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# Coherent control in the classical limit: Symmetry breaking in an optical lattice

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The quantum-to-classical transition of a symmetry-breaking coherent control scenario is computationally demonstrated in an optical lattice arrangement. Control is shown to survive in the classical limit and, for small effective $\hbar$, to be comparable in magnitude to quantum control. Moderate decoherence is seen to eliminate structure from the momentum space distribution, but not to cause loss of control. The proposed scenario is designed so as to be demonstrable experimentally in a moving or shaken one-dimensional optical lattice.

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Group meeting

Feb. 3rd, 2010

by Chao Zhuang

# Motivation

Quantum mechanics

↓ ?

Classical mechanics

$$\hbar_e \longrightarrow 0$$

What happens to
Coherent control ?

What if there is
Decoherence ?

⋮

When does the classicality happen?
Criterion for classicality

Ehrenfest regime -- size of quantum states

Liouville regime -- probability should equal

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# Quantum mechanics of Hyperion

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(Received 14 March 2005; published 19 August 2005)

This paper is motivated by the suggestion [W. Zurek, Phys. Scri. T, **76**, 186 (1998)] that the chaotic tumbling of the satellite Hyperion would become nonclassical within 20 years, but for the effects of environmental decoherence. The dynamics of quantum and classical probability distributions are compared for a satellite rotating perpendicular to its orbital plane, driven by the gravitational gradient. The model is studied with and without environmental decoherence. Without decoherence, the maximum quantum-classical (QC) differences in its average angular momentum scale as $\hbar^{2/3}$ for chaotic states, and as $\hbar^2$ for nonchaotic states, leading to negligible QC differences for a macroscopic object like Hyperion. The quantum probability distributions do not approach their classical limit smoothly, having an extremely fine oscillatory structure superimposed on the smooth classical background. For a macroscopic object, this oscillatory structure is too fine to be resolved by any realistic measurement. Either a small amount of smoothing (due to the finite resolution of the apparatus) or a very small amount of environmental decoherence is sufficient to ensure the classical limit. Under decoherence, the QC differences in the probability distributions scale as $(\hbar^2/D)^{1/6}$, where $D$ is the momentum diffusion parameter. We conclude that decoherence is not essential to explain the classical behavior of macroscopic bodies.

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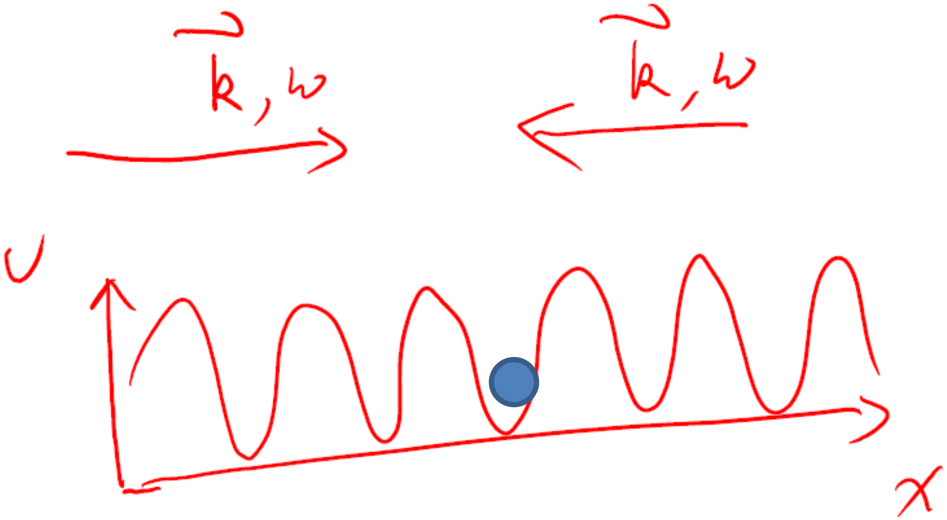
# Outline

- Basics about optical lattice
- Rescale Hamiltonian, effective Plank constant and etc.
- Control knobs: two phases
  - Absolute phase
  - Relative phase
- Pulse parameters
- Initial states: three kinds of probability distribution
- Simulation results and conclusions
  - Dependence on effective hbar
  - Dependence on decoherence

# Basics about optical lattice




$$H = \frac{P^2}{2m} + U_j \qquad \cos[2kx \qquad ]$$

fringe

optical potential for atoms

AC stark effect

# OPTICAL LATTICES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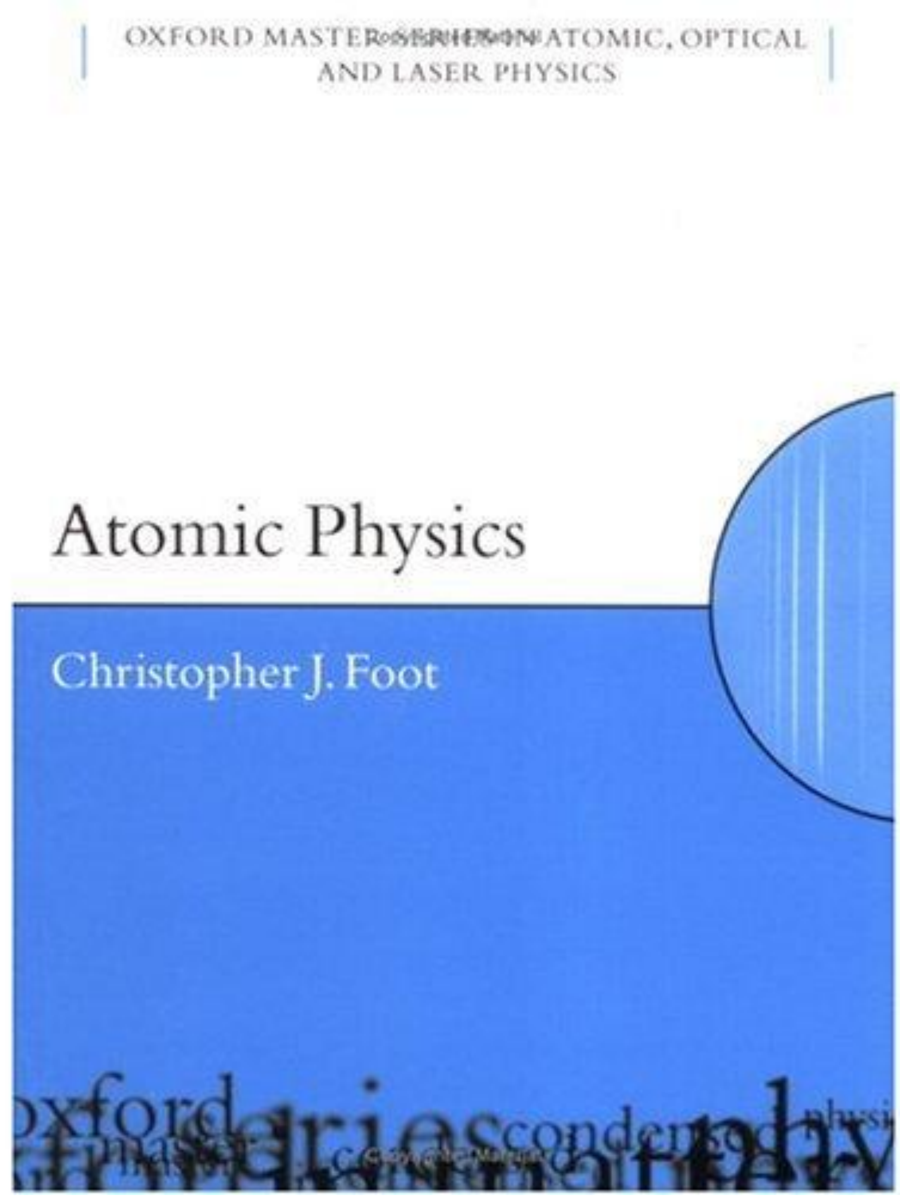
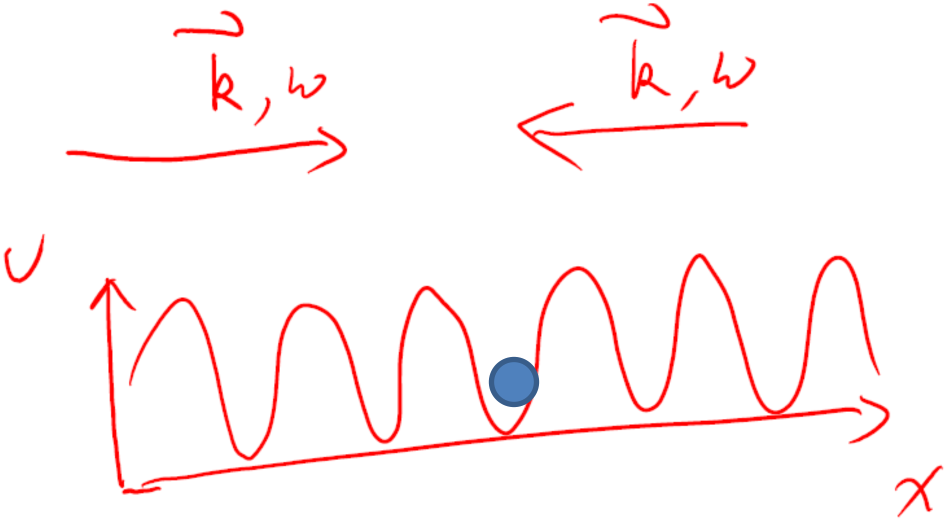
To appear in *Advances in Atomic, Molecular and Optical Physics 37*, eds. B. Bederson and H. Walther. (Academic Press, Cambridge, 1996).

