1999-2000 Physics Olympiad Preparation Program

— University of Toronto —

Solution Set 1: General

1) Pumpkin paradox

a) $M_{big} = 10000*M_{small}$

Therefore, for the same relative strength we need a cross sectional area 10000 times larger than the poles in the small model. So,

$A_{big} = 10000\ A_{small}$

$(1/4)\ \pi\ (D_{big})^2 = 10000\ (1/4)\ \pi\ (D_{small})^2$ or

$D_{big} = 100\ D_{small}$

So Farmer Joe is going to need poles that are 100 times thicker.

b) Mass is proportional to volume which in turn is proportional to area$^{(3/2)}$. Thus, mass is proportional to area$^{(3/2)}$. So,

$(M_{big})^{(2/3)}/(M_{small})^{(2/3)} = A_{big}/A_{small} = (1000\text{kg}/0.1\text{kg})^{(2/3)}$

Therefore, $A_{big} = 464\ A_{small}$

Since the big pumpkin surface area is 464 times greater than the small,

Joe is going to need 4.64 L of paint.

c) $M_{weight}/M_{pumpkin} = 450/1000$ thus,

$M_{weight} = 9/20$

The sum of the moments is zero when balanced, thus,

$R_{weight}\ M_{weight}\ g - R_{pumpkin}\ M_{pumpkin}\ g = 0$

Therefore, $R_{weight} = (20\ R_{pumpkin})/9$ *[Carrie]*

2) Cubic quandry

a) There are 6×4 = 24 equal cubes that can perfectly surround the lamp.so we can say that the fraction of power heating one cube is 1/24.

$$\frac{1}{24}Qt = mc\Delta\theta$$

$$? \quad t = \frac{24mc}{Q} \text{ [seconds]}$$

b) A unit area of a blackbody radiator in temperature T, emits energy by the power of σT^4 Watts. so our cube is emitting $6L^2 \sigma T^4$ J/s by the assumption that our cube is in thermal equilibrium each time. Consider our case that any small change in T, will affect the emission rate and the time needed to increase the temperature accordingly. But, in this case, we are looking at changing the temperature by 1K at room temperature, i.e., from 300K to 301K. The relative change is on the order of 1/300. and the change in T^4 is about $4 \times (1/300)$.

$$301^4 = (300 + 1)^4 = 300^4 (1 + \frac{1}{300})^4 \cong 300^4 (1 + 4 \cdot \frac{1}{300})$$

To a good approximation, we can ignore the changes in dT/dt due to a small change in T, and we can substitute the room temperature instead.

$$- 6L^2\sigma T^4 t + \frac{1}{24}Q t = mc \Delta\theta$$

$$\Rightarrow t = \{mc\} / \left\{\frac{Q}{24} - 6L^2\sigma T^4\right\}$$

c) In thermal equilibrium, the emission rate and heating rate are equal. so we have :

$$- 6L^2\sigma T^4 = \frac{1}{24}Q$$

$$\Rightarrow T = \left\{\frac{Q}{144\sigma L^2}\right\}^{1/4}$$

[Amir]

3) Poles apart, telling

First of all let us note that the North-South (N-S) designations on magnets are a little messed up. You see, the N pole is actually the North-*seeking* pole, meaning that it is actually the South pole (since opposites attract). We will follow these conventions in this document (it doesn't matter how you approach this, so long as you're clear what a North pole of a magnet means to you).

After this lengthy introduction, let us get right to the point. The magnetic field of the magnet will bend the electron orbits into circles. We assume here that the field is uniform throughout the face of the monitor (clearly not true, as the field gets smaller with distance; this solution is thus only valid for electron trajectories not far from the magnet).

i) Suppose we put the N-pole (really the South pole) of the magnet on the right side of the screen. The electrons will be bent according to $\vec{F} = -e\vec{v} \times \vec{B}$ (e is the size of the electron's charge). From this we see that the direction of deflection will be "DOWN" (looking directly at the monitor, $\vec{v}$ is out of the screen, $\vec{B}$ is from left to right).

ii) From the voltage we can find the speed of each electron $Energy = eV = \frac{1}{2}mv^2$, where V = 15 kV. From this, v = 7.3×10^7 m/s (Note that this is actually relativistic… we will ignore this, as the correction is small. Bonus marks will be given to people who realize this).

We also have $\frac{mv^2}{R} = evB$, where R is the radius of rotation of the electron.

