Violation of the Leggett-Garg inequality with weak measurements of photons

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By weakly measuring the polarization of a photon between two strong polarization measurements, we experimentally investigate the correlation between the appearance of anomalous values in quantum weak measurements, and the violation of realism and non-intrusiveness of measurements. A quantitative formulation of the latter concept is expressed in terms of a Leggett-Garg inequality for the outcomes of subsequent measurements of an individual quantum system. We experimentally violate the Leggett-Garg inequality for several measurement strengths. Furthermore, we experimentally demonstrate that there is a one-to-one correlation between achieving strange weak values and violating the Leggett-Garg inequality.

One of the fundamental challenges in understanding the world is addressing the question of whether or not systems have a real state independent of observation. Key ideas addressing this question include the Bell inequality, with its joint assumption of realism and/or locality [1, 2, 3], and contextuality tests, which examine whether or not identical experiments produce results in

disturbance on its subsequent evolution. Thus a violation of the LGI tells us that either earlier measurements perturbed the system, changing the results of later measurements, or macroscopic realism is untenable, or both.

The trick to testing the LGI is to be able to monitor the system without "collapsing" the wavefunction: to make a so-called weak measurement [7]. A weak measurement

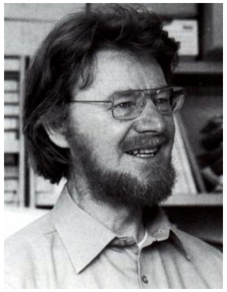
Lee Rozema

QO Group Meeting

Feb 17, 2009

Outline

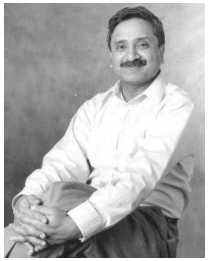
- Intro to Bells Inequalities and realism vs locality
- Leggett-Gargs Inequalities (LGI)
- LGI for a photon
- Weak QND measurement of LGI
- The controlled sign gate (very briefly)
- Results



Bell’s Inequalities

- Bell made 2 assumptions
  1. Objects have a definite state, determining all of their properties
  2. The effects of local actions can’t travel faster than light
- 1+2 = “Objective Local Theories”
- Based on these he showed correlations are bounded
- QM has stronger correlations so we must give up 1 or 2

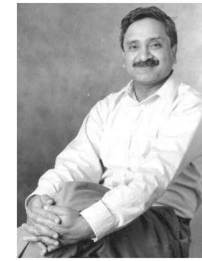
Leggett-Garg Inequalities



- Always need to test realism with another hypothesis:
  1. Macroscopic realism – the observable to be measured has a value at all times
  2. Noninvasive measurability – in principle it is possible to make a measurement without disturbing the system
- Designed to test if there is a limit where quantum mechanics breaks down

Leggett, A.J. Realism and the physical world. *Reports on Progress in Physics* **71**, 022001 (2008).

Leggett-Garg Inequalities



- Quantity Q(t) can take values ±1
- Correlations: $C_{12} = \langle Q(t_1) Q(t_2) \rangle$
- Take 3 measurements and define:

$$B = C_{12} + C_{23} - C_{13}$$

- Assume correlations at 1 & 3: $C_{13} = 1$
  - We can have $C_{12} = 1$ and $C_{23} = 1$ so: $B = 1$
  - Or $C_{12} = -1$ and $C_{23} = -1$ so: $B = -3$
- So the LGI is:

$$-3 \leq C_{12} + C_{23} - C_{13} \leq 1$$

QM violation of LGI

- Spin-1/2 rotating around x: $H = \frac{\hbar\omega}{2}\sigma_x$
- Project into z:

$$\sigma_z(t) = e^{-i\omega t} |\uparrow_x><\downarrow_x| + e^{i\omega t} |\downarrow_x><\uparrow_x|$$