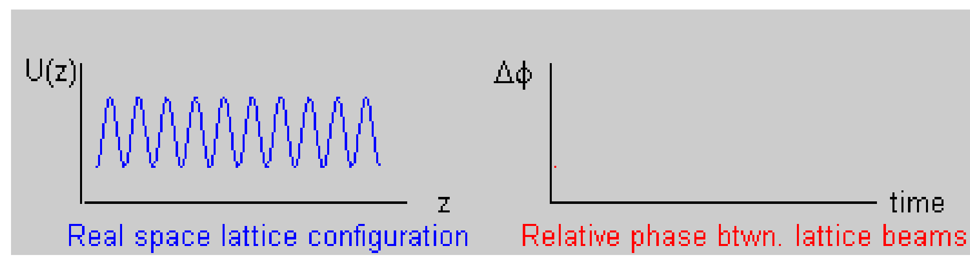





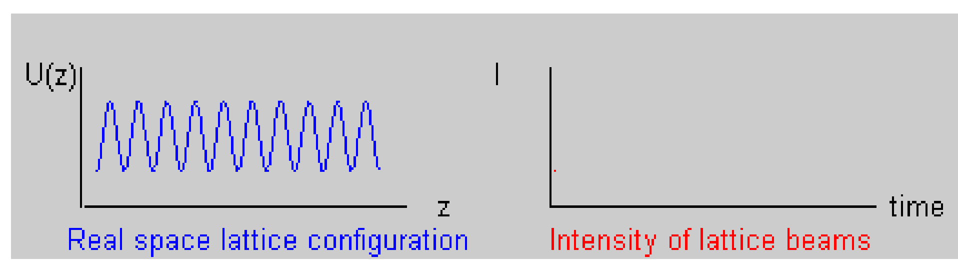

$$H = \frac{P^2}{2m} + Uf_1(t)\cos[2kx - \beta f_2(t)]$$

fringe
optical potential for atoms
AC stark effect

$$f_2(t)$$




$$f_1(t)$$



# Rescale Hamiltonian, effective Plank constant and etc.

$$H = \frac{P^2}{2m} + U f_1(t) \cos[2kx - \beta f_2(t)]$$

the problem in reduced units. For instance, by rescaling the coordinates as

Lattice constant = π/k

$$\underline{\theta = 2kx,} \tag{3a}$$

Effective mass = 1

$$P_\theta = P(2k/\omega m), \tag{3b}$$

$$\underline{\tau = \omega t,} \tag{3c}$$

and defining

$$\mathcal{U} = (2k/\omega)^2 (U/m), \tag{4}$$

the Hamiltonian becomes

$$\mathcal{H} = \frac{P_\theta^2}{2} + \mathcal{U} f_1(\tau) \cos[\theta - \beta f_2(\tau)], \tag{5}$$

# Quantum Mechanics

in the reduced units the Schrödinger equation becomes

$$i\hbar_e \frac{\partial \Psi(\theta,\tau)}{\partial \tau} = \left\{ -\frac{\hbar_e^2}{2}\frac{\partial^2}{\partial \theta^2} + \mathcal{U} f_1(\tau)\cos[\theta - \beta f_2(\tau)] \right\} \Psi(\theta,\tau), \quad (7)$$

where

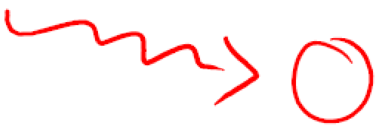
$$\hbar_e = \hbar (2k)^2/(\omega m) \quad (8)$$

Classical limit $\hbar_e \to 0$

$$\hbar_e = \hbar(2k)^2/(\omega m)$$

Recoil frequency

$$\hbar_e = 8\frac{\omega_r}{\omega}$$



$$p = \hbar k$$

$$E_k = \frac{\hbar^2 k^2}{2m}$$

$$= \hbar \omega_r$$


For $\hbar_e \to 0$, shaking of lattice should be much faster than recoil frequency

# Classical Mechanics

The classical equations of motion are then

$$\dot{\theta} = \frac{\partial \mathcal{H}}{\partial P_\theta} = P_\theta, \quad (6a)$$

$$\dot{P}_\theta = -\frac{\partial \mathcal{H}}{\partial \theta} = \mathcal{U} f_1(\tau) \sin[\theta - \beta f_2(\tau)]. \quad (6b)$$

To control: Momentum of atoms, or, atoms go right or left

# Control knobs: two phases

# Absolute phase: between $f_1$ and $f_2$

$$\mathcal{H} = \frac{P_\theta^2}{2} + \mathcal{U} f_1(\tau)\cos[\theta - \beta f_2(\tau + \phi_{\mathrm{abs}})]$$