We find R = 0.41 cm. Now, each electron will actually only be deflected when it's in the field of the magnet. In this case, this would be roughly the size of the magnet, which we cleverly enough didn't tell you. (we assume that the magnetic field drops off really quickly at the sides of the magnet). Seeing that the turning radius is only 4.1 mm, the magnet really can't be bigger, or there would be no picture whatsoever (all electrons would turn back). Let's assume that the width of the magnet is indeed 4.1 mm. Then, electrons will turn by 90 degrees, and the vertical distance traveled is clearly **4.1 mm**.

DISCUSSION: This somewhat makes sense, but I would expect something bigger. This is mostly due to the fact that the magnetic field actually decays slowly with distance (and doesn't become abruptly zero), whereby the force acts over a larger distance and deflects the electrons more.

In the general case, we can resolve the motion into 2 directions: x (towards the screen), y (down the screen). We have:

$$x(t) = -R\sin(\varpi t)$$
$$y(t) = R\cos(\varpi t) \qquad \text{where } v = \varpi R.$$

We want to find a time τ when the electron reaches the screen (solid line) so $-x(\tau)$ = width of magnet, and calculate what the deflection $y(\tau)$ will be.

[Peter]

4) Physics Rocks!

a) To determine the critical height one must consider the equation of motion of the stone in the water. The forces acting on the stone in water are,

(i) the downward force of gravity = –m*g

(ii) the upward drag on the stone = $0.5\ b\ p\ A\ v^2$

(b = drag coefficient, p = density of water)

and from Archimedes' principle,

(iii) the upward buoyant force = p V g

(V = volume of fluid displaced)

Summing the forces we get,

$$p\ V\ g + 0.5\ b\ p\ A\ v^2 - m^*g = 0$$

The sum of the forces is zero at equilibrium, when the velocity is the terminal velocity. Given that the diameter of the stone is 5 cm and that the density of quartz is 2600 kg/m^3 it is easy to find the following,

$$A = 1.96 \times 10^{-3}\ m^2$$

$$V = 6.54 \times 10^{-5}\ m^3$$

$$m = 0.17\ kg$$

Plugging in the values yields a terminal velocity of 1.6 m/s (of the stone in water). Thus if the stone is dropped in air with V_o = 0 m/s then under the acceleration of gravity the distance it travels before reaching a velocity of 1.6 m/s is,

$$D = Vt^2/(2\ g) = 0.13\ m$$

So the stone should be dropped from a height of 0.13 m above the surface of the water.

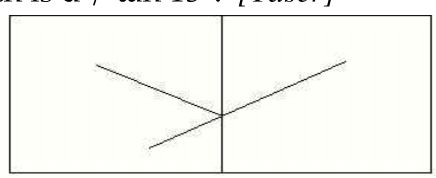
b) At terminal velocity the water-drag problem is trivial! After 6 seconds the rock has fallen, 6*(1.6 m/s) = 9.6 m. *[Carrie]*

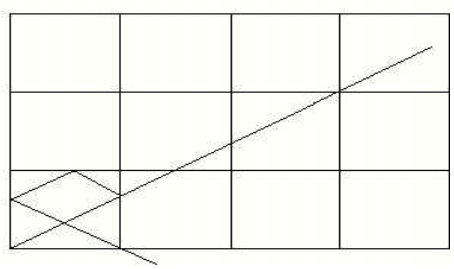
5) Light diversions

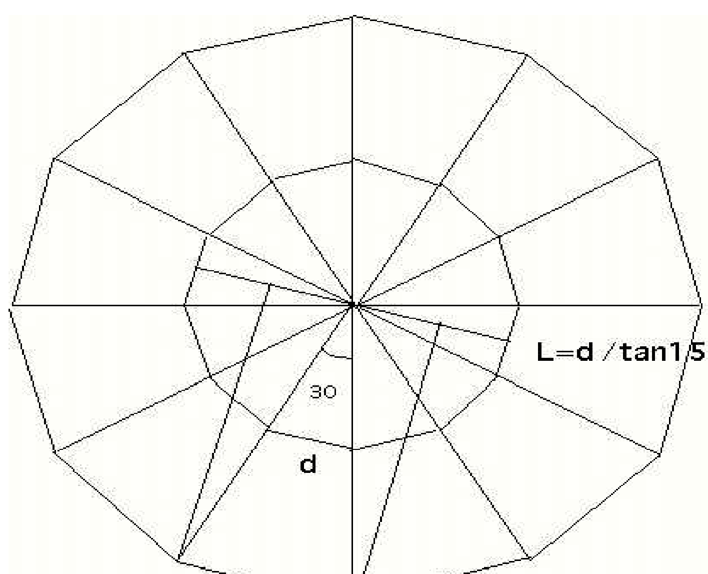
Consider a ray of light reflected from one of the mirrors. The reflected ray is a mirror image of the continuation of the incident ray. Therefore, an easy way to trace the ray path is to make another square beside it and follow the continuation of the incident ray in that square.