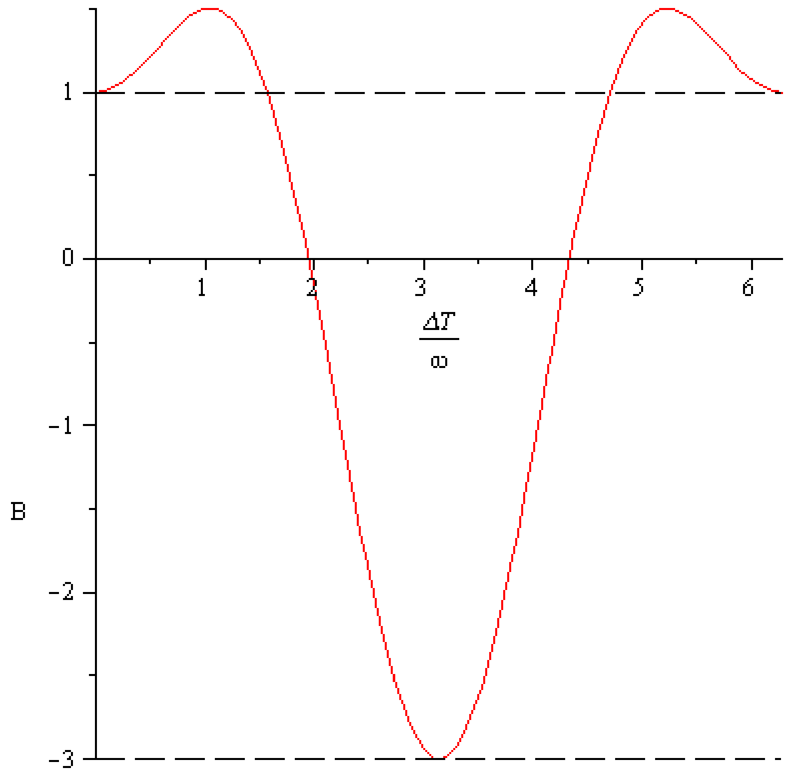
- Can show:

$$\langle \sigma_z(t_1)\sigma_z(t_2) \rangle = \cos\left[\omega\left(t_1 - t_2\right)\right]$$

- So the LGI becomes:

$$-3 \leq 2\cos[\omega\Delta t] - \cos[2\,\omega\Delta t] \leq 1$$

QM violation of LGI



$$-3 \leq 2\cos[\omega\Delta t] - \cos[2\,\omega\Delta t] \leq 1$$

LGI for a photon

- Need to make 3 measurements:

$$-3 \leq \langle M_1 M_2 \rangle + \langle M_2 M_3 \rangle - \langle M_1 M_3 \rangle \leqslant 1$$

- State preparation is M1 with outcome 1

$$-3 \leq \langle M_2 \rangle + \langle M_2 M_3 \rangle - \langle M_3 \rangle \leqslant 1$$

- M2 measure first Stokes parameter (H-V)
- M3 measure second Stokes parameter (D-A)

$$-3 \leq \langle S_1 \rangle + \langle S_1 S_2 \rangle - \langle S_2 \rangle \leqslant 1$$

LGI for a photon

- Prepare state: $\cos\frac{\theta}{2}|H> + \sin\frac{\theta}{2}|V>$
- Easy to show: $\langle S_1 \rangle = \cos\theta$
  $\langle S_2 \rangle = \sin\theta$
  $\langle S_1 S_2 \rangle = 0$
- So $-3 \leq \langle S_1 \rangle + \langle S_1 S_2 \rangle - \langle S_2 \rangle \leqslant 1$ becomes:

$$-3 \leq \cos\theta - \sin\theta \leqslant 1$$

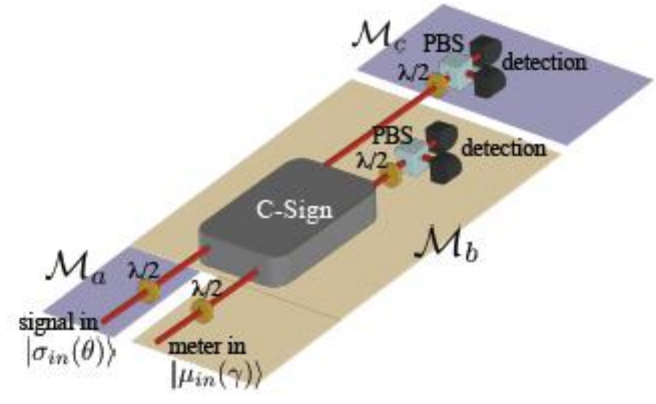
- But for $\frac{3\pi}{2} \leq \theta \leq 2\pi$ this is greater than 1

So what?

- We must admit that either:
  - The photon has no deterministic evolution
  - OR the evolution can’t be monitored no matter how weak the measurement

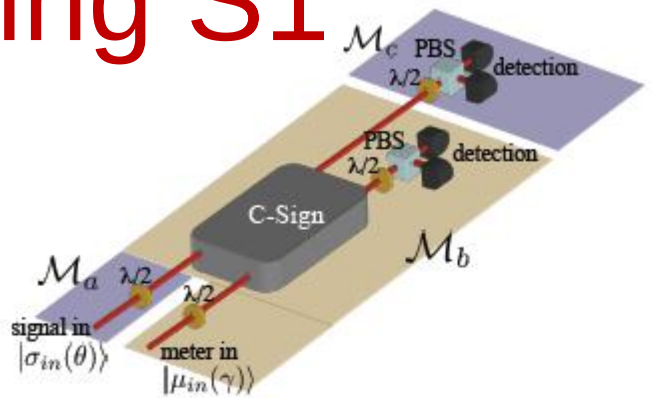
How do we measure S1 and S2?

