The Quantum Random Walk

Dylan Mahler

Discrete single-photon quantum walks with tunable decoherence

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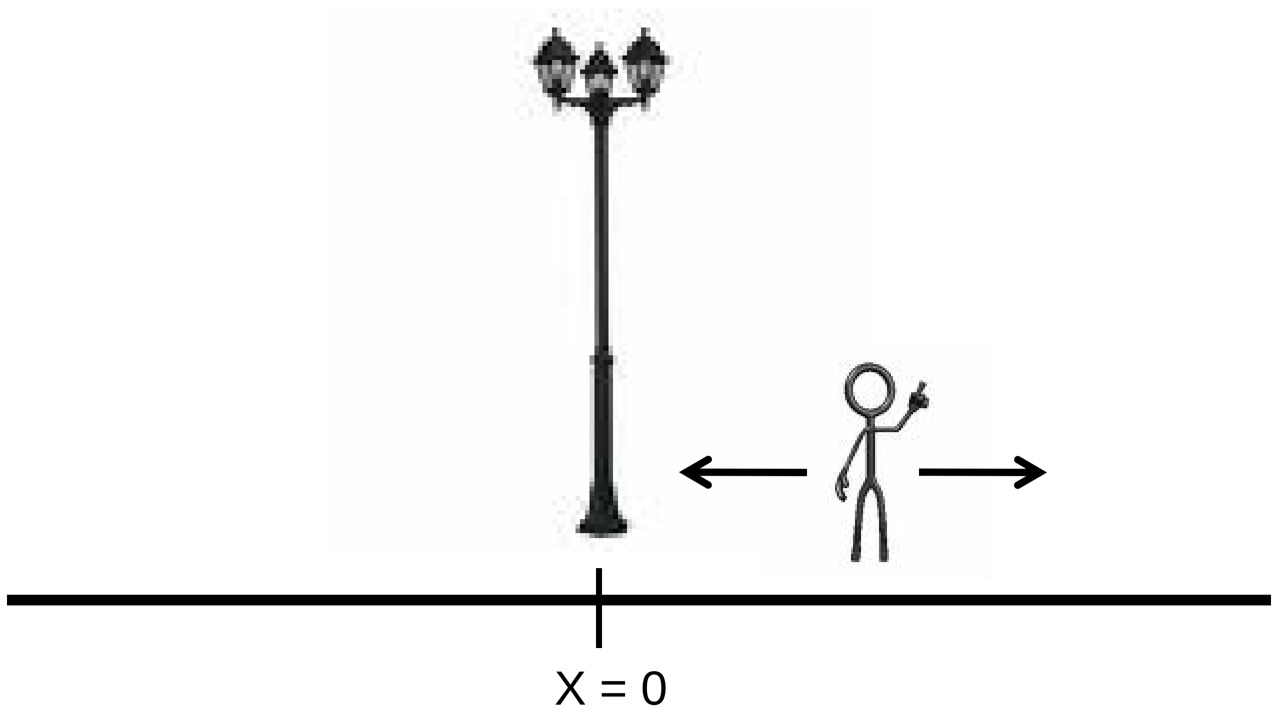
Quantum walks have a host of applications, ranging from quantum computing to the simulation of biological systems. We present an intrinsically stable, deterministic implementation of discrete quantum walks with single photons in space. The number of optical elements required scales linearly with the number of steps. We measure walks with up to 6 steps and explore the quantum-to-classical transition by introducing tunable decoherence. Finally, we also investigate the effect of absorbing boundaries and show that decoherence significantly affects the probability of absorption.

Agenda

- The Classical Random Walk
- Quantum Walks
- The Experiment

The Classical Random Walk

- The drunk guy and the lamp post:



Classical Random Walk

$$W_N(m) = \frac{N!}{(\frac{N-m}{2})!(\frac{N+m}{2})!} p^{(\frac{N+m}{2})}(1-p)^{(\frac{N-m}{2})}$$

- The binomial distribution!

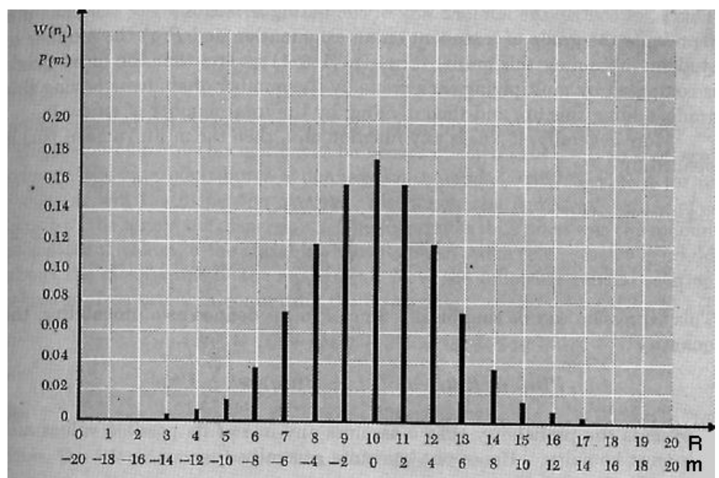


Fig. 1·2·3 Binomial probability distribution for $p = q = \frac{1}{2}$ when $N = 20$ steps. The graph shows the probability W_N(R) of R right steps, or equivalently the probability $P_N(m)$ of a net displacement of m units to the right.