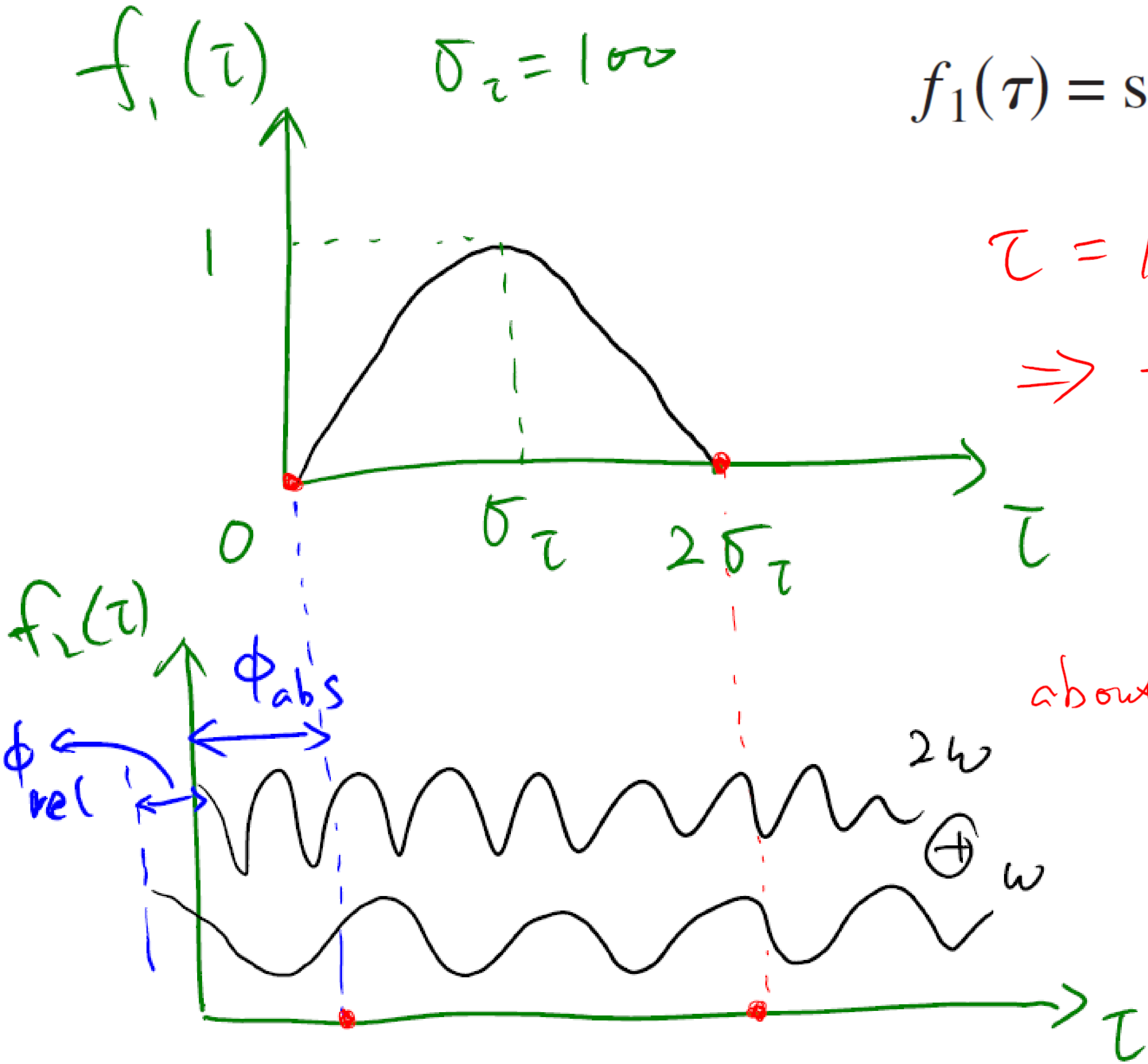
# Relative phase: between ω and 2ω

$$f_2(t) = \cos(\omega t + \phi_{\mathrm{rel}}) + s\cos(2\omega t)$$

$$f_1(\tau) = \sin^2\left(\frac{\pi\tau}{2\sigma_\tau}\right)$$

$\tau = 100 \quad (\tau = \omega t)$

$\Rightarrow t = \frac{100}{\omega}$




about 30 cycle of ω

60 cycle of 2ω

$$f_2(t) = \cos(\omega t + \phi_{\text{rel}}) + s\cos(2\omega t)$$

# Pulse parameters

$$\mathcal{U} = \left(\frac{2k}{\omega}\right)^2 \left(\frac{U}{m}\right), \quad \hbar_e = \frac{\hbar (2k)^2}{\omega m} \Rightarrow \mathcal{U} = \frac{\hbar_e U}{\hbar \omega}$$

$$\hbar_e = 8\frac{\omega_r}{\omega} \Rightarrow \mathcal{U} = \frac{\hbar_e^2 U}{8\hbar\omega_r}$$

$$\Rightarrow U = \frac{8\mathcal{U}}{\hbar_e^2}\hbar\omega_r = \frac{8\mathcal{U}}{\hbar_e^2}E_r$$

$$\mathcal{U} = 0.1 \Rightarrow U = \frac{0.8}{\hbar_e^2}E_r$$

**For $\hbar_e \to 0$, perturb with very deep lattice**

$$\mathcal{H} = \frac{P_\theta^2}{2} + \mathcal{U} f_1(\tau)\cos[\theta - \beta f_2(\tau + \phi_{\text{abs}})], \qquad (15)$$

where $\phi_{\text{abs}}$ determines the temporal shift between the envelope $f_1(\tau)$ and the underlying oscillations $f_2(\tau)$. Computations below assume $\mathcal{U}=0.1$ and $\beta=1$.

$$\mathcal{U} = (2k/\omega)^2(U/m)$$
$$\hbar_e = \hbar(2k)^2/(\omega m)$$

ω and 2ω have the same amplitude

$$f_2(t) = \cos(\omega t + \phi_{\mathrm{rel}}) + s\cos(2\omega t), \qquad (2)$$

where $s$ controls the ratio of field amplitudes ($s=1$ in all calculations below) and $\phi_{\mathrm{rel}}$ is the relative phase

and the amplitude is 1 for each,

or 1/(2π) of the lattice constant

$$\mathcal{H} = \frac{P_\theta^2}{2} + \mathcal{U} f_1(\tau)\cos[\theta - \beta f_2(\tau + \phi_{\mathrm{abs}})], \qquad (15)$$

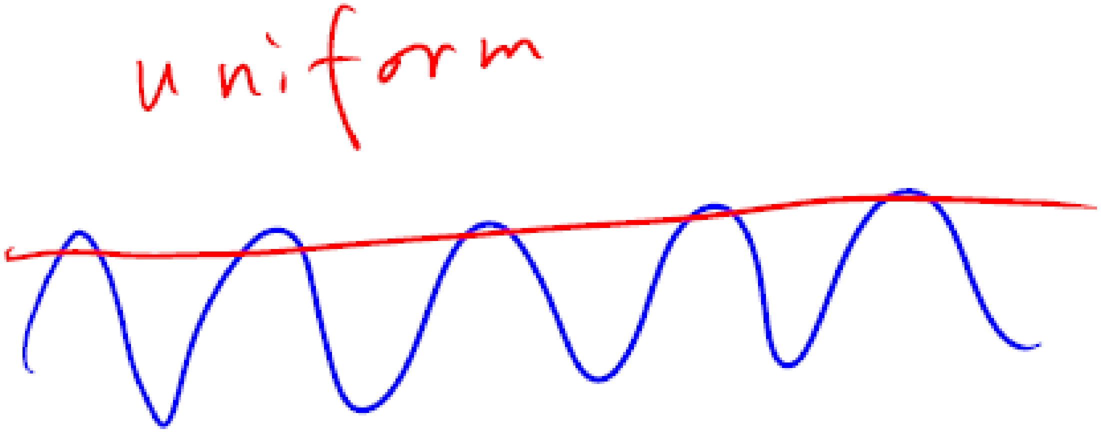
where $\phi_{\mathrm{abs}}$ determines the temporal shift between the envelope $f_1(\tau)$ and the underlying oscillations $f_2(\tau)$. Computations below assume $\mathcal{U}=0.1$ and $\beta=1$.

# Initial states: three kinds of probability distribution classical and quantum are same

# Classical ←→ Quantum

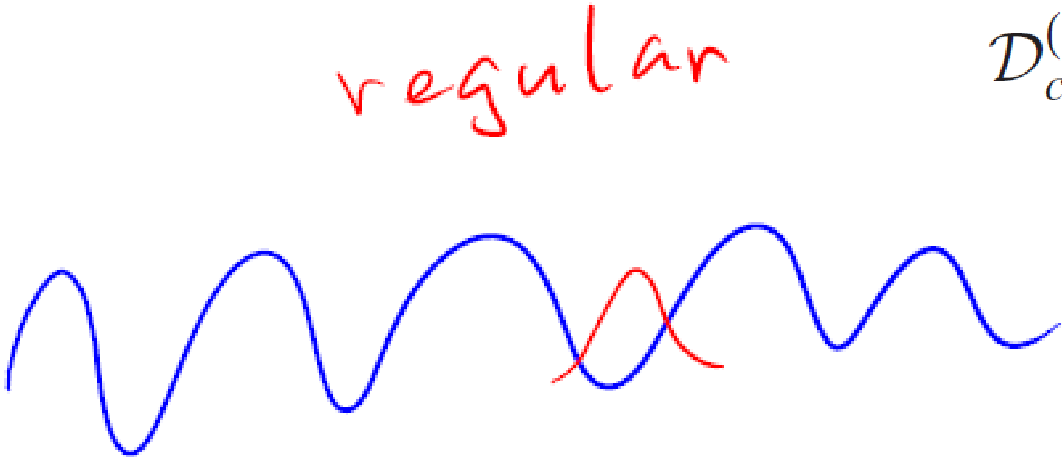
$$\Psi_0(\theta) = \sqrt{\mathcal{D}_c(\theta)}, \qquad (16)$$

where $\Psi_0(\theta)$ is the initial quantum wave function, $\mathcal{D}_c(\theta)$ is the distribution of initial classical trajectory positions, which all have zero initial momentum $P_\theta=0$. With this definition $|\Psi_0(\theta)|^2=\mathcal{D}_c(\theta)$, i.e., the initial quantum and classical spatial probability distributions are the same. The initial quantum states are chosen to be real and hence have zero initial momentum in the semiclassical sense $|\Psi_0(\theta)|e^{ip(x)x}$, where $p(x)=0$.