- Weak measurement and quantum logic gates
- C-Sign: $|HH> \rightarrow |HH>$
  $|HV> \rightarrow |HV>$
  $|VH> \rightarrow |VH>$
  $|VV> \rightarrow -|VV>$



- Weakly measure S1 with the meter photon
- Strong measurement of S2 on signal

Weakly measuring S1



- S1 = P(H)-P(V)
- Input:

$$\left\{\cos\frac{\theta}{2}|H> + \sin\frac{\theta}{2}|V> \right\}\otimes\left\{\gamma\,|D> + \gamma'\,|A> \right\}$$

- Output of C-Sign:

Strong ($\gamma=1, \gamma'=0$): $\cos\frac{\theta}{2}|HD> + \sin\frac{\theta}{2}|VA>$

Weak ($\gamma=\gamma'=1/\sqrt{2}$): $\left\{\cos\frac{\theta}{2}|H> + \sin\frac{\theta}{2}|V>\right\}\otimes|H>$

- Get S1 by measuring meter photon:

$$S1=P(D_m|D_s)-P(A_m|D_s)$$

But we can't measure it strongly

- Still need to measure S2 on the signal photon (for strong $\gamma' = 0$) :

$$\langle S_2 \rangle \sim \gamma \gamma' \sin\theta$$

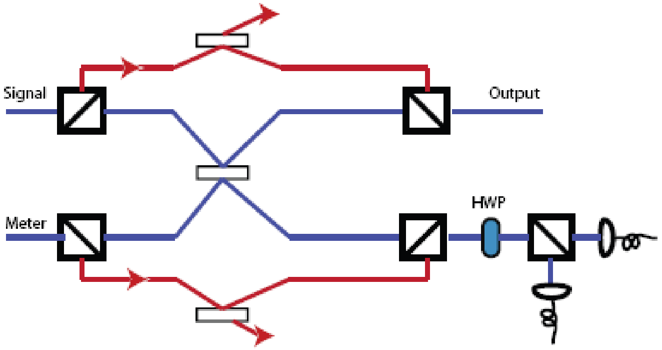
The C-Sign gate

- Post-select on 1 photon in each output
- Reflectivity: η= 1/3

$$
\begin{aligned}
|HH> &\rightarrow r^2|HH> + t^2|HH> + \ldots. \\
&\rightarrow -\frac{1}{3}|HH> + \frac{2}{3}|HH> + \ldots \\
&\rightarrow \frac{1}{3}|HH>
\end{aligned}
$$

- Need loss modes take other inputs:

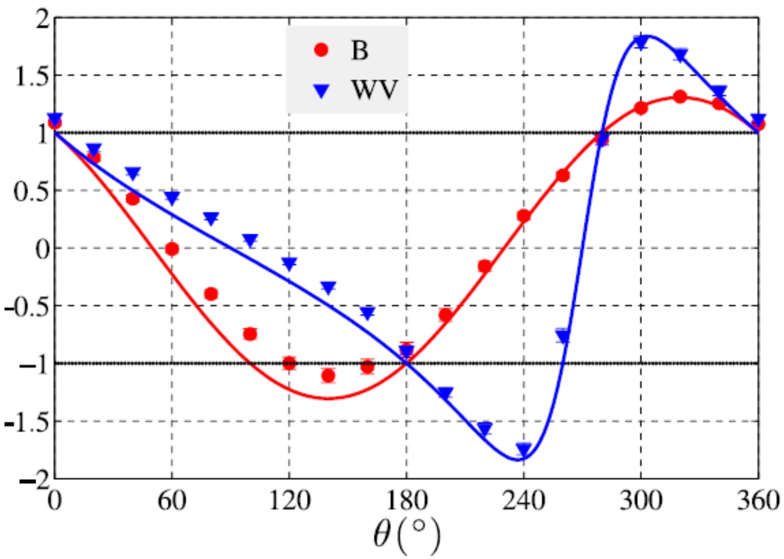
$$
\begin{aligned}
|HV> &\rightarrow r^2|HV> + \ldots \\
&\rightarrow -\frac{1}{3}|HV>
\end{aligned}
$$