Classical Random Walk

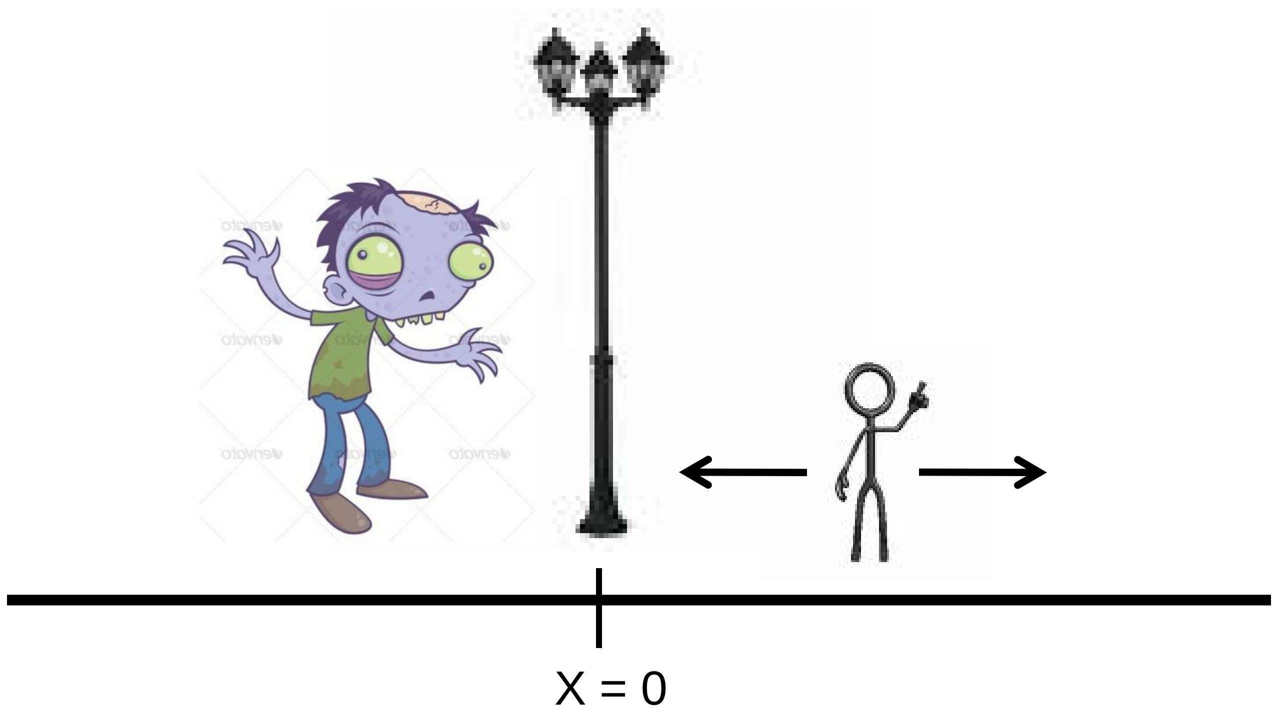
- Things to note:
  - Dispersion of the total displacement after N steps scales linearly with N:

$$\sqrt{\overline{(\Delta x)^2}} = \sqrt{N}$$

(for p=q=1/2)

Classical Random Walk

- Also, survival probability -> 0 as
- N -> Infinity for case of absorber



Quantum Random Walk

Quantum random walks

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We introduce the concept of *quantum random walk*, and show that due to quantum interference effects the average path length can be much larger than the maximum allowed path in the corresponding classical random walk. A quantum-optics application is described.

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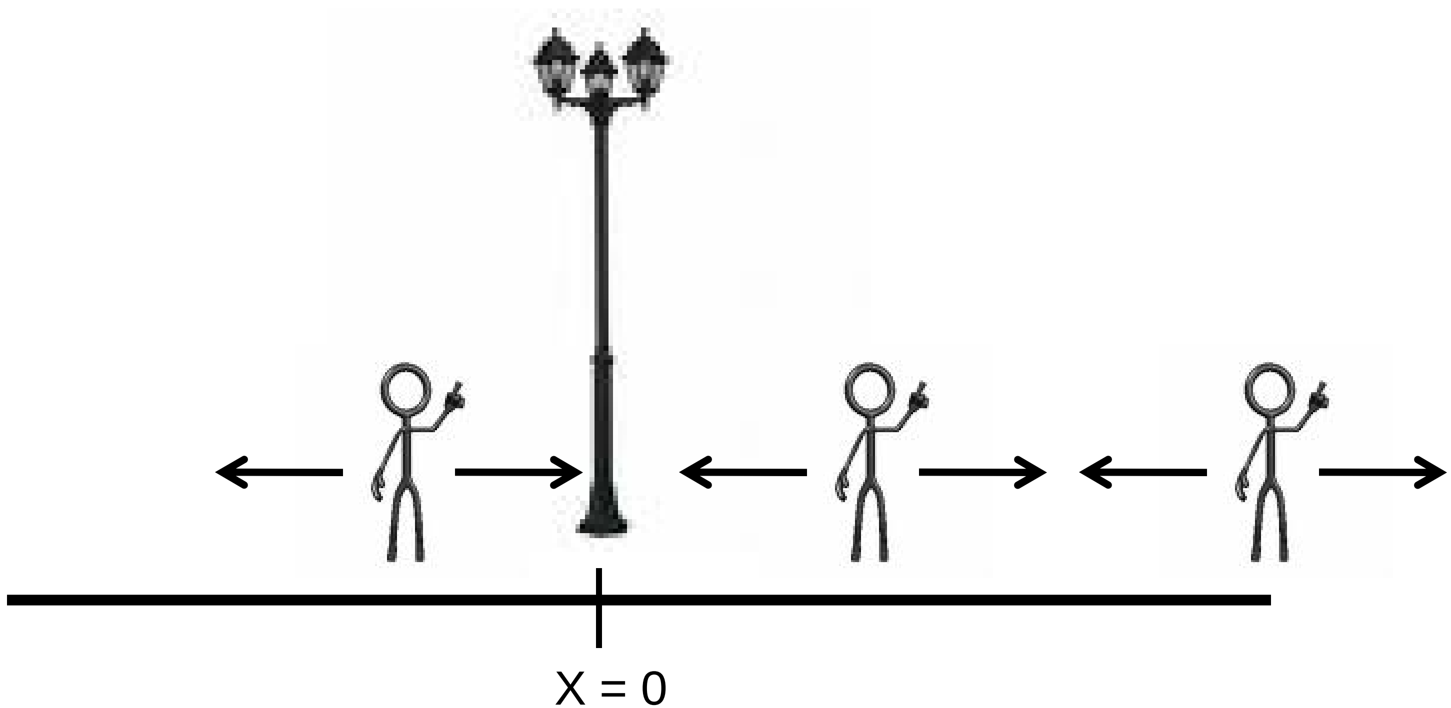
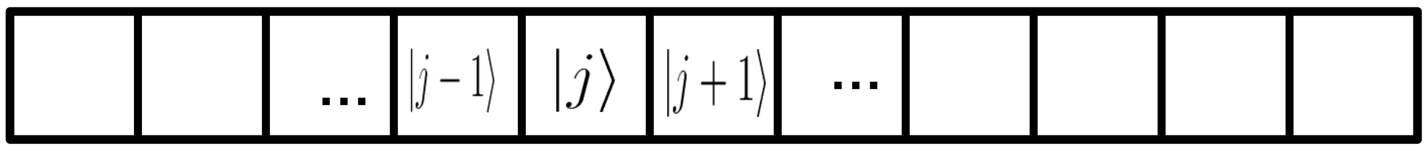
Quantum Random Walk