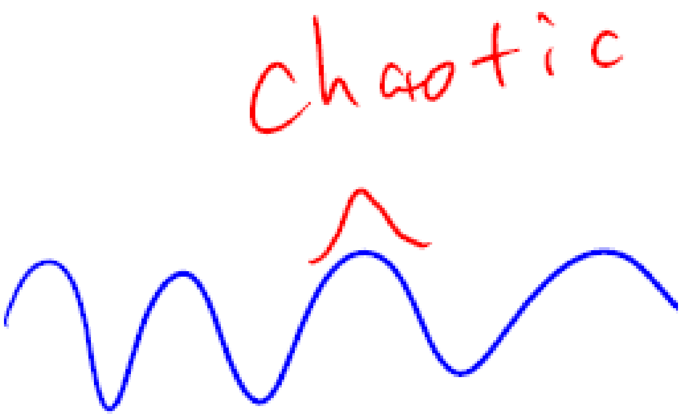

## Uniform

$$\mathcal{D}_c^{(u)}(\theta) = 1/(2\pi)$$




## Regular

$$\mathcal{D}_c^{(r)}(\theta) = \eta^{(r)} \exp[-7(\theta - \pi)^2]$$




## Chaotic

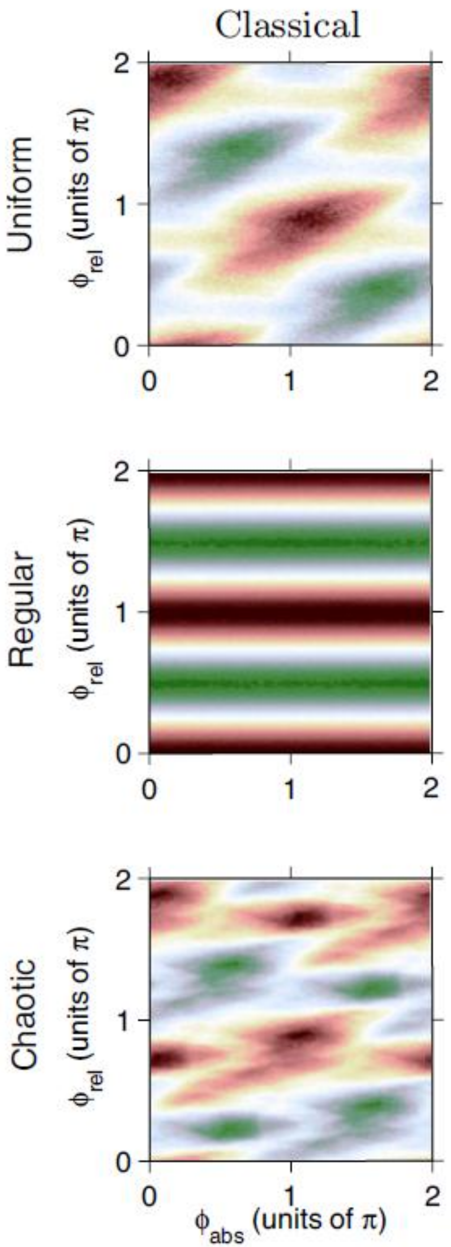
$$\mathcal{D}_c^{(c)}(\theta) = \eta^{(c)} \exp[-7\theta^2]$$

Specifically, the classical system displays chaotic dynamics for some regions of the initial phase space. These chaotic regions lie on the crests of the optical lattice and correspond to bifurcation instabilities (i.e., small perturbations cause rapid oscillations between falling off the crest to the left or right).

# Results

# Dependence on hbar

# General character




# The color stands for final average momentum

- Regular 1 vs. 2 survives in CM
- Uniform ← Chaotic
- $\phi_{abs}$ initial condition

$$f_2(t) = \cos(\omega t + \phi_{\rm rel}) + s\,\cos(2\omega t)$$

$$\mathcal{H} = \frac{P_\theta^2}{2} + \mathcal{U} f_1(\tau)\cos[\theta - \beta f_2(\tau + \phi_{\rm abs})]$$

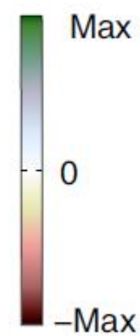
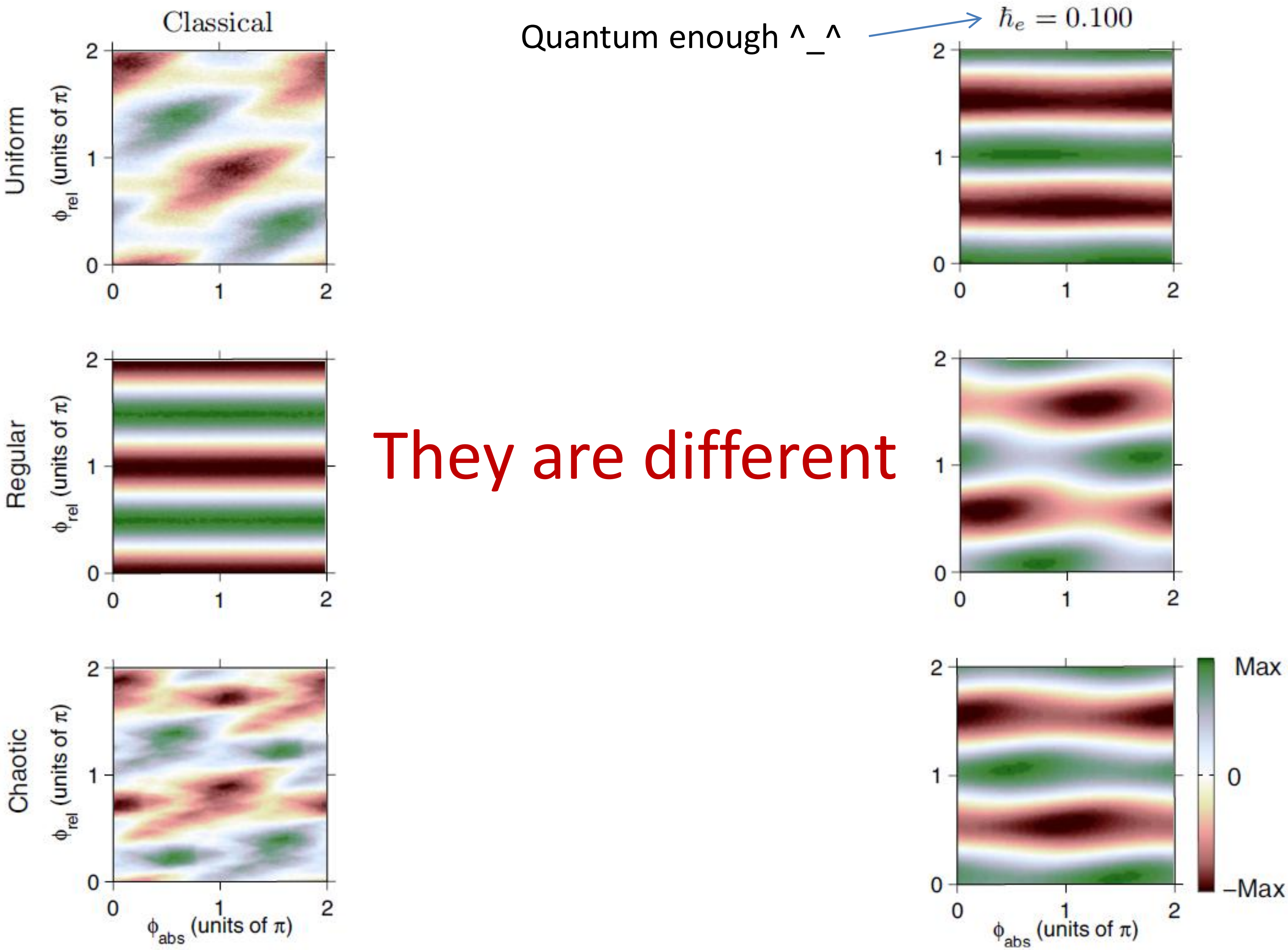



