma t}{m}}} \sin\left(\frac{1}{2}\sqrt{4\frac{k}{m} - \left(\frac{\gamma}{m}\right)^2}\, t\right)}{\sqrt{4\frac{k}{m} - \left(\frac{\gamma}{m}\right)^2}}$$

which gives (not to scale!):



Here, the left ball is black, right ball red and centre of mass is green (straight line).

Clearly, the last model is most realistic. After a body collides with something, we might expect its molecules to vibrate for a little bit, but not forever as in ii) or iii). ii) is most unrealistic as it experiences infinitely long vibrations and the bond between molecules is most likely not a rigid string (this implies that the force acting on the molecules occurs over an infinitely short amount of time, leading to infinite accelerations, etc.).

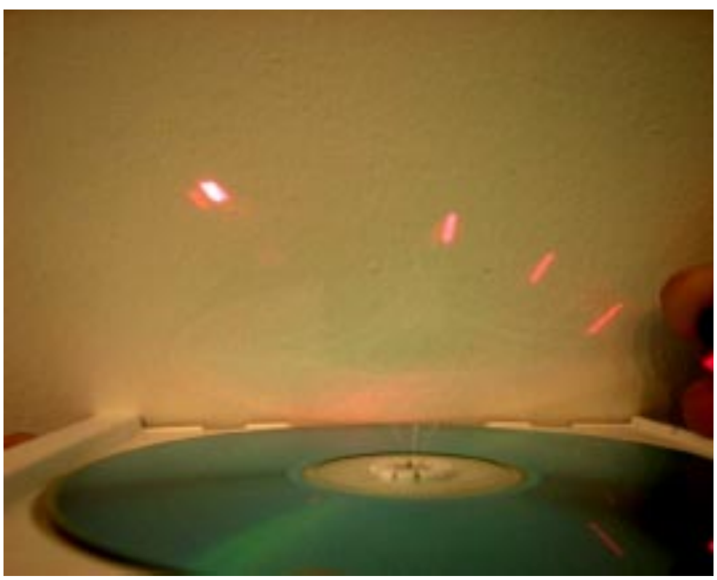
The dissipation mechanism is due to radiation – an accelerating charge radiates away its energy [see problem set 1]. *[Peter]*

6) Making a shoebox spectrograph, using a compact disc

As given in the 'mini tutorial' under InfoBits™, the input and output angles of light diffracted from a diffraction-grating depend on wavelength, and on the diffraction grating itself:

$$n\lambda = 2d \bullet (\sin\theta_i + \sin\theta_d)$$

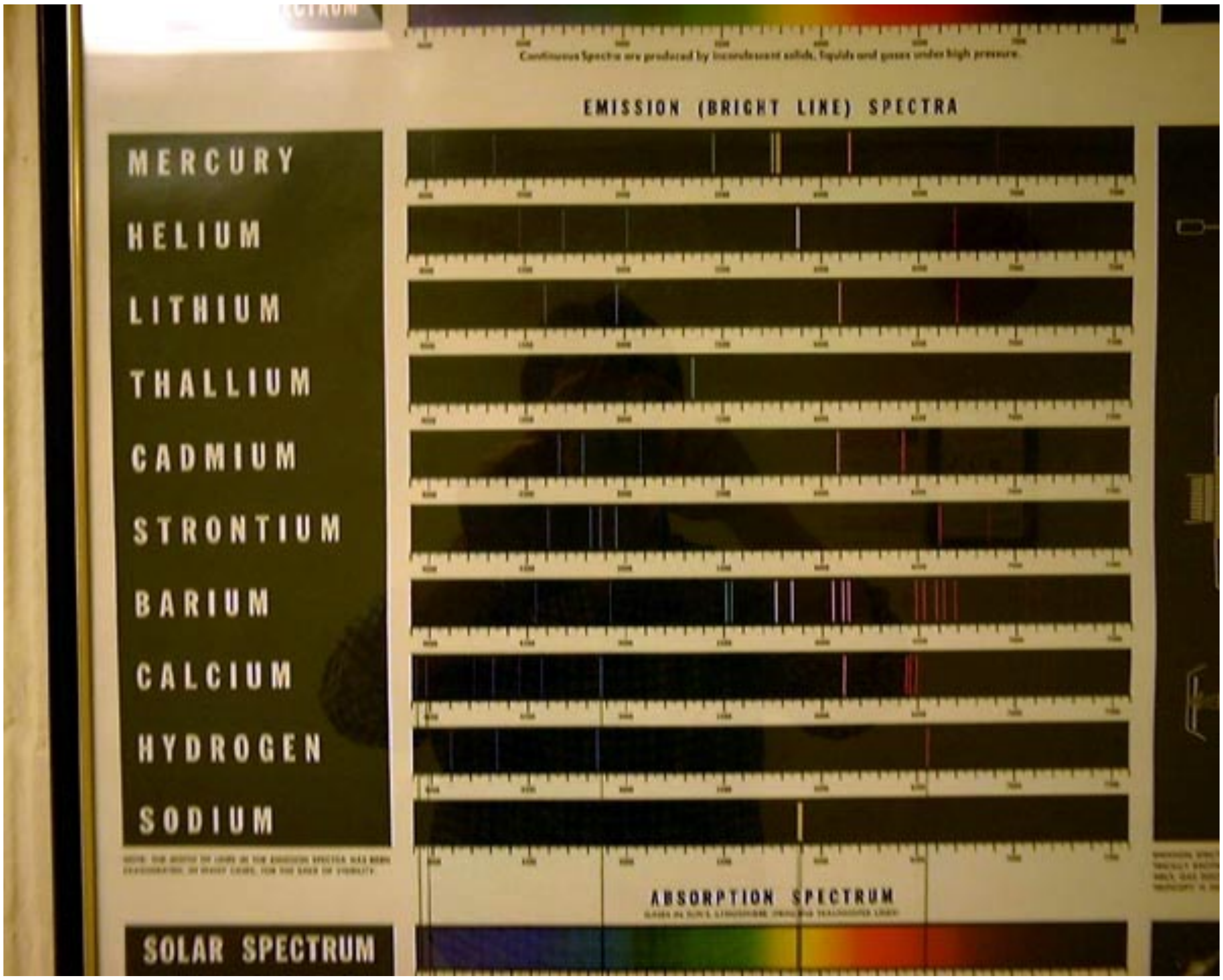
where θ_i is the input angle, measured from the perpendicular to the surface (the surface *normal*), and θ_d is the angle of the diffracted light, measured similarly. *2d* is the spacing between grooves of the diffraction-grating, or tracks of the CD (rather than *d*, as given — this is something of a convention). The *n* is an integer which gives the *order* of diffraction, which you can investigate.



Diffraction orders: This figure was made using a CD and a laser-pointer. You can see the laser-pointer, held at the right. The brightest spot of light is the undiffracted specular reflection (sometimes called the 'zeroth order,' and then to the right are three orders or diffracted light, labelled n = –1, –2, –3. Try the equation yourself to see why they're negative.

We used a HeNe laser from the lab, and a CD that was kicking around, and found that with normal incidence (θ_i =0), the 1st order diffracted angle was found to be 22 deg. Since the HeNe wavelength is 632.8 nm, the spacing between adjacent grooves on the CD works out to be approximately 1.7 μm.

If you look at fluorescent lights, you can easily see the different spectral lines, of different colours, that make the fluorescent light look white. The colours come from different phosphors coated on the inside of the glass tube — each spectral line comes from a quantum transition in the atoms making up the phosphor. The atoms are excited by absorbing ultraviolet light, which comes from mercury atoms which are themselves excited by electrons accelerated up and down the tube. So fluorescent lights are *doubly* fluorescent — first the mercury vapour inside, and then the phosphor coating. Your school may have a wall-chart that shows characteristic spectral lines of different materials (these are available through Edmund Scientific, among other places); a picture of part of one of ours is below.



If you look at street lights, you may find that there are different kinds. Incandescent lamps give a whole or continuous spectrum, but low-pressure and high-pressure sodium lamps are different: the low-pressure lamps give spectral lines, but in high-pressure lamps the much more frequent collisions between atoms make the spectral lines spread out or broaden. Astronomical observatories care very much which sort of lights a city installs, since too much scattered light (light pollution) can wreck observations. If low-pressure sodium lamps are used, the scientists can use the frequencies between spectral lines to peer through to the deep sky. *[Robin]*