- What if now, we allow our walker to exist in superposition states?



(Discrete) Quantum Random Walk



- State is prepared initially at j=0
- With the state of the coin stored in a polarization qubit

(Discrete) Quantum Random Walk

- Every step consists of two actions:
- 1. Flip a coin to tell us what direction to go
- 2. Move based on the coin’s result

Quantum Random Walk

- Prepare the state in:

$$|\Psi\rangle = |0\rangle(|H\rangle + |V\rangle)$$

- We then "flip the coin" with the "coin operator".

- If we use an unbiased coin:

$$C = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$ (Hadamard)

Quantum Random Walk

- Then, we “walk the drunk guy”, with the shift operator, which has the form:

$$S = \sum_j |j-1\rangle\langle j| \otimes |H\rangle\langle H| + |j+1\rangle\langle j| \otimes |V\rangle\langle V|$$

- Reads: “If H, move left, if V, move right.

Quantum Random Walk

- So each step is equivalent to acting with the operator:

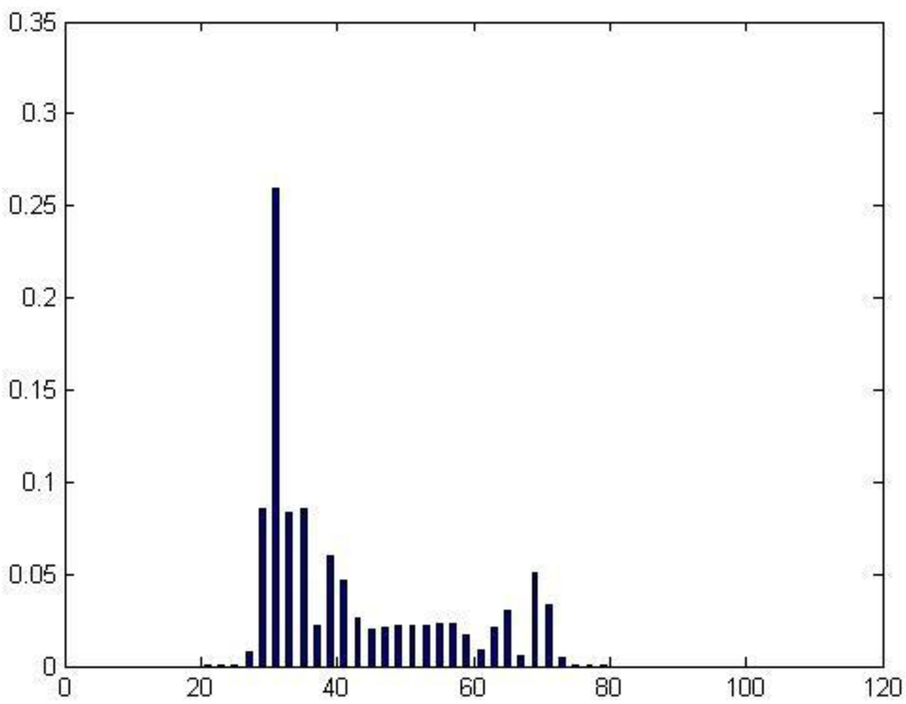
$$M = SC$$

- And after N steps we are in the state:

$$|\psi\rangle = M^N |0\rangle(|H\rangle + |V\rangle)$$

Quantum Random Walk

- Problem!! Not quite the behaviour we were looking for: (its biased to the left!)



Quantum Random Walk

- This is because the Hadamard is actually a biased coin flip; it treats H and V differently