FIG. 1. (Color online) Control dynamics. Final average momentum for the classical and quantum systems. The top row is for the uniform initial state, while the middle and bottom rows correspond to the regular and chaotic initial state. The first column plots the classical results, while the remaining columns plot the quantum results for three values of $\hbar_e$=0.001, 0.01, and 0.1.

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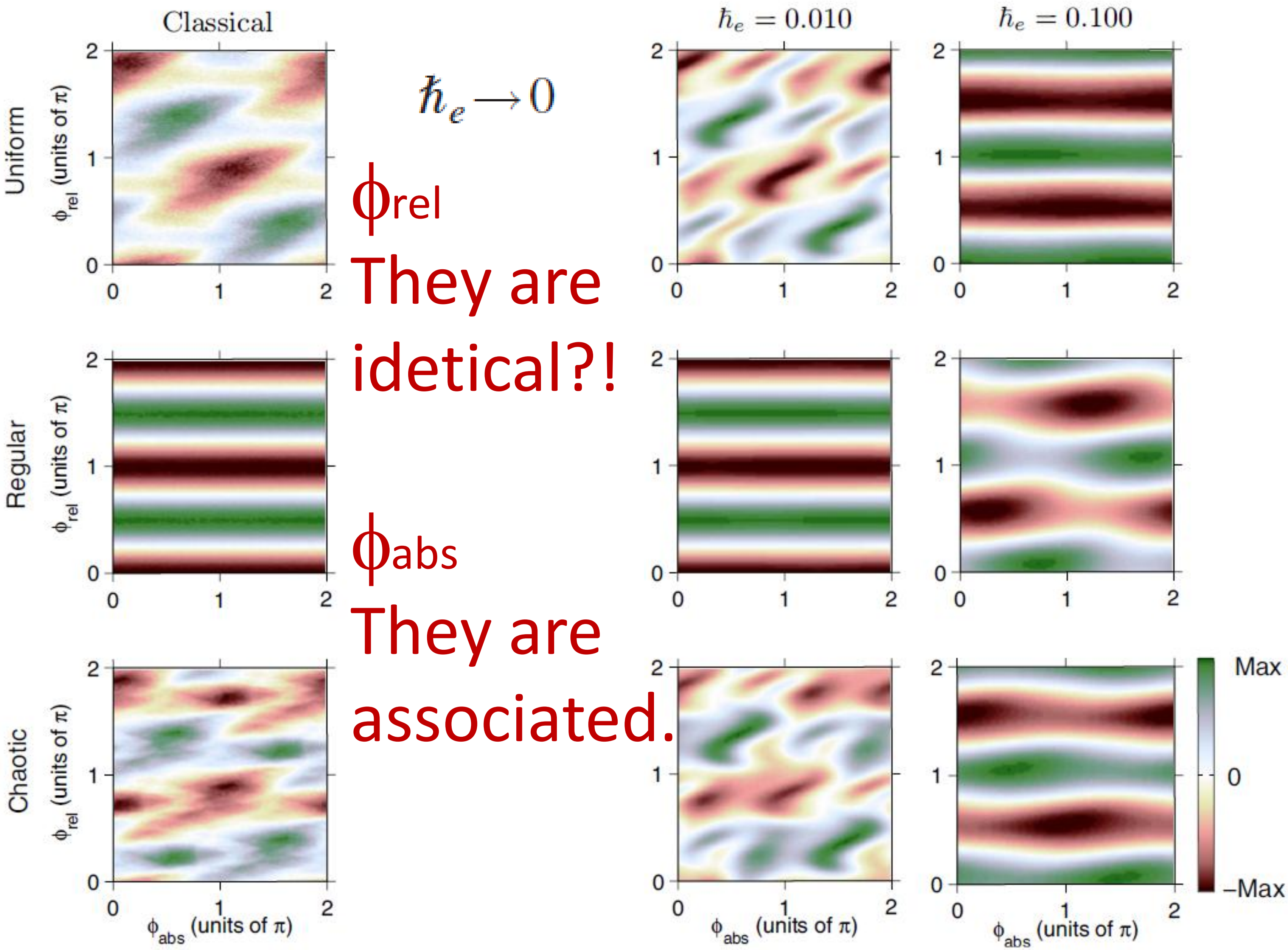


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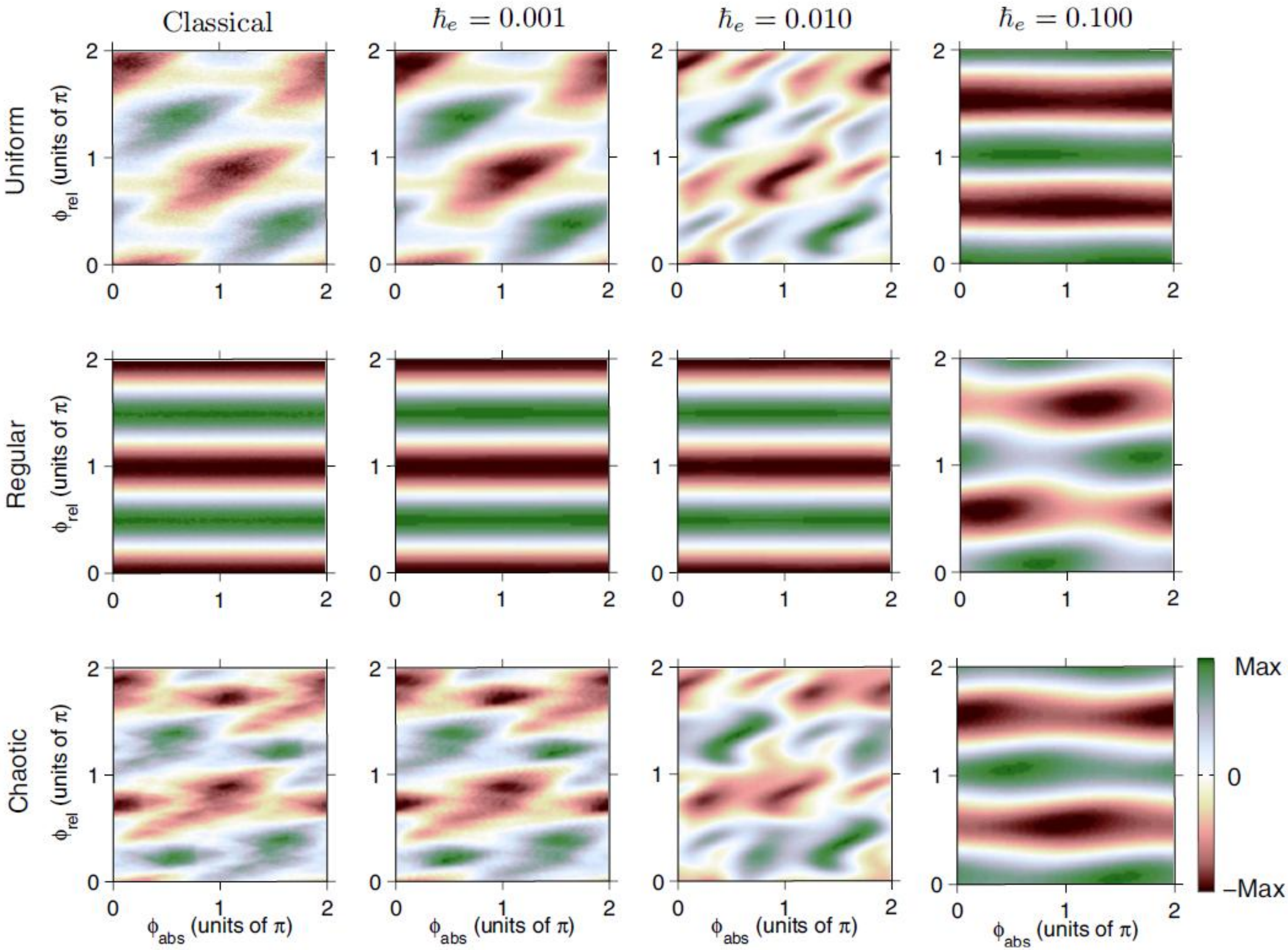