For more than one reflection , more squares should be drawn as shown. It is observed from the figure that as the continuation of the initial ray pass through any corners, it means that the light have gone out from the square. Therefore, the necessary condition would be tan(θ) = m /n which m and n are two arbitrary integers.

Doing the first part, makes solving this part much easier. We have to build up some triangles which are image of one another. Since the cone angle is 30°, last triangle fir exactly the first one. Two regular polygons with 12 sides is produced. Analogous to condition that the rays leave through the upper hole after many reflections is that the continuation of the incident ray pass through the smaller polygon. After some simple mathematics, we figure out that D must be smaller than the width of the polygon which is d / tan 15°. *[Yaser]*







6) Physicists' pipe dreams

a) The water column will get narrower and narrower because the flow is constant and the speed is increasing, therefore the surface area of the water will decrease until it breaks up into droplets.

b) If we consider low viscosity for water, molecules of water are freely falling down.

$$v^2 - v_0^{\ 2} = 2gh$$

$$v\pi r^2 = v_0 \pi R^2$$

$$r = R\sqrt{\frac{\nu}{\nu}}$$

$$r = R\ \{ v_0^{\ 2}/(v_0^{\ 2} + 2gh)\}^{\ 1/4}$$

c) The stream will break up into droplets. It cannot get arbitrarily skinny, the way the formula gives, because:

1 . The radius can't be less than the radius of a water molecule!

2 . Before you reach the limit (1) above, *surface tension* of water won't let you make it as skinny as possible. In fact, surface tension could affect the result which we obtained , but we are neglecting that!

Consider the droplets have radius *a* . What is the height of a same-volume cylinder of radius *r*? The energy of surface tension for these two area (as you could see in many books & specially at the end off this problem set :)

$$\frac{4}{3}\pi a^3 = \pi r^2 \Delta x$$

$$? \quad \Delta x = \frac{4}{3} a^3/r^2$$

The surface-tension energy of these two states (water tube & droplet) are :

$$\sigma\, 2\pi r\, \Delta x \quad and \quad \sigma\, 4\pi a^2$$

substituting delta-x, we have and comparing them:

$$\sigma\, 8\pi\frac{a^3}{3r} \quad \text{and} \quad \sigma\, 8\pi\frac{a^3}{3r}\langle\, \frac{3r}{2a}$$

it means that, if 3r/2a is less than 1 , then the surface energy of droplets is less than that of a water cylinder, so it's energetically favourable for the water to break into droplets instead of staying in a cylindrical shape. What does determine the actual radius of a droplet? Several factors may affect that — for example the circular motion of water flow. But the most important one is density of water. It will break into droplets when the weight of the droplet has the same order of magnitude as the surface tension. It means:

$$\rho = 4\pi a^3 / 3\, g = 2\pi r\sigma$$

using previous result, we will get:

$$r = \sqrt{\frac{4\sigma}{4\rho g}} = 2 \times 10^{-3}\ \text{m} = 2\ \text{mm}$$

and then the height at which we reach this redious is:

$$h = v_0^2 / 2gr^4 \quad (R^4 - r^4)$$

d) For the experimental part, I just used a tap with diameter of 6 mm and tried to see at what height the water breaks up. After that, I fixed the height and measured the volume of the poured water and the time interval.

Here are my results:

Height of break up (cm)	**Volume (ml)**	**Time needed (s)**	$\mathbf{v_0 = V/(\pi R_0^2 T)}$
5-6 cm	100 ml	78	4.5 cm/s
7-10 cm	100 ml	62	5.7 cm/s
16-20 cm	200 ml	50	14.7 cm/s
26-30 cm	300 ml	61	17.4 cm/s

Due to large error I have, the experimental behaviour is not exactly the same as our expectation, but \`h' is still proportional to v_0^2. *[Amir]*