Quantum Random Walk

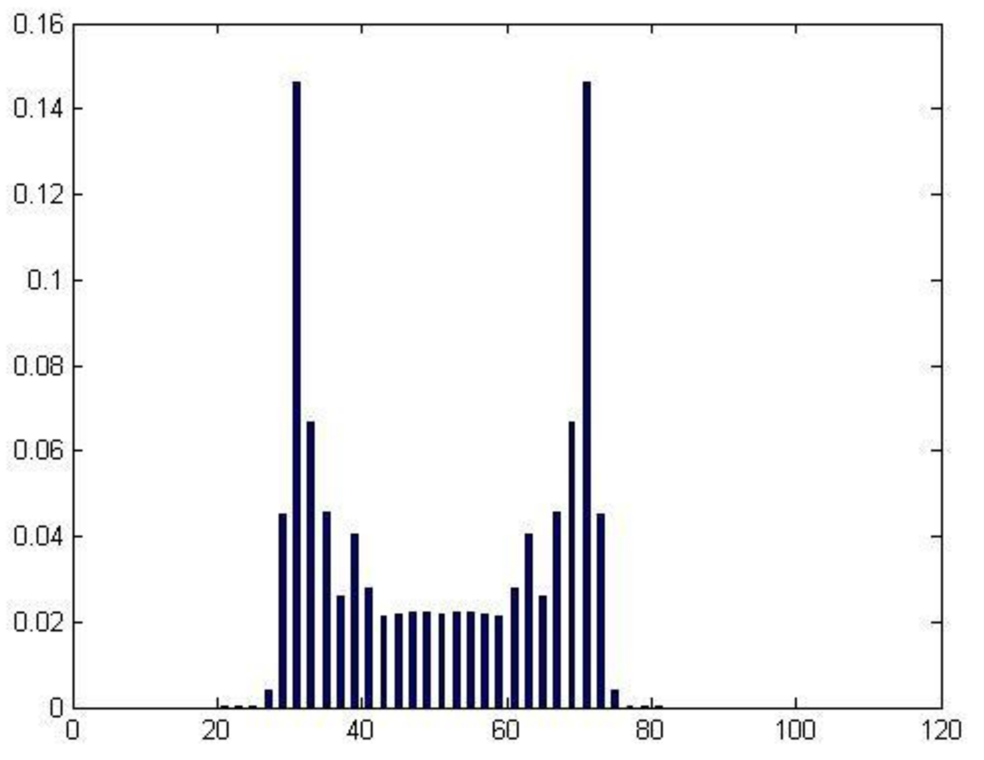
- To fix this, make sure that ‘left’ and ‘right’ don’t interfere by starting in the initial state:

$$|\Psi\rangle = |0\rangle(|H\rangle + i|V\rangle)$$

- This ensures the left path and right path don’t interfere
- Alternatively, we could use a truly unbiased coin

Quantum Random Walk

- Now we get the behaviour we want:



Differences

- Quantum distribution is bimodal, which leads to a larger variance in the distance

$$\sqrt{\overline{(\Delta x)^2}} = N$$

- Placing an absorber (zombie) still yields a finite survival probability as n->Infinity

Uses?

Quantum random-walk search algorithm

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(Received 9 October 2002; published 23 May 2003)

Quantum random walks on graphs have been shown to display many interesting properties, including exponentially fast hitting times when compared with their classical counterparts. However, it is still unclear how to use these novel properties to gain an algorithmic speedup over classical algorithms. In this paper, we present a quantum search algorithm based on the quantum random-walk architecture that provides such a speedup. It will be shown that this algorithm performs an oracle search on a database of N items with $O(\sqrt{N})$ calls to the oracle, yielding a speedup similar to other quantum search algorithms. It appears that the quantum random-walk formulation has considerable flexibility, presenting interesting opportunities for development of other, possibly novel quantum algorithms.

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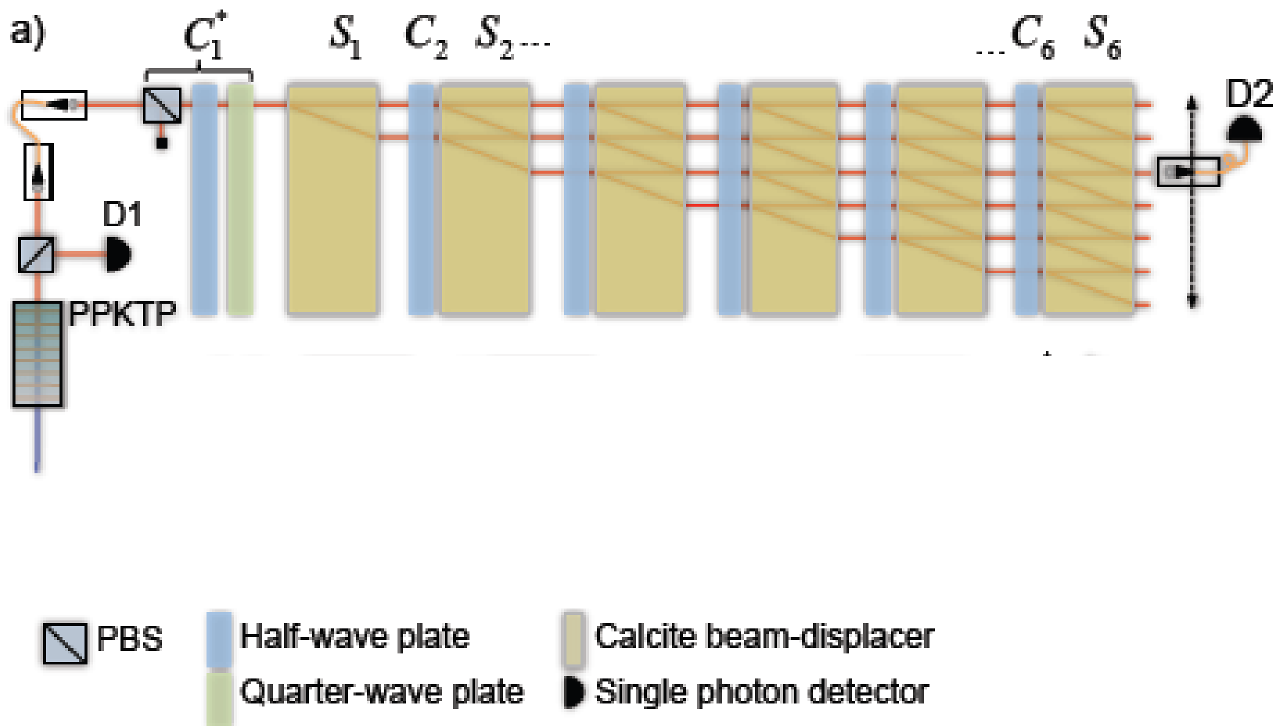
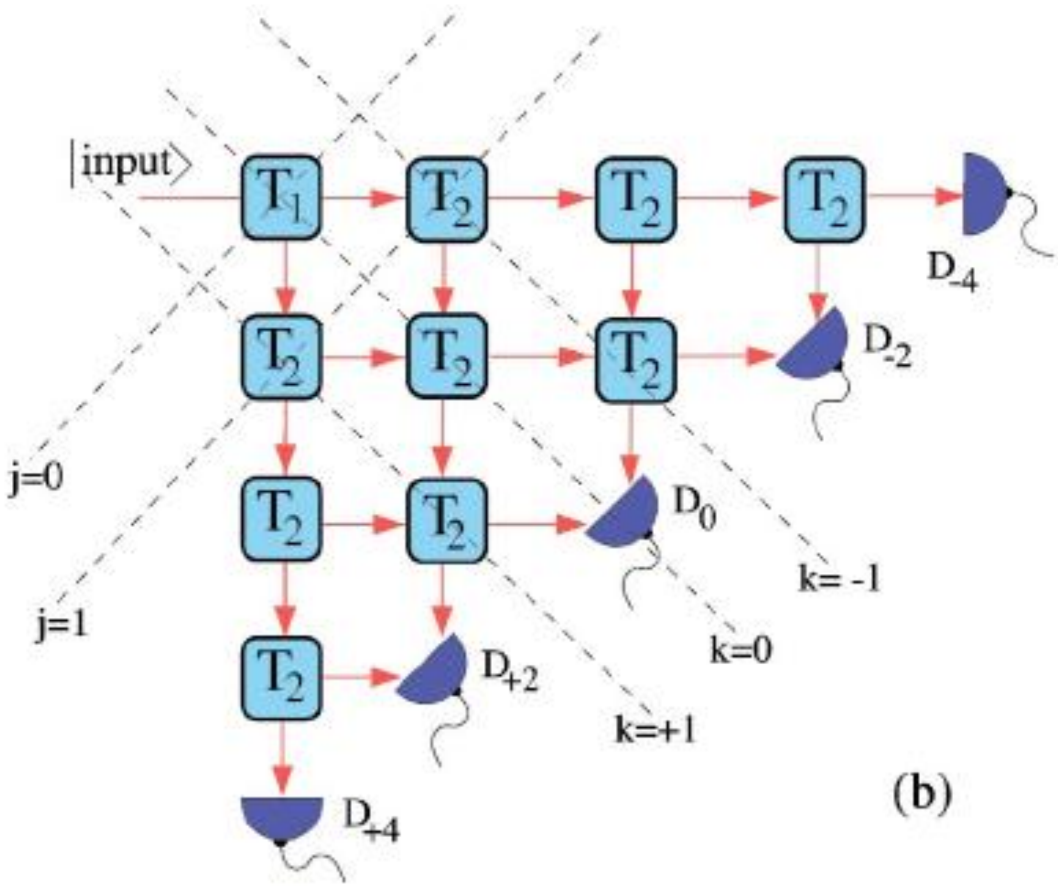
The Experiment