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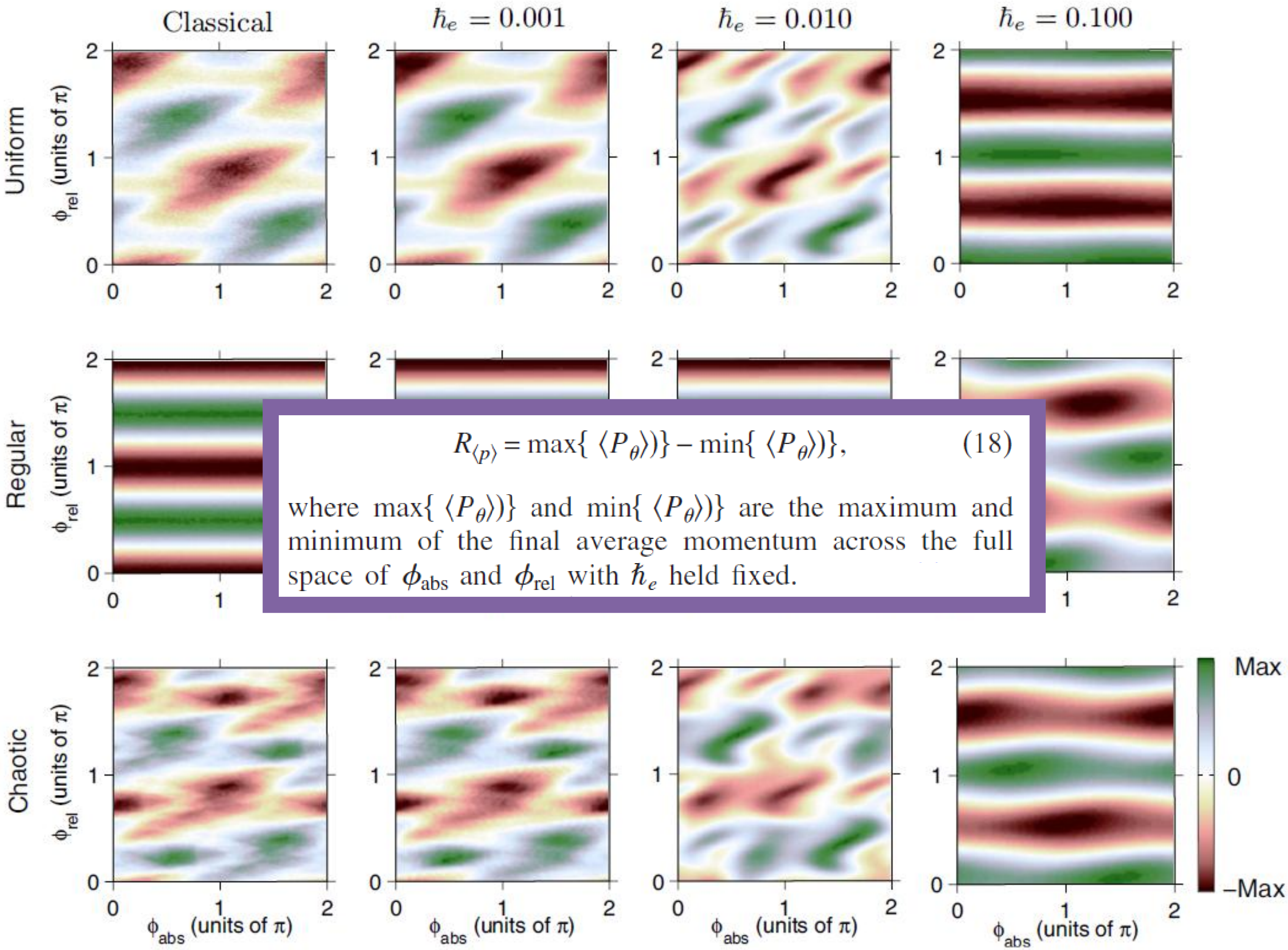
# Magnitude of control

$$R_{\langle p\rangle} = \max\{\langle P_\theta \rangle\rangle\} - \min\{\langle P_\theta \rangle\rangle\}, \tag{18}$$

where $\max\{\langle P_\theta \rangle\rangle\}$ and $\min\{\langle P_\theta \rangle\rangle\}$ are the maximum and minimum of the final average momentum across the full space of $\phi_{\rm abs}$ and $\phi_{\rm rel}$ with $\hbar_e$ held fixed.

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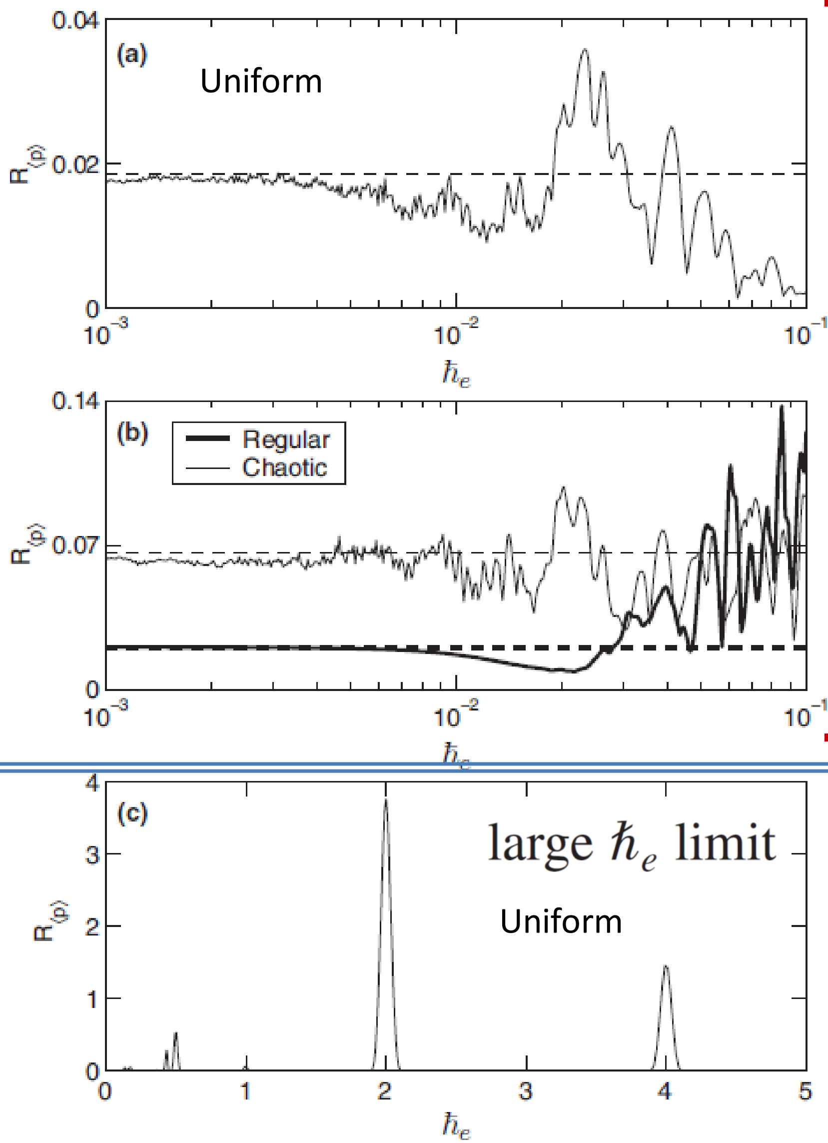



FIG. 2. Magnitude of the control ratio $R_{\langle p \rangle}$ (see text for definition) for (a) the uniform initial state for $\hbar_e \rightarrow 0$, (b) the chaotic and regular initial states for $\hbar_e \rightarrow 0$, and (c) the uniform initial state in the large $\hbar_e$ regime. The dashed lines in (a) and (b) denote the corresponding classical values of $R_{\langle p \rangle}$.