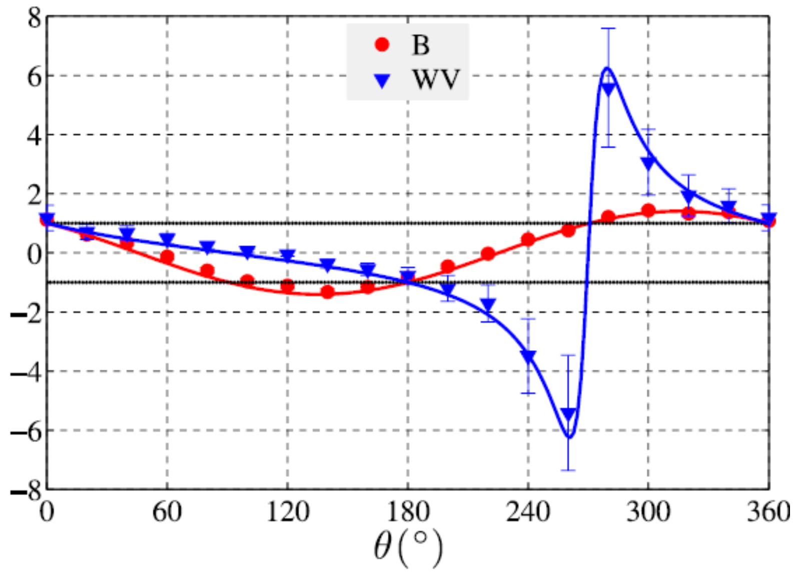
Pryde, G.J., O'Brien, J.L., White, A.G., Bartlett, S.D. & Ralph, T.C. Measuring a Photonic Qubit without Destroying It. *Phys. Rev. Lett.* **92**, 190402 (2004).

Want to violate:

$$\langle S_1 \rangle + \langle S_1 S_2 \rangle - \langle S_2 \rangle \leqslant 1$$



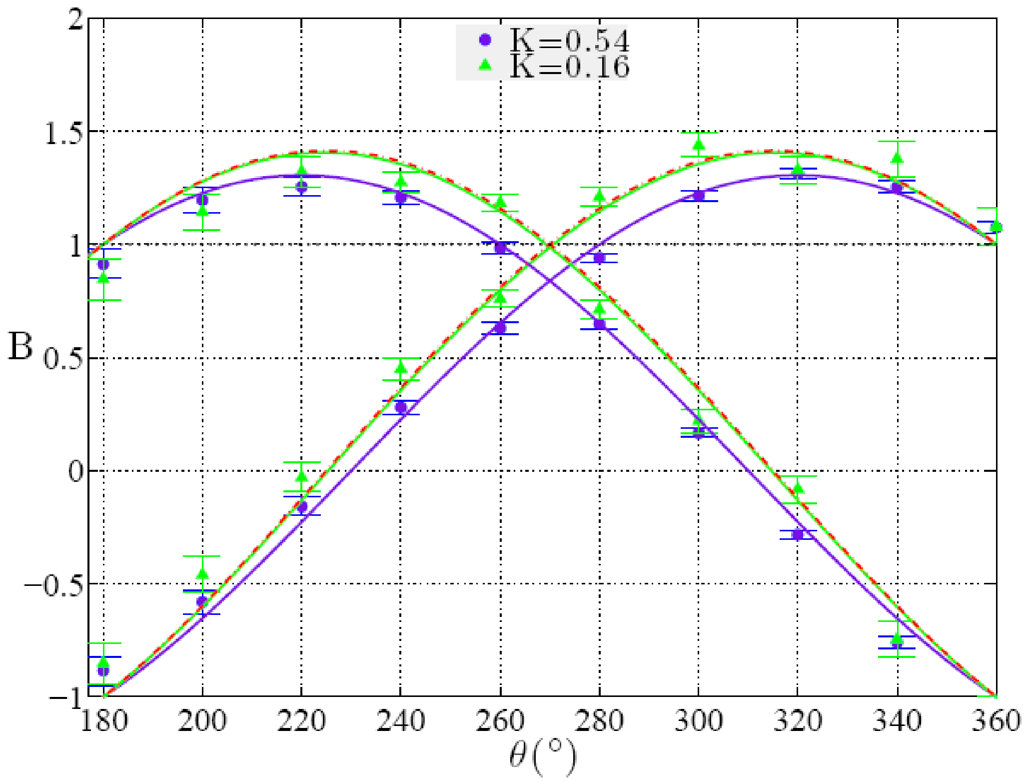
K=0.54



K=0.16

- Measurement strength: K= γ^2 - 1
- Weak value (blue) is <S1>
- B (red) is the inequality

Effect of Measurement Strength



- Violation is larger, and covers a wider range of input states
- Green dashed line K=0

Conclusions

- The LGI was violated by a sequence of measurements on a single photon
- This means any theory (not just QM) which describes measurements on a photon must admit that either the photon has no definite evolution or it cannot be monitored.