The Experiment

- Benefits:
  - Easy to align by successively adding beam displacers
  - Scales linearly with the number of steps (goes like 2N as opposed to the PBS version - by Zwickl et al - that scales like N^2)

(Not) The Experiment



(Not) The Experiment

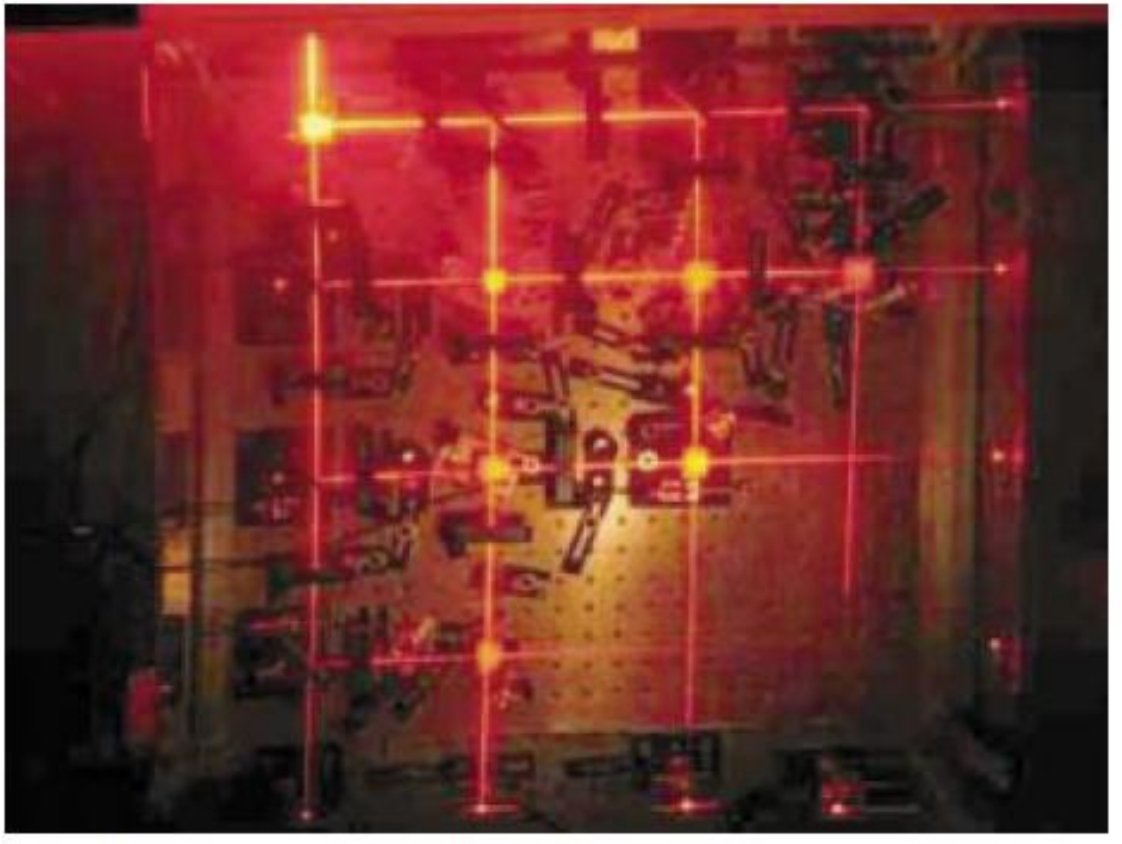


Fig. 4. Photograph of the apparatus in operation.