Dashed lines are classical value

**Same order of magnitude**

- Regular goes to classical fast and smooth
- Chaotic oscillates and is the cause of oscillation in Uniform

The localized regular and chaotic states were not considered since they carry nonzero momentum $\Delta P_\theta \sim \hbar_e / \Delta\theta$.

- presence of control strongly depends on value of $\hbar_e$
- the magnitude is about 2 orders larger than $\hbar_e \rightarrow 0$

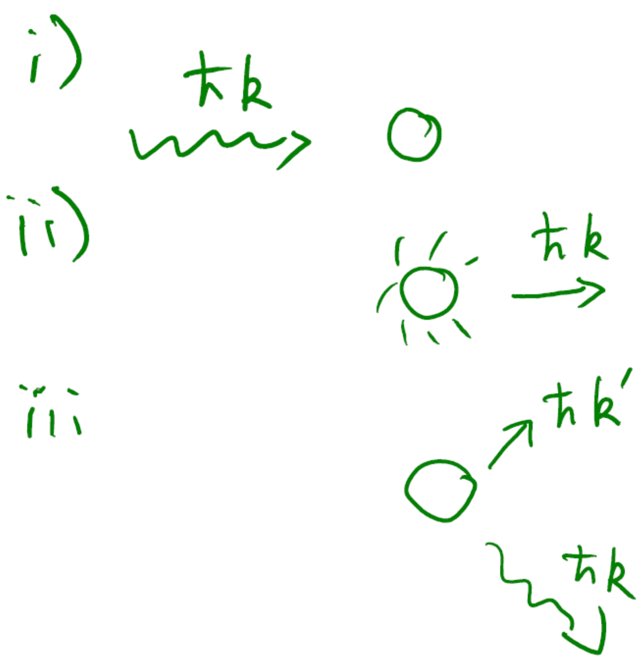
?

First, the *presence* of control in the large $\hbar_e$ limit is now strongly dependent on the value of $\hbar_e$. This is because in the regime of large $\hbar_e$ the spacing between two adjacent momentum states coupled by the field is large (on the order of $\hbar_e$), and, thus, the allowed transitions depend strongly on the relationship between $\hbar_e$ and the driving frequencies. Second, when control is present, the magnitude of the control is about two orders of magnitude larger than in the $\hbar_e \to 0$ limit. This happens because in the large $\hbar_e$ limit, the momentum transfer happens in a highly resonant manner between very few states, contrary to the $\hbar_e \to 0$ limit where many momentum states are coupled and populated.

# Dependence on decoherence

# Decoherence model

- Simple model of <u>*spontaneous emission*</u>
- Random momentum jumps <u>*on the order*</u> of recoil momentum
- Probability of a jump occurring is uniformly distributed in time
- Schrodinger equation is solved many times for different realization of the random momentum shifts
- All observables are averaged over these different realizations
- Also works for classical motion equations




$$\hbar k' \sim \hbar k$$

$$p_\theta = p \frac{2k}{\omega m}$$

$$p \sim \hbar k$$

$$\Rightarrow p_\theta \sim \frac{2\hbar k^2}{\omega m} \sim 2\hbar_e$$

# Decoherence in the $\hbar_e \to 0$ limit

Adding weak decoherence via spontaneous emission (1 momentum jump per cycle of the driving field and of magnitude $\delta P_\theta = \hbar_e$)

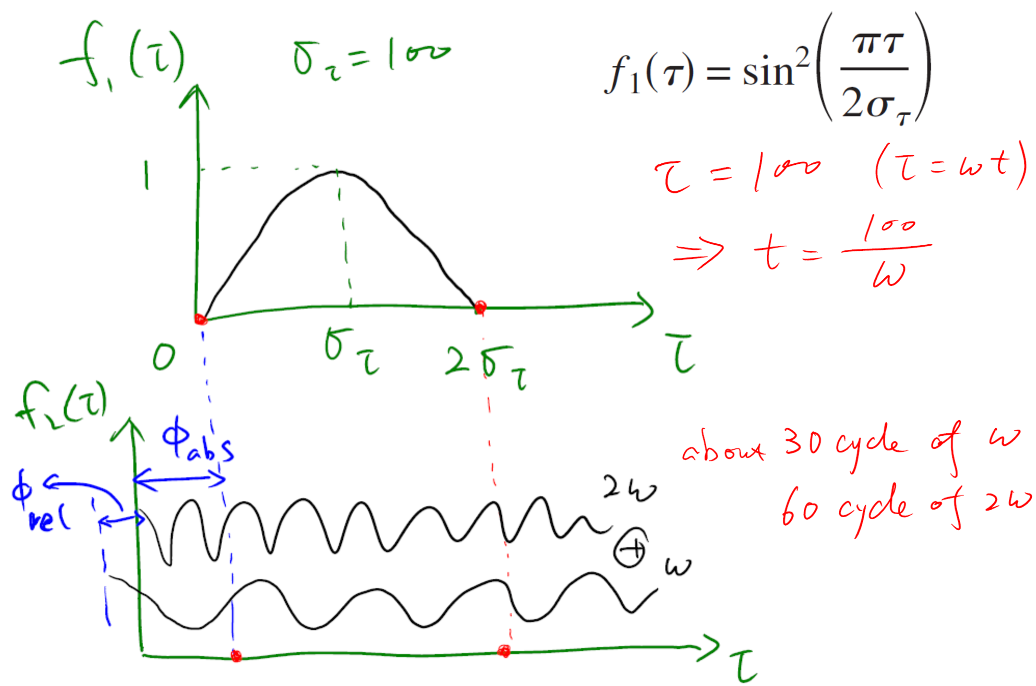
That's 100% chance of decoherence per cycle?!!

Weak?

Or, because $\hbar_e \to 0$

$$\hbar_e = 0.000\ 1$$

About 30 jumps?

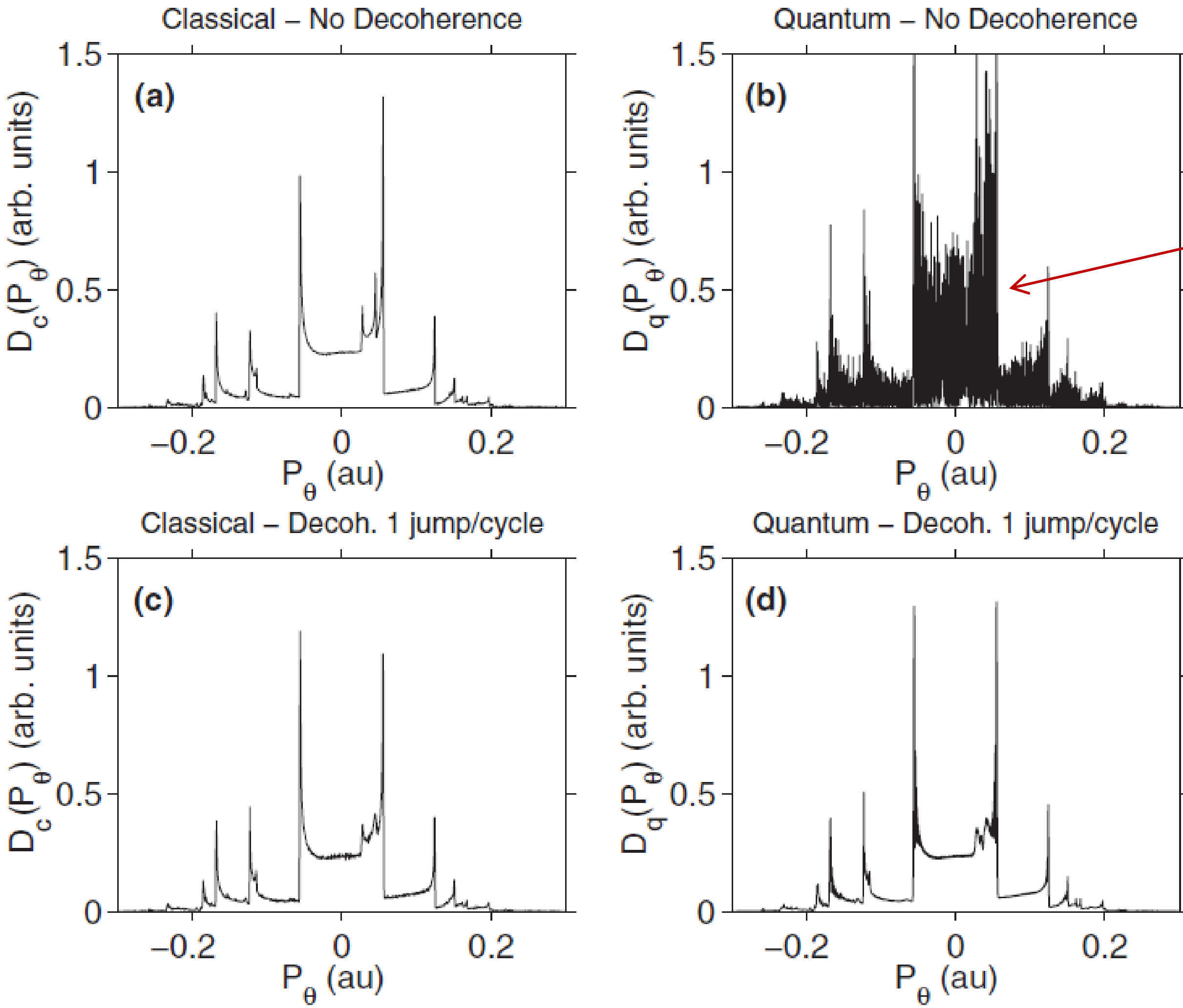



$$f_1(\tau) = \sin^2\left(\frac{\pi\tau}{2\sigma_\tau}\right)$$

$$f_2(t) = \cos(\omega t + \phi_{\text{rel}}) + s\ \cos(2\omega t)$$

Not all properties of the quantum scenario match the classical scenario perfectly in the $\hbar_e \to 0$ limit.

?? Don't know:
Initial state
What phases




Decoherence get rid of quantum interference
But not detrimental

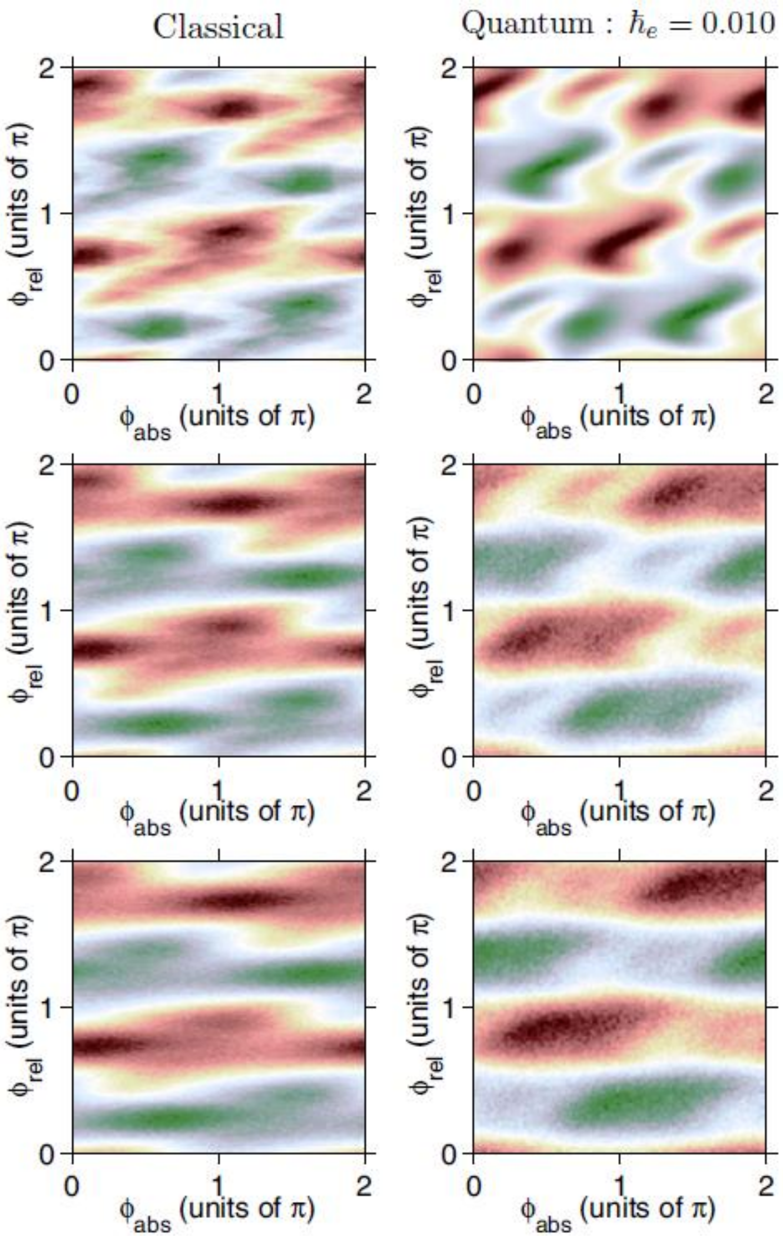
?????
Coherent control
but not quantum control
?????
Classical coherent control
Quantum coherent control

FIG. 3. Classical and quantum momentum distribution (where $\hbar_e = 10^{-4}$) with (panels c and d) and without (panels a and b) decoherence.

# Decoherence in the "large" $\hbar_e$ regime

The magnitude of the momentum jump was set to the value of $\hbar_e$ used, i.e., $\delta P_\theta = 0.01$.

Can the presence of decoherence accelerate the emergence of quantum-classical correspondence when $\hbar_e$ is not close to zero?




The top plots correspond to no decoherence, the middle plots are for 10% chance of a momentum jump per lattice shaking oscillation, and the bottom plots are for a 20% chance of a momentum jump per lattice shaking oscillation.

about 3 jumps in total and about 6 jumps in total?

Initial state is Chaotic

- They are different
- to have quantitative agreement, decoherence modifies the classical dynamics strongly -- "an agreement which has not yet set in for the result of Fig. 4"

?? Meaning the answer is 'No' ??
Or 'Yes', but you have to modify the dynamics a lot.

In the $\hbar_e < 1$ regime, the effects of small decoherence are fairly general, and as in the previous section, the conclusions reached here are independent of the type of noise used to induce decoherence. However, in the $\hbar_e > 1$ regime, preliminary computations (not shown) show that ......

FIG. 4. (Color online) Classical and quantum ($\hbar_e$=0.010) control results for (top) no decoherence, (middle) 10% chance of a momentum jump per cycle, and (bottom) 20% chance of a momentum jump per cycle.

# That's it!

The results are of fundamental significance to the general area of coherent control and motivate additional work. First, the results emphasize the fact that quantum-based coherent control scenarios can persist in the classical limit, albeit that the numerical values of the control can be vastly different in the quantum and classical regimes.

Second, the decoherence results shown here are relevant to the optical lattice setup, where decoherence results from spontaneous emission. Other types of open system interactions, in accord with the decoherence literature, can be expected to induce decoherence to other preferred bases