The Experiment

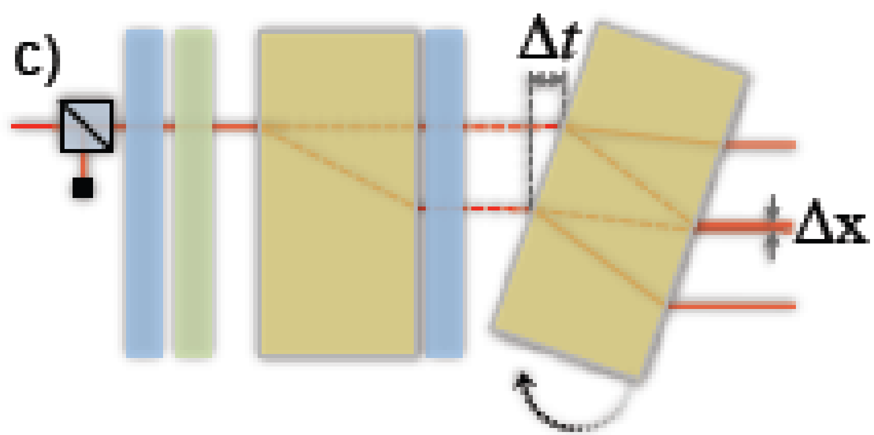
- We can introduce tuneable decoherence which yields:

$$\rho_{N+1} = (1-q)W\rho_N W^\dagger + q\sum_i K_i W \rho_N W^\dagger K_i^\dagger, \qquad (2)$$

- Where $K_i = |i\rangle\langle i|$
- This is called “dephasing”, but really corresponds to making a path measurement at each step

The Experiment

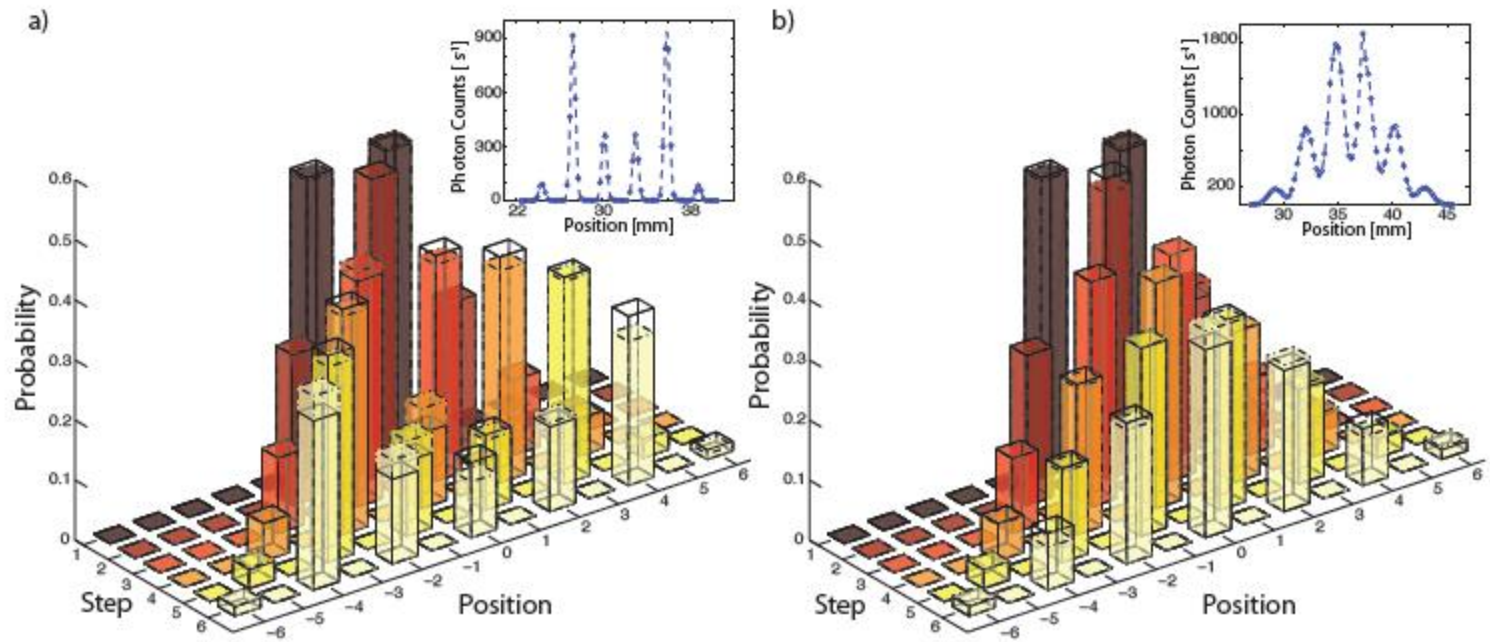
- By tilting one of the beam displacers, we add a time delay that destroys the interference



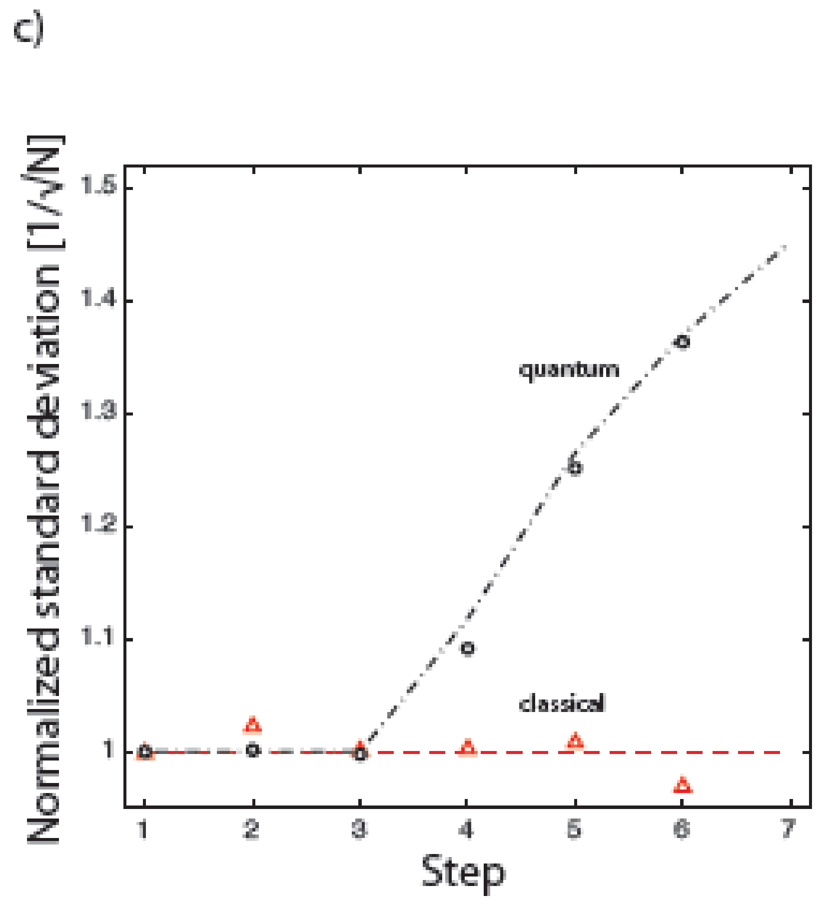
The Experiment

- To ensure photon collection:
  - Set the delay window to be much larger than the decoherence delay
  - Couple into multimode fiber

Results

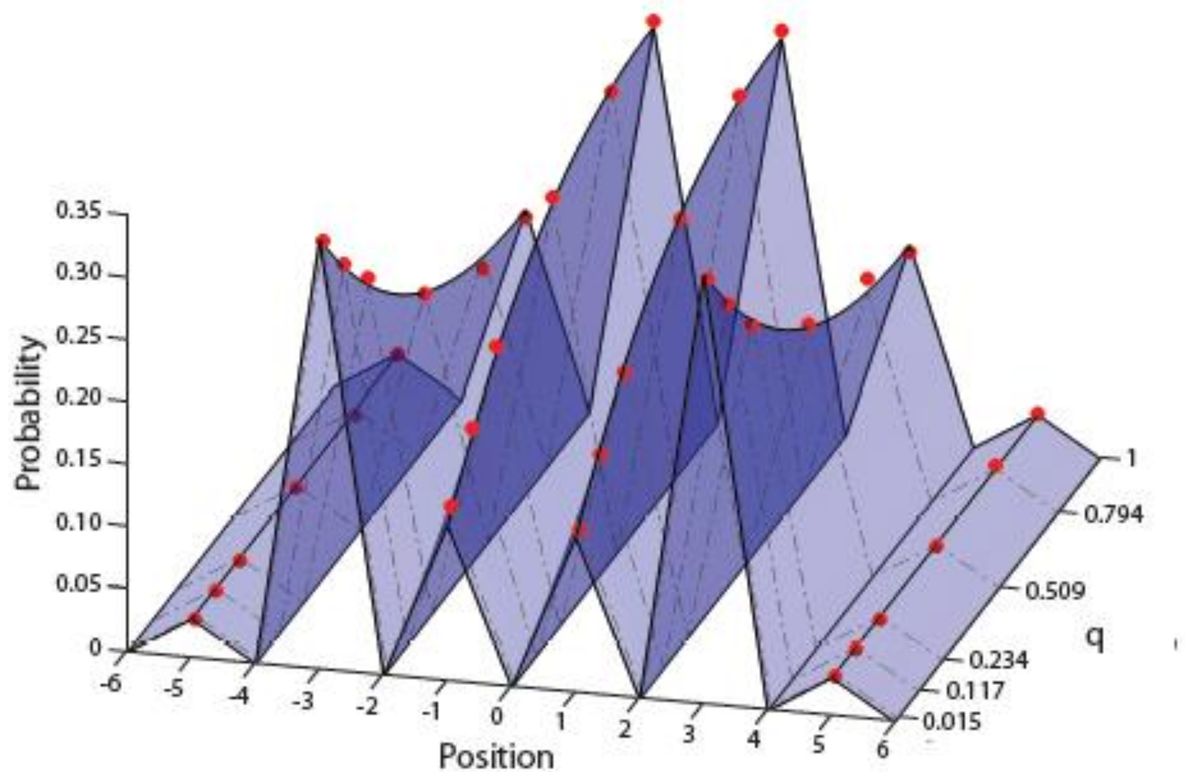


Results



c)
Normalized standard deviation [1/√N]
1.5
1.4
1.3
1.2
1.1
1
quantum
classical
1
2
3
4
5
6
7
Step

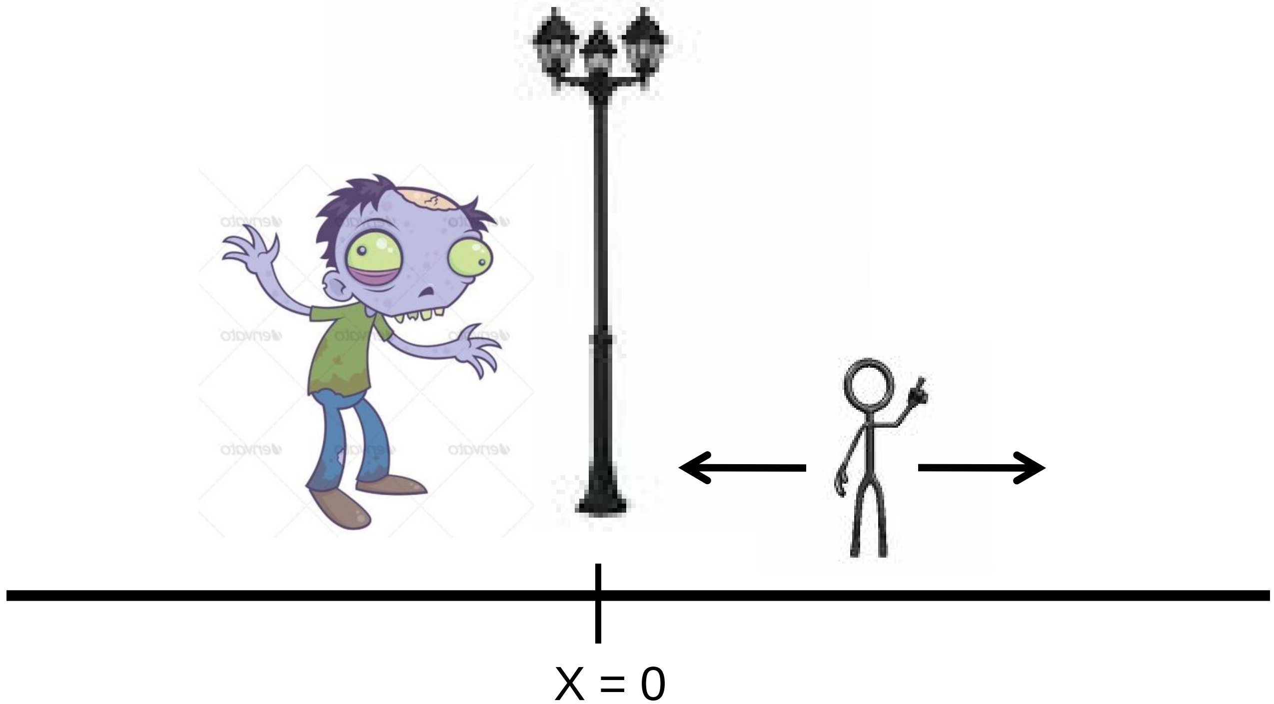
Results



Probability
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
Position
q
1
0.794
0.509
0.234
0.117
0.015

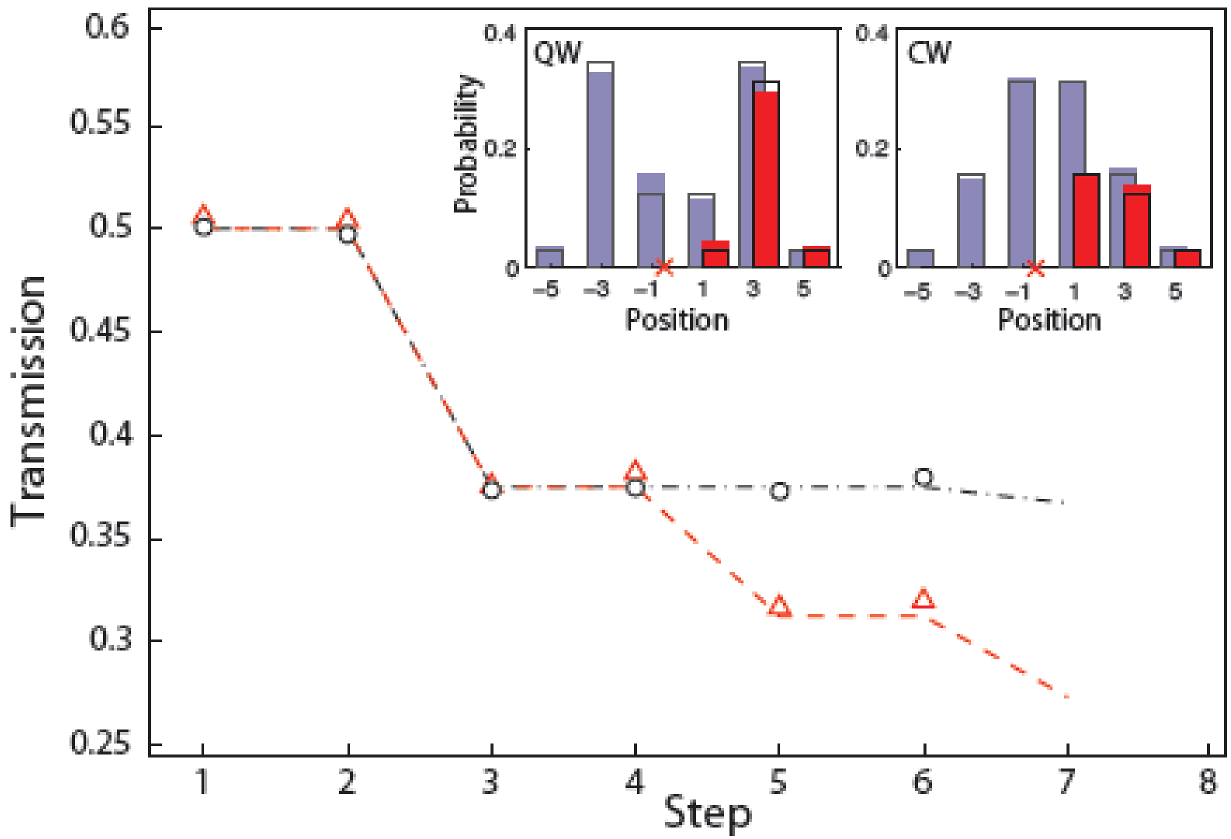
Results

- Now add an absorber!



- (add a beam block)

Results



In Summary

- Have a scalable, stable quantum walk apparatus with a tuneable decoherence mechanism to study the quantum-to-classical transition

Thank you,